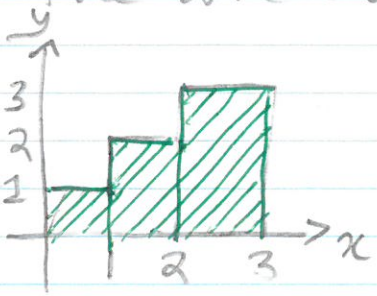
Area and Finite Sums (§5.1)

Question: what is the area of 

?

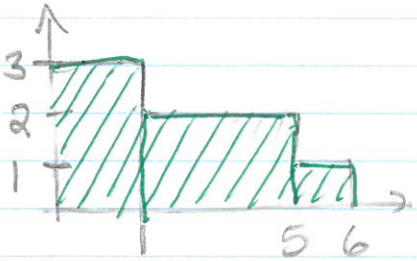
$$A = \ell w$$

Ex: Find the area of the shaded region.



$A = 1\cdot 1 + 1\cdot 2 + 1\cdot 3 = 6$

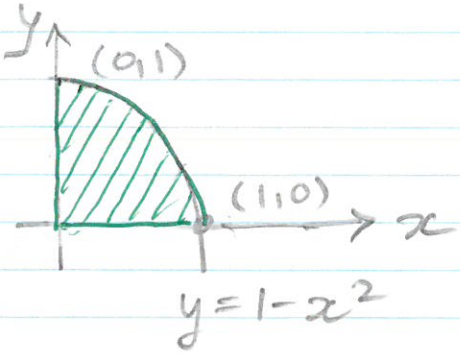
Ex: Find the area of the shaded region.



$A = 1\cdot 3 + 4\cdot 2 + 1\cdot 1$

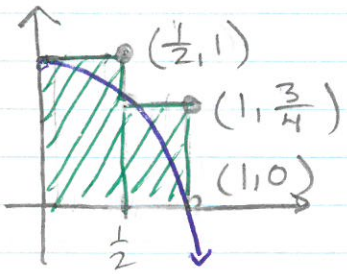
$= 3 + 8 + 1 = 12$

Ex: Find the area of the shaded region



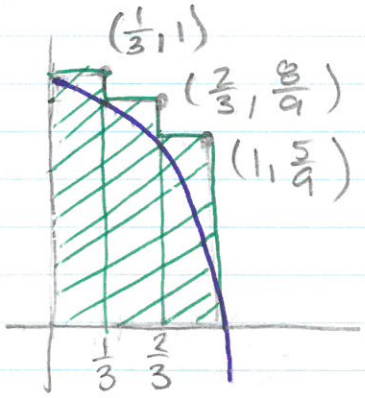
(!) The curved edge makes our method of rectangles difficult to apply.

Ex: Find the area of the shaded region.



$$A = (\tfrac{1}{2} - 0)\cdot 1 + (1 - \tfrac{1}{2})(\tfrac{3}{4})$$
$$= \tfrac{1}{2} + \tfrac{3}{8} = \tfrac{7}{8}$$

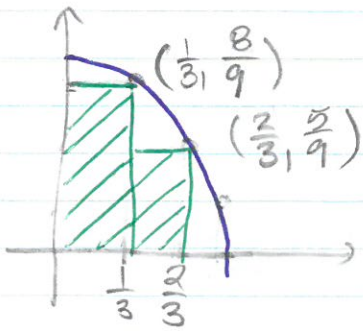
Ex: Find the area of the shaded region



$$A = \tfrac{1}{3}\cdot 1 + \tfrac{1}{3}\cdot\tfrac{8}{9} + \tfrac{1}{3}\cdot\tfrac{5}{9} = \frac{9+8+5}{27}$$
$$= \frac{22}{27}$$

Defn: The **UPPER SUM** is where we take the largest value of $f(x)$ as the height of the rectangle.

Ex: Find the area of the shaded region.



$$A = \tfrac{1}{3}\cdot\tfrac{8}{9} + \tfrac{1}{3}\cdot\tfrac{5}{9} = \frac{13}{27}.$$

Defn: The **LOWER SUM** is where we take the least value of $f(x)$ has the height of the rectangle.

MAT 134 — Week 13c

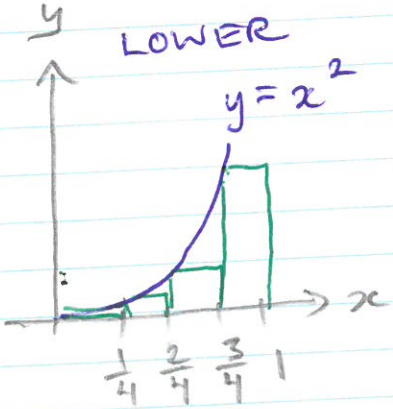
Ex: Suppose you drive with speed $s(t) = 2t$ km/h at time t given in hours.

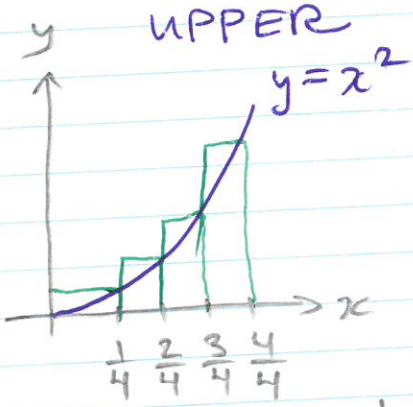
Estimate how far you travel in 6 h.

- From $t=1$ to $t=2$ we travel at least 2 km
  - $t=2$ $t=3$ at least 4 km
  - $t=3$ $t=4$ 6 km
  - $t=4$ $t=5$ 8 km
  - $t=5$ $t=6$ 10 km

Thus, we travel at least 30 km.

Ex: Suppose you travel at $f(t) = t^2$ m/s for t in $[0,1]$ seconds. Estimate how far you travel by dividing $[0,1]$ in to four equal parts and calculating the upper and lower sums.





$$A = \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot\left(\frac{1}{4}\right)^2 + \frac{1}{4}\left(\frac{1}{2}\right)^2 + \frac{1}{4}\left(\frac{3}{4}\right)^2$$

$$= \frac{1}{64} + \frac{4}{64} + \frac{9}{64} = \frac{14}{64}$$

$$A = \frac{1}{4}\left(\frac{1}{4}\right)^2 + \frac{1}{4}\left(\frac{1}{2}\right)^2 + \frac{1}{4}\left(\frac{3}{4}\right)^2 + \frac{1}{4}1^2$$

$$= \frac{1}{64} + \frac{4}{64} + \frac{9}{64} + \frac{16}{64} = \frac{20}{64}$$

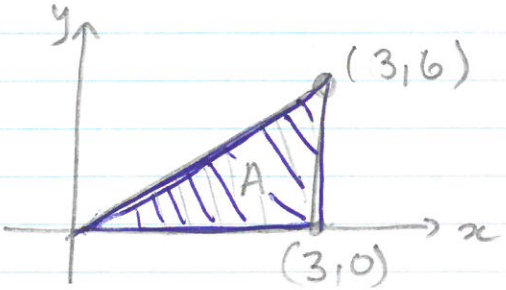
MAT 134 – Week 13c

Average Value

Ex: What is the average value of

$$f(x) = 2x \quad \text{on } [0,3]$$

Find the area under $f(x)$ on $[0,3]$.



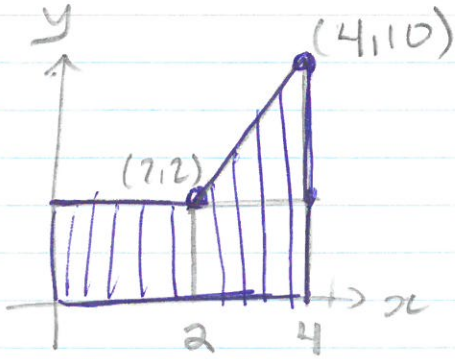
$$A = \frac{1}{2} \cdot 3 \cdot 6 = 9$$

Divide area by length.

$$Avg = \frac{1}{3} \cdot A = 3$$

Thus, the average value is $Avg = 3$.

Ex: What is the average value of the function given below.



$$A = 2 \cdot 2 + 2 \cdot 2 + \frac{1}{2} \cdot 2 \cdot 8$$

$$= 4 + 4 + 8 = 16$$

$$Avg = \frac{1}{4} \cdot 16 = 4.$$