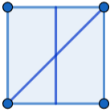
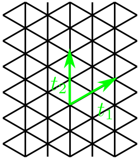
MAT 402: Classical Geometry

Groups

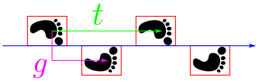


$$\mathrm{Symm}(\square) = \langle r, s : r^2 = s^2 = (rs)^4 = e \rangle$$

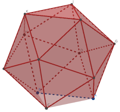
Tilings



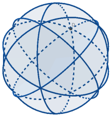
Friezes



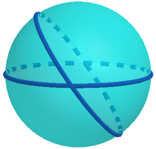
Platonic Solids



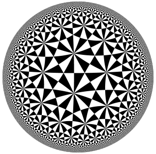
Coxeter



Spherical



Hyperbolic

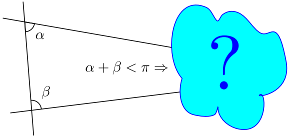


Trigonometry

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

Parallels



MAT 402: Friday December 4th 2020

Learning Objectives:

- Get ready for the exam!
- Chat about the course.

These questions are available as a handout:
https://pgadey.ca/teaching/2020-2021/mat-402-fall-2020/Exam_Questions.pdf

Task

The cube's rotational symmetry $^{+}(I^{3})$ *is isomorphic to which permutation group? Why?*

Task

Describe the fundamental domain of the symmetry group of the octahedron. Draw a diagram to illustrate the fundamental region and calculate the angles of the triangles on the faces of the fundamental domain.

Task

Show that the composition of two reflections of the sphere in planes passing through its center is a rotation. Determine the axis of rotation and, if the angle between the plane is given, the angle of rotation.

Task

What are all the possible orders of the subgroups of D_n and what are they? Which subgroups are normal?

Task

Prove that any dihedral group can be generated by two elements of order two.

Task

Consider the Symmetry Group of a Rhombus in $\mathbb{R}^2$ *given by* $(\Diamond)$. *List all subgroups of* $(\Diamond)$ *and prove that they satisfy the subgroup properties. Are any of these subgroups normal?*

Task

If G is the isometry group of the plane, and P is the subgroup of parallel translations, intuitively, what does the quotient group G/P represent? Why?

Task

Consider a pyramid with vertices $(\pm 2, \pm 2, 0)$ *and* $(0, 0, 4)$*. Sketch such a solid, and determine its symmetry group.*

Task

Describe a subgroup of the symmetry group of the octahedral which is not cyclic. Write the permutations of the faces which generate this subgroup.

Task

Find all the cosets of $\{e, r\}$ *in the symmetry group of the square where* r *is a rotation of 90 degrees.*

Task

If the cube has sidelength L, what is the sidelength of the inscribed octohedron?

Task

In the motion group of the cube, find all groups isomorphic to $\mathbb{Z}_n$ and D_n for various values of n.

Task

Let S be a cube inscribed into a sphere. Find all the planes of reflection of S and then find all the great circles that have each plane intersecting the sphere.

Task

An L tetromino consists of four squares of unit side length glued together to form an L. Create two friezes using L tetrominoes and describe why they have different symmetry groups.

Task

Given a triangle in hyperbolic geometry, take $\Theta(\Delta)$ to be the sum of all the angles of the triangle. Calculate the infimum and supremum of $\Theta(\Delta)$.

Task

Prove the second cosine theorem on the sphere $\mathbb{S}^2$*:*

$$\cos(\alpha) + \cos(\beta)\cos(\gamma) = \sin(\beta)\sin(\gamma)\cos(a)$$

Task

Proving the equivalent of the Pythagoras theorem in spherical space.

Task

Prove the surface area of a unit sphere is 4π with the multivariable calculus.

Task

Using the Poincaré half plane model, find the measures of the hyperbolic sidelength and angles of the hyperbolic triangle whose vertices are $(0,1)$, $(1,2)$ *and* $(2,4)$.

Task

Construct a triangle in the Cayley-Klein model with angle sum less than a given positive ε.