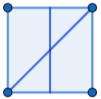
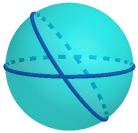
# MAT 402: Classical Geometry

## Groups

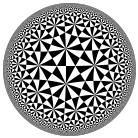


$$\mathrm{Symm}(\square) = \langle r, s : r^2 = s^2 = (rs)^4 = e \rangle$$

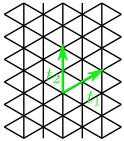
## Spherical



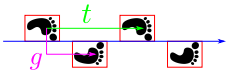
## Hyperbolic



## Tilings




## Friezes

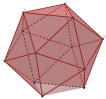



## Trigonometry

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

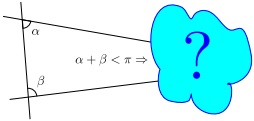
$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

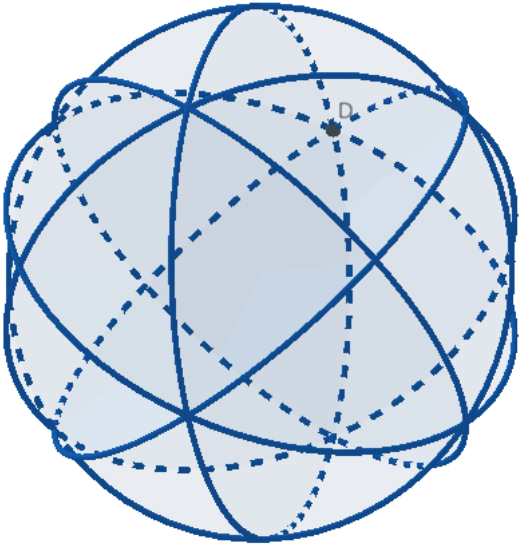
## Platonic Solids



## Coxeter

## Parallels






**How was Reading Week? Questions? Funny stories?**

# MAT 402: Monday October 19th 2020

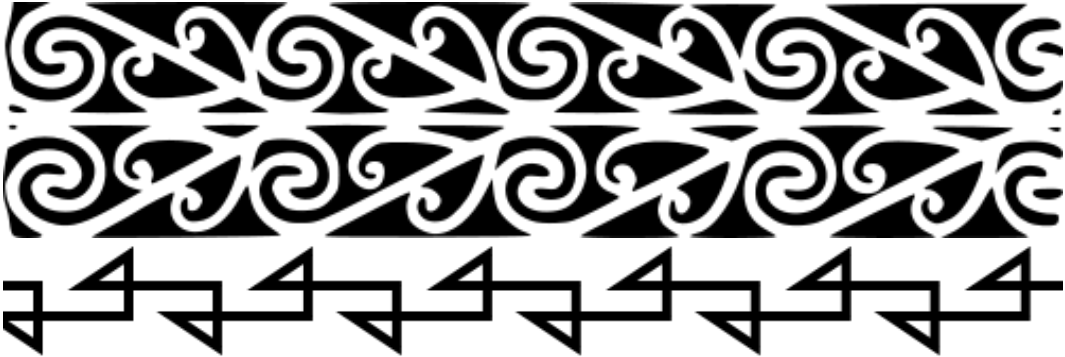
**Learning Objectives:**

- Determine the symmetry group of a frieze.
- Construct a frieze with a particular symmetry group.
- Write group presentations for friezes.

# Friezes

**Definition**

A frieze is a repeating pattern in $\mathbb{R} \times [-1, 1]$ with a discrete symmetry group.



All images borrowed from Anneke Bart and Bryan Clair's EscherMath Wiki.

# Friezes

## Question

*Which symmetries of* $\mathbb{R}^2$ *preserve* $\mathbb{R} \times [-1, 1]$*? Alternatively, what are the possible symmetries of a frieze?*

# Frieze Exploration I

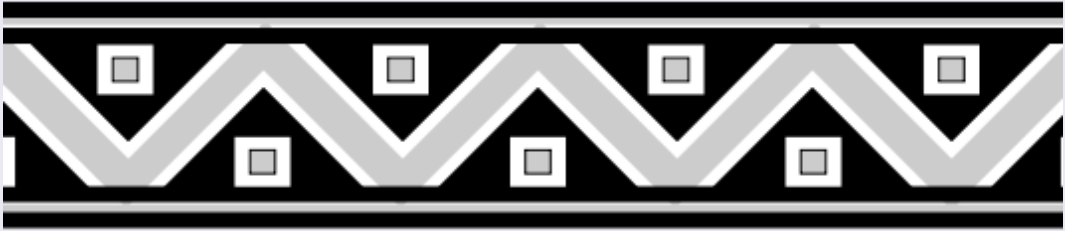
## Task

*What are the symmetries of this frieze?*

# Frieze Exploration II

## Task

*What are the symmetries of this frieze?*



# Frieze Construction I

**Task**

*Construct a frieze with only translation and horizontal mirror symmetry.*

# Frieze Construction I

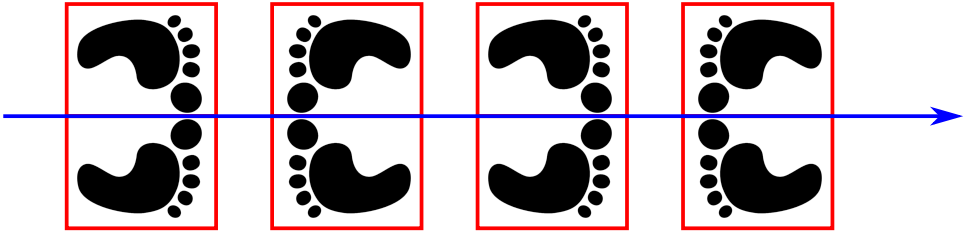
**Task**

*Construct a frieze pattern with only translation and $\theta = \pi$ rotational symmetry.*

# Frieze Group

## Task

*Write a presentation for the symmetry group of this frieze.*



# Symmetry

## Question

*Consider a rotation $R_O(\theta)$ centered at $O$ by angle $\theta = \pi$ and parallel translation $T_{\vec{v}}$ vector $\vec{v}$. What kind of isometry is $R_O(\pi) \circ T_{\vec{v}}$? What about $T_{\vec{v}} \circ R_O(\pi)$?*