Question

Experiment!

Toss a coin three times. How many heads did you get?

A. 0
B. 1
C. 2
D. 3

$P(E_1) = \frac{|E_1|}{8} = \frac{3}{8}$ 10 people!

$E_0 = \{TTT\} \to 1/8$

on! $E_1 = \{HTT, THT, TTH\} \to 3/8$

off. $E_2 = \{HHT, HTH, THH\} \to 3/8$

$E_3 = \{HHH\} \to 1/8$

As the samples go to infinity.

If you do not have a coin, visit justflipacoin.com/

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

1000 people
≈ 375
with two heads.

$P(E_2) = \frac{|E_2|}{8} = \frac{3}{8}$

5 people had 2 heads.

$26 \cdot \frac{3}{8} = 9.75$

Question

Experiment!

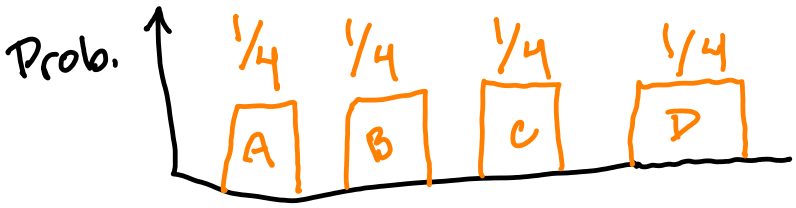
Think of a random number n from one to a thousand. Where was your number n?

A. $1 \leq n < 250$

B. $250 \leq n < 500$

C. $500 \leq n < 750$

D. $750 \leq n \leq 1000$.



small numbers

As the number of samples $\longrightarrow \infty$ these numbers go closer to $1/4$.

Nada:

$$\mathbb{P}(A) = \frac{250}{1000} = \frac{1}{4}$$

$$\mathbb{P}(B) = \frac{250}{1000} = \frac{1}{4}$$

Question

Experiment!

Make an educated guess, without actually calculating:

What's the probability that two people in our class share a birthday?

~ 26 people

A. Between 0% and 25%

B. Between 25% and 50%

C. Between 50% and 75%

D. Between 75% and 100%

Maheen + Chan!

If you have > 365 people then $\mathbb{P} = 1$.

Question

Five "letters": C O R C MEME

Suppose that we randomly re-arrange the letters of the word COMMERCE. What is the probability that the word MEME appears somewhere in the shuffled letters?

A. $1/5040 = \dfrac{1}{8!/(2!)^2}$

B. $1/1680 = \dfrac{4!}{8!}$

C. $1/84 = \dfrac{5!/2!}{8!/(2!)^3}$ oops! typo.

D. $1/60 = \dfrac{1}{5 \cdot 4!/2!}$

$P = \dfrac{\text{MEME ways}}{\text{Total ways}} = \dfrac{5!/2!}{8!/(2!)^3}$ ← repeated C; ← repeated M, E, C.

Question

three 20-sided dice

Suppose that you roll 3d20. Given that all dice are fair, what is the probability of rolling any specific triple (i, j, k)?

A. ~~1/20~~ ℙ of one die

B. 1/400

C. 1/800

D. 1/8000

(1,3,3)

$$\mathbb{P}((i,j,k) = (1,3,3)) = \mathbb{P}(i=1, j=3, k=3)$$
$$= \mathbb{P}(i=1)\,\mathbb{P}(j=3)\,\mathbb{P}(k=3)$$
$$= \frac{1}{20} \cdot \frac{1}{20} \cdot \frac{1}{20} = \frac{1}{8000}$$

Question

A chess tournament has twelve players, including Beth and Benny. The players are randomly paired up. What's the probability that Beth and Benny face each other?

A. $({}_{10}C_2 \cdot {}_8C_2 \cdot {}_6C_2 \cdot {}_4C_2 \cdot {}_2C_2)/6!$ in m.

B. $({}_{10}C_2 \cdot {}_8C_2 \cdot {}_6C_2 \cdot {}_4C_2 \cdot {}_2C_2)/5!$ in m.

C. ${}_{12}C_2$ in m.

D. 6 in m.

where $m = ({}_{12}C_2 \cdot {}_{10}C_2 \cdot {}_8C_2 \cdot {}_6C_2 \cdot {}_4C_2 \cdot {}_2C_2)/6!$

Question

Roll 2d6. Let A be the event that the sum is seven, and let B be the event that **one of the dice** is odd. Which of the following is true?

A. A and B are independent.

B. A and B are mutually exclusive.

C. A and B are both independent and mutually exclusive.

D. A and B are neither independent nor mutually exclusive.

Question

Roll 2d6. Let A be the event that the sum is seven, and let B be the event that the **first die** is odd. Which of the following is true?

A. A and B are independent.

B. A and B are mutually exclusive.

C. A and B are both independent and mutually exclusive.

D. A and B are neither independent nor mutually exclusive.

Question

{Can we finish early?}

4 midterms next week :(

Roll 3d6 and record the sum. What is the sample space of this experiment?

A. $\{(1,1,1),(1,1,2),(1,2,1),(2,1,1),\ldots,(6,6,6)\}$ ← we write (x,y,z)

~~B. $\{\{1,1,1\},\{1,1,2\},\{1,1,3\},\ldots,\{6,6,6\}\}$~~

*C. $\{3,4,5,\ldots,16,17,18\}$ ← we write $x+y+z$.

D. $\{1,2,3,\ldots,17,18\}$

The results of the roll are ordered.
We take the sum of the results.

$p(3) = \frac{1}{36}$ $\quad p(18) = \frac{1}{36}$

$p(4) = \frac{3}{36} = \frac{1}{12}$

cliff hanger!

Question

$\mathbb{P}(E) = 1 - \mathbb{P}(E^c)$

Roll 3d6. What is the probability that the sum of the dice is at least six?

A. $5/12 = 1 - \frac{7}{12}$

B. $7/12 = 1 - \frac{5}{12}$

* C. $103/108 = 1 - \frac{10}{6^3}$

D. $211/216 = 1 - \frac{5}{6^3}$

at least six ≥ 6

What is the prob. that 3d6 is < 6? Outcomes: 3, 4, 5.

3: 111

4: 112 121 211

5: 221 212 122 113 131 311

Ten outcomes give total < 6.

$1 - \frac{10}{216} = \frac{216-10}{216}$:)

$= \frac{206}{216} = \frac{103}{108}$

Event: $216 = 6^3$ outcomes

Question

Suppose that we draw four cards from a deck of fifty-two cards. What is the probability that at least one of the cards is an ace?

A. 38916/54145 $= \frac{{}_{52-4}C_4}{{}_{52}C_4}$

B. 15229/54145 $= \frac{{}_{52}C_4 - {}_{52-4}C_4}{{}_{52}C_4} = \frac{{}_{52}C_4}{{}_{52}C_4} - \frac{{}_{48}C_4}{{}_{52}C_4}$

C. 38916/54145 $= \frac{{}_{52-4}P_4}{{}_{52}P_4}$

D. 15229/54145 $= \frac{{}_{52}P_4 - {}_{52-4}P_4}{{}_{52}P_4}$

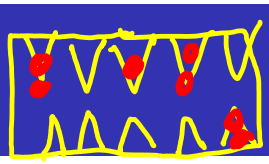
all hands

hands w no ace

Although the fractions are equal, we have unordered cards.

Question

Gr: Tavli Ταβλη



In the game of Backgammon, you roll 2d6. It is often strategically helpful to a pair of consecutive numbers such as 3 and 4. What is the probability of rolling a pair of consecutive numbers?

A. $\frac{6}{36}$

B. $\frac{9}{36}$

C. $\frac{10}{36}$

D. $\frac{18}{36}$

5/18:

• 12 21
• 23 32
• 34 43
• 45 54
• 56 65

10 possible outcomes.

$\frac{10}{6^2}$ → $6^2 = 6 \cdot 6$ → ordered

first second

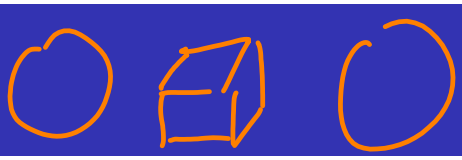
Unordered:

${}_6C_2 + 6$

differ same

$= 15 + 6 = 21$

Question



You flip a fair coin, roll a d6, then flip another fair coin. Let E be the event that the number of heads (H) shown is larger than or equal to the number on the d6. Which statement is true?

A. $|E| = 1$
B. $|E| = 2$
C. $|E| = 3$
D. $|E| = 4$

# of heads $\leq$ 2

	H	die
H 2 H	2	2
H 1 T	1	1
T 1 H	1	1
H 1 H	2	1

Check the events that you write really work!

Question

Important!

Consider the experiment of rolling 2d6, but the first die is fair, while the second follows this distribution:

$$p(y) = \begin{cases} 1/12, & y \text{ is odd} \\ 1/4, & y \text{ is even.} \end{cases}$$

If x denotes the number that shows up on the fair die, while y is the number on the second, compute $P(x \text{ is odd and } y \leq 3)$.

11 12 13

31 32 33 ✓

51 52 53 ✓

A. $\frac{1}{8}$

B. $\frac{5}{24}$

C. $\frac{1}{4}$

D. $\frac{11}{12}$

$$P(E) = \left(\frac{1}{6}\cdot\frac{1}{12} + \frac{1}{6}\cdot\frac{1}{4} + \frac{1}{6}\cdot\frac{1}{12}\right)\cdot 3 = \frac{3}{6}\left(\frac{1}{12}+\frac{1}{4}+\frac{1}{12}\right) = \frac{5}{24}$$

Question

Suppose that we have two bags of marbles. Bag #1 has two red and five blue marbles. Bag #2 has 97 red and three blue. We pick a bag randomly, and select a marble. What is the probability that it is blue?

A. $\frac{1}{2}$

B. $\frac{8}{107} = \frac{5+3}{7+100}$

C. $\frac{\cancel{512}\ 521}{1400} = \frac{1}{2}\frac{5}{7} + \frac{1}{2}\frac{3}{100}$ *

D. $\frac{879}{1400} = \frac{1}{2}\frac{2}{7} + \frac{1}{2}\frac{97}{100}$

= Prob(Red)

$\mathbb{P}(\text{Blue})$
$= \mathbb{P}(\text{Blue} \mid \#1)\,\mathbb{P}(\#1) +$
$\mathbb{P}(\text{Blue} \mid \#2)\,\mathbb{P}(\#2)$
$= \frac{5}{7}\cdot\frac{1}{2} + \frac{3}{100}\cdot\frac{1}{2}$

No week 11 on the test!