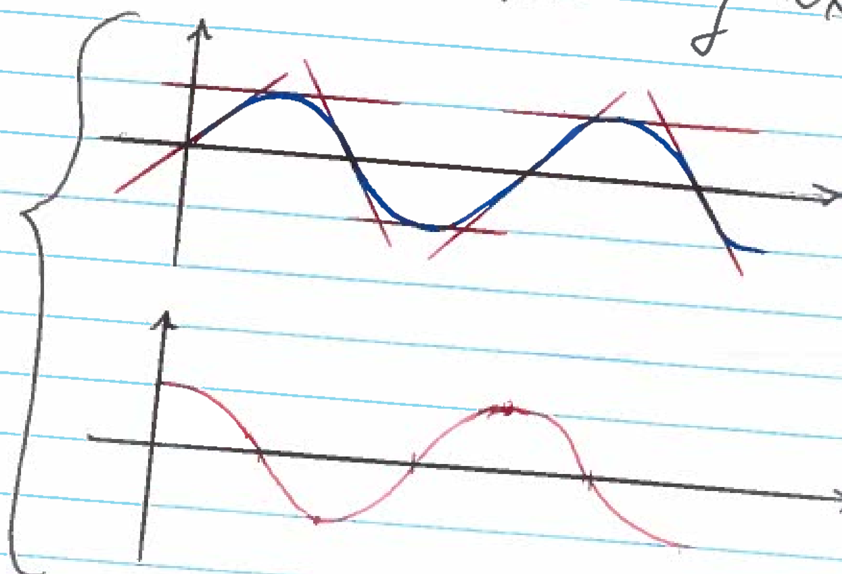
MAT A31 — Wk 7a

This week: Calculus! (Derivatives §2.1–2)

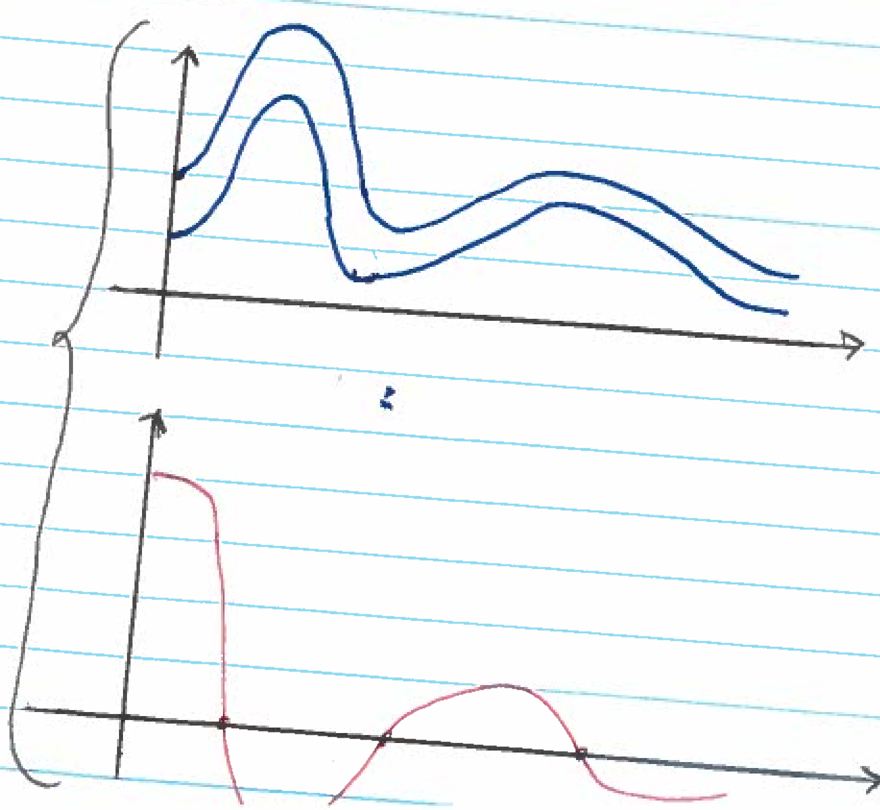
The Slope Function

Consider the following examples



The function f

The slope function of f.

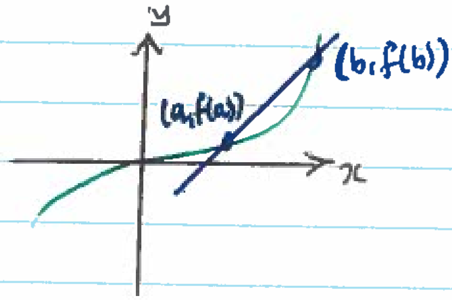


Discuss:
How is the slope function of $f(x)$ related to $f(x)$?

Consider,
- zeros
- increasing
- vertical shift
- rate of growth

Secants

Defn: A SECANT LINE to a graph $y=f(x)$ is a line between $(a,f(a))$ and $(b,f(b))$



Ex: Calculate the slope of the secant line joining $(2,4)$ to $(3,9)$ on $y=x^2$

$$m = \frac{\text{“ rise ”}}{\text{“ run ”}} = \frac{9-4}{3-2} = 5$$

Ex: Calculate $m(h)$ the slope of the secant line from (x,x^2) to $(x+h,(x+h)^2)$

$$m(h) = \frac{(x+h)^2 - x^2}{(x+h)-x}$$

$$= \frac{x^2+2xh+h^2-x^2}{h}$$

$$= 2x+h$$

Defn: The TANGENT LINE to $y=f(x)$ at (x,y) passes through (x,y) and has "the same slope" as $y=f(x)$.

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Ex: Find the tangent line to $y = x^3 + 2x$ at $(1,3)$.

Calculate slope

\# [redacted] of secant lines

$$m(h) = \frac{(x+h)^3 + 2(x+h) - [x^3 + 2x]}{(x+h) - x}$$

$$= \frac{\cancel{x^3} + 3x^2h + 3xh^2 + \cancel{2x} + 2h - \cancel{x^3} - \cancel{2x}}{h}$$

$$= \frac{3x^2h + 3xh^2 + 2h}{h} = 3x^2 + 3xh + 2$$

\# Find the slope at $(1,3)$

\# Take a limit of slopes of secant lines.

$$m = \lim_{h \to 0} m(h) = 3x^2 + 2$$

$$\Rightarrow m = 5$$

\# Construct the tangent line.

Suppose $y = mx + b$ is the tangent line.
We have $m = 5$.

[redacted] $3 = 5 \cdot 1 + b \Rightarrow b = -2$.

Thus, $y = 5x - 2$ is the tangent line.

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3.5

Def^n: The LEFT DERIVATIVE of $f(x)$ is:

$$f'_-(x) = \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h}$$

The RIGHT DERIVATIVE of $f(x)$ is:

$$f'_+(x) = \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h}$$

The DERIVATIVE of $f(x)$ is:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Ex: Compute $f'_-(0)$ and $f'_+(0)$ for $f(x) = |x|$

$$f'_-(0) = \lim_{h \to 0^-} \frac{|0+h| - |0|}{h} = \lim_{h \to 0^-} \frac{|h|}{h} = -1.$$

$$f'_+(0) = \lim_{h \to 0^+} \frac{|0+h| - |0|}{h} = \lim_{h \to 0^+} \frac{|h|}{h} = 1.$$

Thus, $f'(0)$ does not exist for $f(x) = |x|$.

And now for something completely different!

Ex: An object dropped on Earth accelerates at $9.8\,m/s^2$.

How fast does an object travel once it has dropped 10 m?

Determine distance travelled at time t.
Determine speed at time t.

$$s(t) = 9.8\,t$$

NB: Speed is change in distance.

$$d(t) = \frac{9.8}{2}t^2$$

We obtain: An object dropped on earth travels $d(t) = \frac{9.8}{2}t^2$ in time t.

Find time needed to travel 10 m

$$10 = \frac{9.8}{2}t^2 \Rightarrow t = \sqrt{\frac{2\times10}{9.8}}$$

$\sim\sqrt{2}\,s$

We obtain,

$$s\left(\sqrt{\frac{2\times10}{9.8}}\right) = 9.8\sqrt{\frac{2\times10}{9.8}} = \sqrt{2\times10\times9.8} = 14$$

$$\left(= \sqrt{196}\right)$$

Thus, you are travelling at 14 m/s.