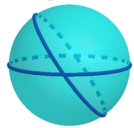
MAT 402: Friday September 11th 2020

MAT 402: Classical Geometry

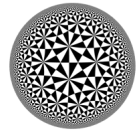
Groups

$\mathrm{Symm}(\square) = \langle r, s : r^2 = s^2 = (rs)^4 = e \rangle$

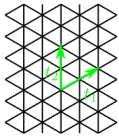
Spherical



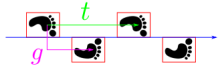
Hyperbolic



Tilings



Friezes

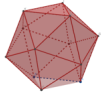


Trigonometry

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

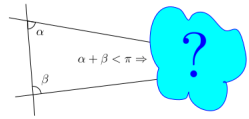
$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

Platonic Solids



Coxeter

Parallels



Plan for today's session:

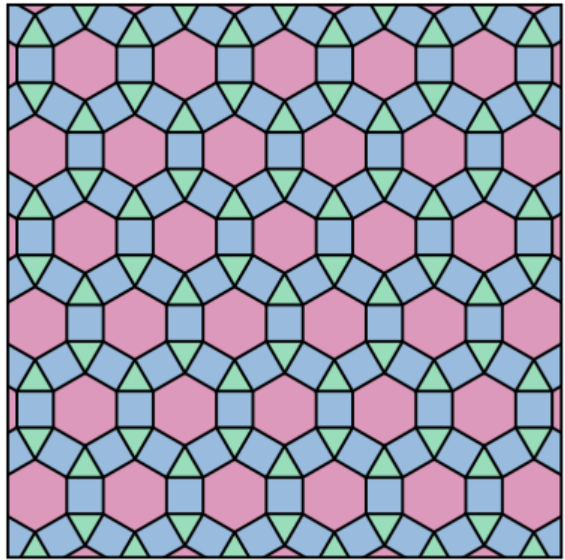
- What is a geometry?
- Syllabus question hunt.
- Questions about the course?
- A bit of group theory.

What is a Geometry?

Question

In break-out groups find examples of different geometries.
We are aiming for lots of examples! Anything counts!

Is this a geometry?



Small Rhombitrihexagonal Tiling by R. A. Nonenmacher.

What is a Geometry?

Question

In break-out groups, try to formulate a one sentence definition of a geometry.
Use words like: axioms, rules, theorems, distance, shapes, symmetry.

Syllabus Hunt

Question

Open up the Quercus page and read through it.

1. *What is our textbook and where can you find it?*
2. *Where can you find readings for the course?*
3. *What is a "notice and wonder" assignment?*

Syllabus Hunt

Question

Submissions and deadlines.

1. *What are we using for submitting homework?*
2. *Why does Quercus say homework is due on Wednesday and Crowdmark say homework is due on Thursday?*
3. *When are things due?*
4. *What happens if you submit something at 12:05pm on Thursday?*

Syllabus Hunt (Extreme Mode!)

Question

1. *Who is Fusako Aizawa?*
2. *Where can you find Group Think by Steven Strogatz?*

Questions about the Course?

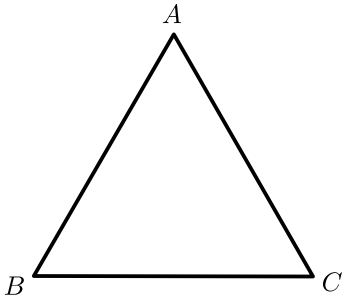
Task

In break-out groups, decide on questions that you want to ask about the course.
Put your questions in the chat box.

Symmetry

Question

What is a symmetry of an equilateral triangle?



Symmetry

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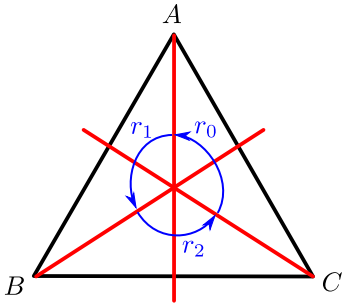

Question

What algebraic properties does the collection of symmetries have?

Groups

With symmetries we are able to:

Groups

With symmetries we are able to:

- Do and undo operations

Groups

With symmetries we are able to:

- Do and undo operations
- Do nothing

Groups

With symmetries we are able to:

- Do and undo operations
- Do nothing
- Regroup the terms

Definition

A *group* is a set G and a map $* : G \times G \to G$ such that:

- There is an *identity element* $e \in G$:

$$e * g = g * e = g$$

- For any $g \in G$ there is $g^{-1} \in G$:

$$g * g^{-1} = g^{-1} * g = e$$

- The map is associative:

$$g * (h * k) = (g * h) * k$$

Example

In $G = \langle r : r^3 = e \rangle$ compute r^5.

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Example (Another Finite Group)

$*$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

- What is a^{-1}?
- Calculate $abbaaca$.