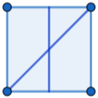
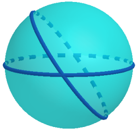
MAT 402: Classical Geometry

Groups

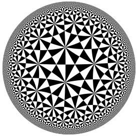


$$\mathrm{Symm}(\square) = \langle r, s : r^2 = s^2 = (rs)^4 = e \rangle$$

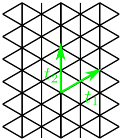
Spherical



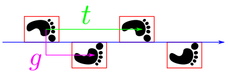
Hyperbolic



Tilings



Friezes

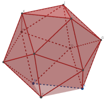


Trigonometry

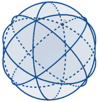
$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

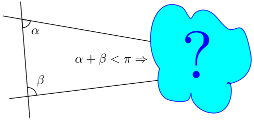
Platonic Solids



Coxeter



Parallels



Any question about the homework? Comments?

MAT 402: Monday September 21st 2020

Learning Objectives:

- Perform calculations in the symmetric group
- Write a permutation as a product of cycles
- Find a given group embedded in a symmetric group

The Symmetric Group

Definition

The symmetric group is $S_n = \{f : \{1, 2, \dots, n\} \to \{1, 2, \dots, n\} \ : \ f \text{ bijective}\}$.
The group operation is function composition.

Task (5 min)

Consider the bijections $f = [2, 4, 3, 1]$ *and* $g = [3, 2, 1, 4]$. *Compute* $f \circ g$ *and* $g \circ f$.

The Symmetric Group

Task

Decompose $f = [2, 4, 3, 1, 5]$ *as a product of cycles.*

The Symmetric Group

Theorem (Cayley)

For any finite group G, there is an n such that $G \subseteq S_n$ is a subgroup of S_n.

Task (5 min)

Find an n such that $\mathbb{Z}_3 \subseteq S_n$. How can you represent $\mathbb{Z}_3$ in S_n?

The Symmetric Group

Task (5 min)

How can you represent D_4 in S_4?

The Symmetric Group

Task (2 min)

Consider a regular pentagon with vertices labelled clockwise $\{1, 2, 3, 4, 5\}$*.*
What is the axis of rotation which permutes the vertices according to $(4\ 5)(3\ 1)$*?*

The Symmetric Group

Task (2 min)

Consider a regular hexagon centered at $(0, 0)$ *with vertices labelled clockwise* $\{1, 2, 3, 4, 5, 6\}$. *Write the permutation of the vertices induced by the map* $f(\vec{x}) = -\vec{x}$.