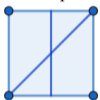


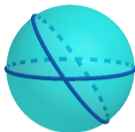
# MAT 402: Classical Geometry

Groups

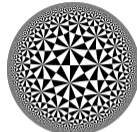


$$\text{Symm}(\square) = \langle r, s : r^2 = s^2 = (rs)^4 = e \rangle$$

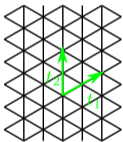
Spherical



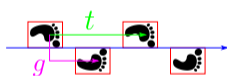
Hyperbolic



Tilings



Friezes

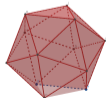


Trigonometry

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$$

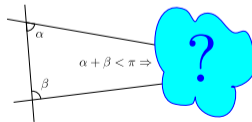
Platonic Solids

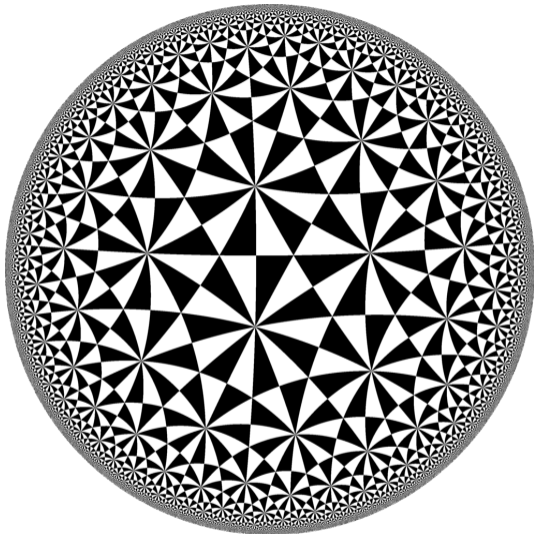


Coxeter



Parallels





**What tiling is this? What can we notice?**

## Learning Objectives:

- ▶ Poincaré's argument about the un-knowability of the intrinsic metric of space
- ▶ Discuss the distinction between a model of a geometry and a geometry
- ▶ Show that the sum of angles in the disk model is less than  $\pi$ .

## Poincaré's Thought Experiment (§7.8)

*In his famous book *Science et Hypothèse*, Henri Poincaré describes the physics of a small “universe” and the physical theories that its inhabitants would create. The universe considered by Poincaré is Euclidean, plane (two-dimensional), and has the form of an open unit disk. Its temperature is  $100^\circ$  Fahrenheit at the center of the disk and decreases linearly to absolute zero at its boundary. The lengths of objects (including living creatures) are proportional to temperature.*

– Sossinsky, *Geometries*.

### Task

*What conclusions would scientists in this world come to?*

# Hilbert's Theorem

Theorem (Hilbert (1901))

*There is no complete isometric embedding  $F : \mathbb{H}^2 \rightarrow \mathbb{R}^3$ .*

# Real World Applications of Hyperbolic Geometry

## Task (15 min)

*Find some real world applications of hyperbolic geometry.  
Feel free to Google and use Wikipedia, look around a bit.*

# Sum of Angles

## Task

*Show that the sum of angles in the disk model is less than  $\pi$*

## Exam Discussion

### Task

*What would you like on the exam? Suggestions for the structure?  
How many questions? What length?*

*(Parker will try to accomodate these are far as possible.)*