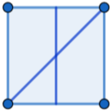
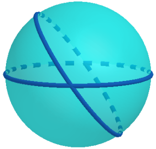
MAT 402: Classical Geometry

Groups

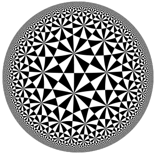


$$\mathrm{Symm}(\square) = \langle r, s : r^2 = s^2 = (rs)^4 = e \rangle$$

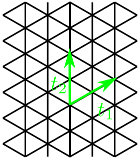
Spherical



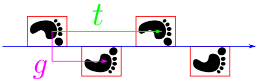
Hyperbolic



Tilings



Friezes

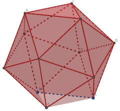


Trigonometry

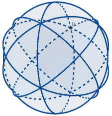
$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

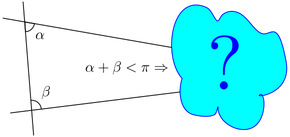
Platonic Solids



Coxeter



Parallels



MAT 402: Friday September 18th 2020

Learning Objectives:

- Select a notion of distance on a space
- Determine whether a linear map is an isometry
- Compute orbits and stabilizers of a given transformation group

Isometries

Definition

Suppose that a space X has a notion of distance $d(x, y)$.
An isometry is a map $f : X \to X$ such that $d(x, y) = d(f(x), f(y))$.

Question (3 min)

What's the notion of distance on the vector space $\mathbb{R}^2$?

Linear Maps

Question (5 min)

Which of the following linear transformations is distance preserving?

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

Prove your claim.

Linear Maps

Question (5 min)

Consider a line $\ell \subset \mathbb{R}^2$. Pick a coordinate system and an orthonormal basis of $\mathbb{R}^2$ and write a matrix for the reflection about ℓ. Prove that reflection is an isometry.

Isometries of the Circle

Question (2 min)

What's the natural notion of distance on the circle $\mathbb{S}^1 = \{(x, y) : x^2 + y^2 = 1\}$*?*

Isometries of the Circle

Question (5 min)

What are the isometries of the circle?

Isometries of the Circle

Question (5 min)

What are the isometries of the circle?

Definition

We have two version of $\mathbb{S}^1 = \{(x, y) : x^2 + y^2 = 1\} = \{e^{i\theta} : 0 < \theta \leq 2\pi\}$.

Isometries of the Circle

Question (5 min)

Show that

$$R_\theta = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \qquad R_\theta = e^{i\theta}$$

both rotate $\mathbb{S}^1$ *by* θ *radians.*

Orbits

Definition

The orbit of a point x under a transformation group G is: $\mathrm{Orb}(x) = \{xg : g \in G\}$.

Task (2 min)

Suppose that $G = \{R_{\pi/2}, R_{\pi}, R_{3\pi/2}, R_{2\pi}\}$ *and* $x = (1, 0)$. *Calculate* $\mathrm{Orb}(x)$.

Stabilizers

Definition

The stabilizer of a point x is: $\mathrm{St}(x) = \{g \in G : xg = x\} \subseteq G$.

Question (5 min)

Why is $\mathrm{St}(x)$ *a subgroup?*

Stabilizers

Question (5 min)

Consider a square $\Box = ABCD$. *What is* $\mathrm{St}(A)$ *for* $G = \mathrm{Symm}(\Box)$*?*