MAT A29 wk 2a

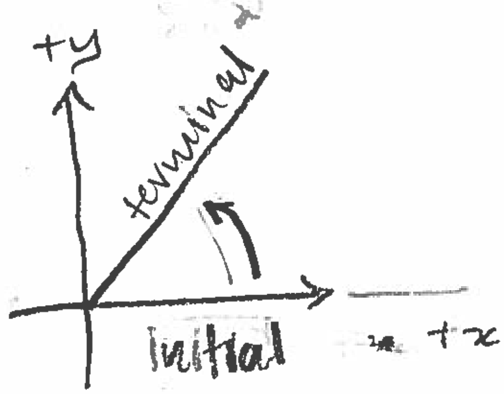
Welcome to Camp SOH CAH TOA!

This week we will learn:

- How to measure angles
- How to measure big things using triangles
- The art of trigonometry

## Angles

Defn: An ANGLE is:




Application:

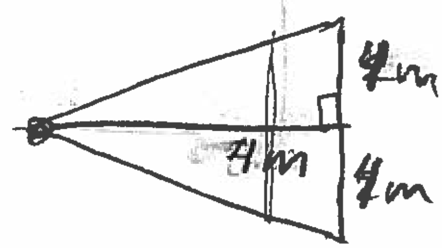
How big is an object you see?

Exp: Squish Parker!

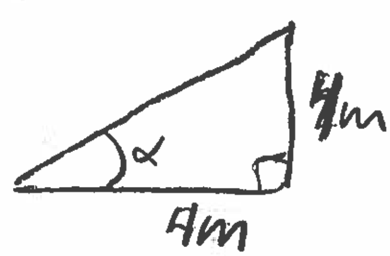
EX:

If you are standing 4m away from a from a 8m wide object: how big does it look?

# Draw the set up.




# Measure the angle



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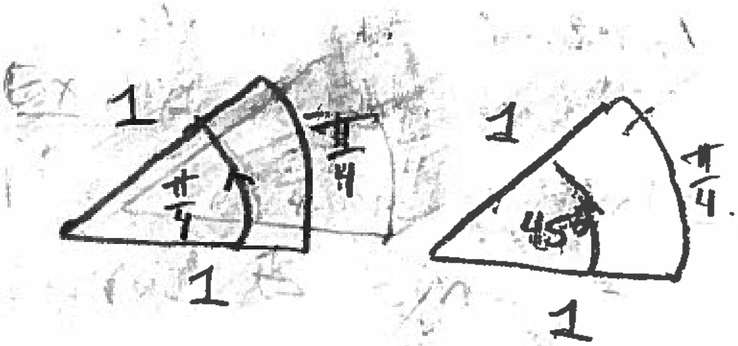
## Radians

<u>Def<sup></sup></u>

Def$^n$: Two measures:

DEGREES : $360°$

RADIANS : $2\pi$




Ex: convert $30°$ to radians.

# Express $30°$ using $360°$

$$30° = \frac{12 \cdot 30°}{12} = \frac{360°}{12}$$

# use $360° = 2\pi$.

$$30° = \frac{360°}{12} = \frac{2\pi}{12} = \frac{\pi}{6}$$

$\square$.

Ex:

convert $\frac{\pi}{5}$ to degrees.

# Express $\frac{\pi}{5}$ using $2\pi$.

$$\frac{\pi}{5} = \frac{2\pi}{2 \cdot 5} = \frac{2\pi}{10}.$$

# use $360° = 2\pi$.

$$\frac{\pi}{5} = \frac{2\pi}{10} = \frac{360°}{10} = 36°$$

$\square$.

In general,

Fact:

$$\frac{\text{radians}}{\pi} = \frac{\text{degrees}}{180°}$$

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EX: The UTSC Math racetrack is 30m in radius.

Suppose you walk $10^\circ$ around it

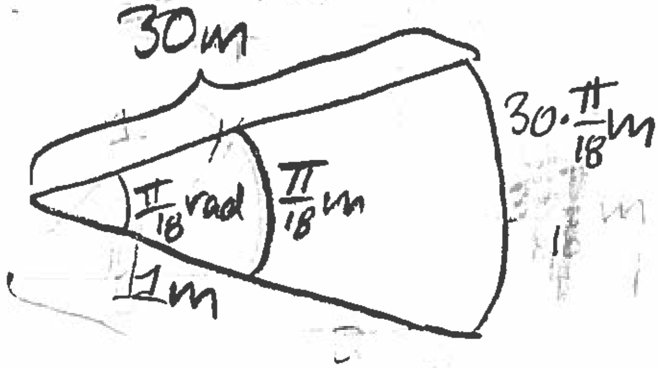
How far did you go?

# Convert $10^\circ$ to radians

$$10^\circ = \frac{36 \cdot 10^\circ}{36} = \frac{360^\circ}{36}$$

$$= \frac{2\pi}{36} = \frac{\pi}{18}.$$

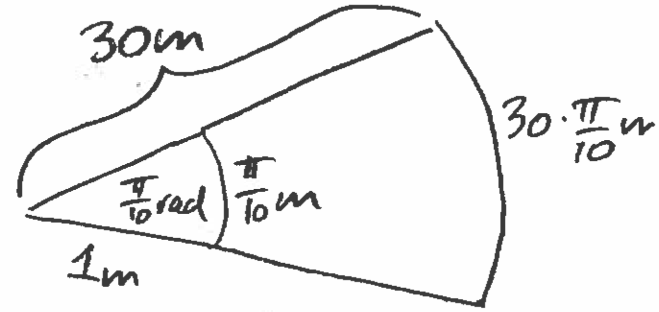
# Measure using the definition of radian




Thus, you walk $\frac{5\pi}{3}$ m.

EX: How far do you go if you walk $\frac{\pi}{10}$ radians

# Measure the length using radians




You walk $30 \cdot \frac{\pi}{10} = 3\pi$ m

In general,

We will use radian because they make things easier to measure in real life

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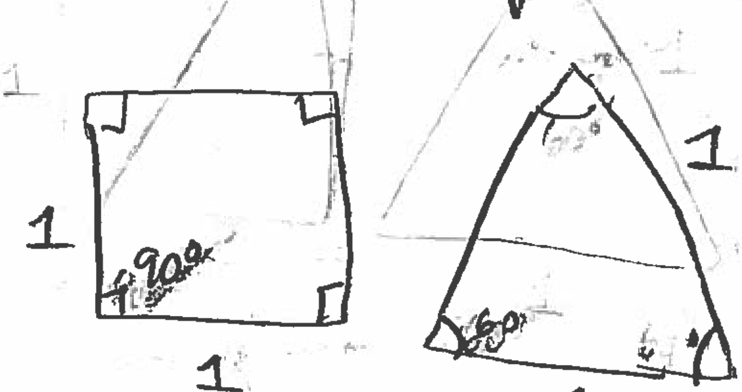
## The Special Triangles

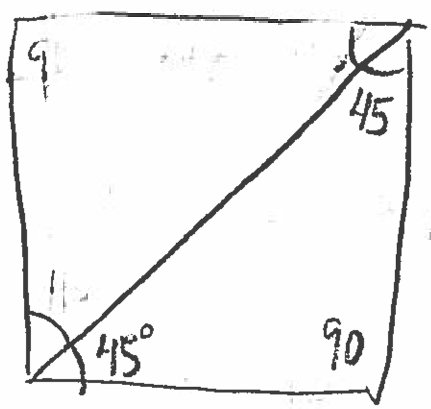
**Fact**: These are special:

Please memorize these!

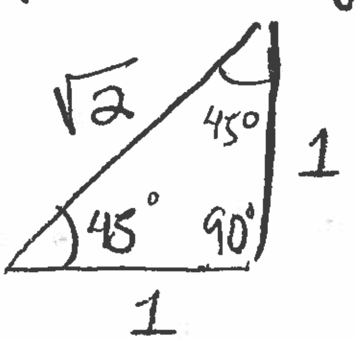



Square Triangle

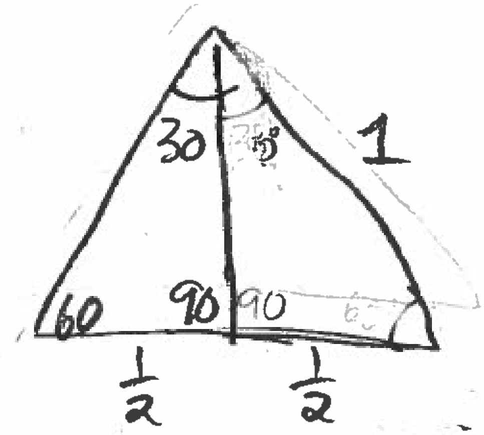
### The Square



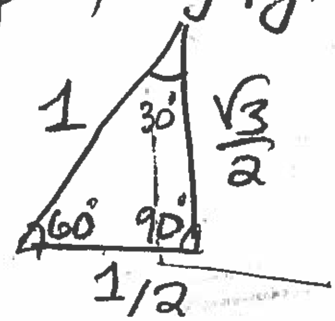

We get, by Pythagoras,




### The Triangle



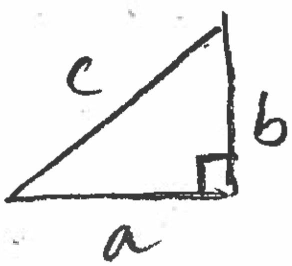

We get, by Pythagoras,



## The Trig Functions

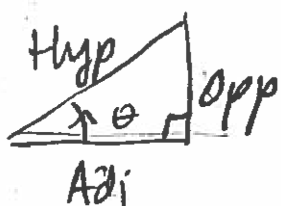
**Defn**: TRIGONOMETRY is the study of triangles.

**Thm** (Pythagoras)
Given a right angled triangle




we have $a^2+b^2=c^2$

**Defn**: Given




We have:

$$\sin\theta = \frac{\text{Opp}}{\text{Hyp}}$$

$$\cos\theta = \frac{\text{Adj}}{\text{Hyp}}$$

$$\tan\theta = \frac{\text{Opp}}{\text{Adj}}$$

angles
$\Updownarrow$
right triangles

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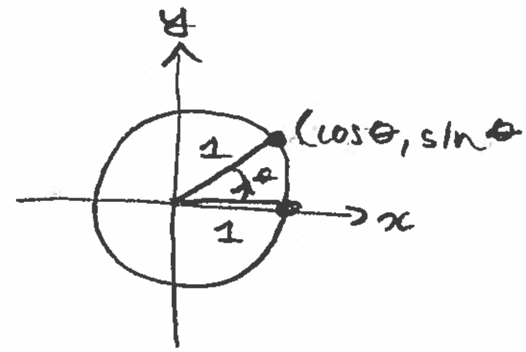
**Fact**:
In the unit circle we have:




Because $(\sin\theta, \cos\theta)$ is always on the unit circle we get:

$$\sin^2\theta + \cos^2\theta = 1.$$

This is called:

The Pythagorean Identity

**Fact**:

$$\sin(\theta) = \sin(\theta \pm 2\pi)$$
$$\cos(\theta) = \cos(\theta \pm 2\pi)$$

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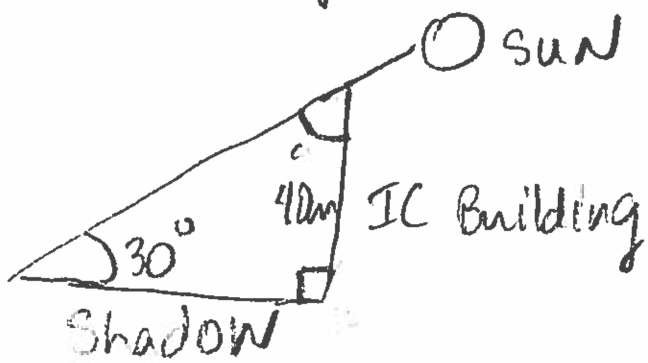
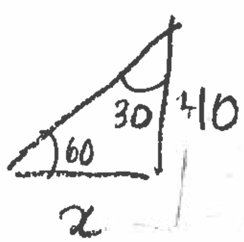
Ex:

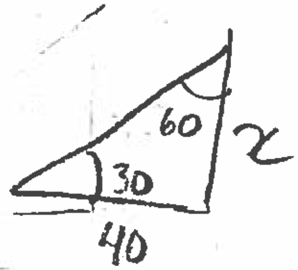
If the sun is 30° above the horizon and the IC is 40m tall, then how long is its shadow?

# Draw the picture




# Find the unknowns




# use trigonometry

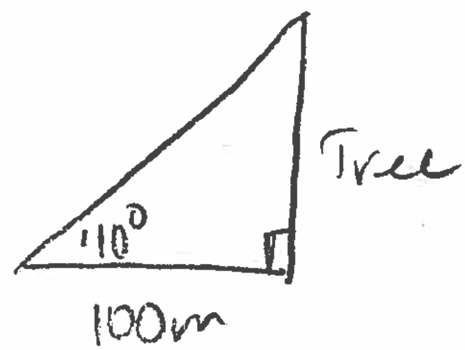
$$\tan 30 = \frac{Opp}{Adj} = \frac{x}{40}.$$

Ex:

A great red wood pine is observed to produce an angle of 40° at 100m distance.

How tall is this tree?

# Draw the picture



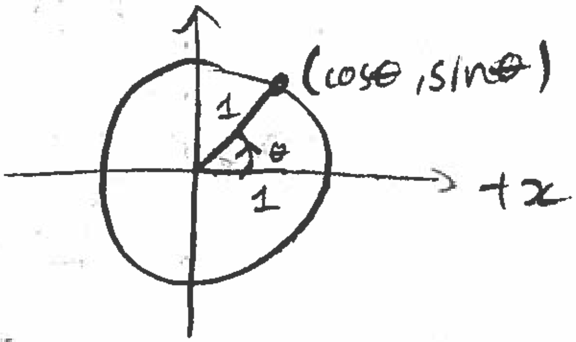

# calculate

$$\text{Tree} = 100m \cdot \tan(40°)$$

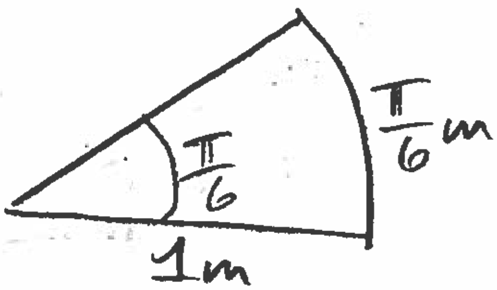
$$\simeq 83.90\ m$$

Fact: Redwood trees are huge!

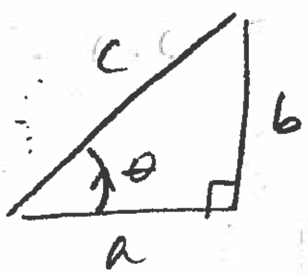
Recall, from last lecture




Measure angles in RADIANS.




For a right angled triangle




$a^2+b^2=c^2$ SOH CAH TOA

$\sin(\theta)=\frac{b}{c}$

$\cos(\theta)=\frac{a}{c}$

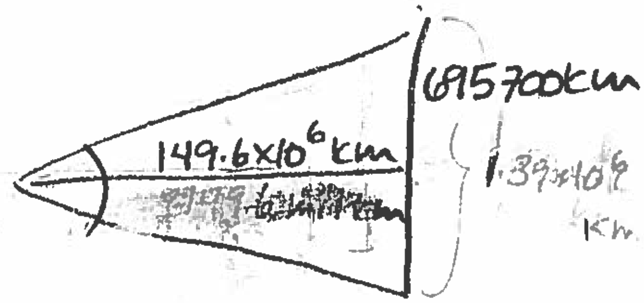
$\tan(\theta)=\frac{b}{a}$

Ex: The sun has radius 657,600km

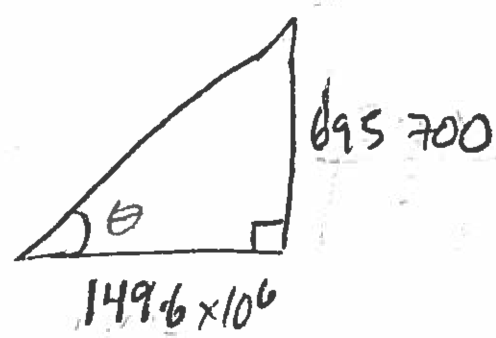
It is approx 149.6 mil km away

How big does the sun look?

# Draw the diagram




# Identify the unknown.



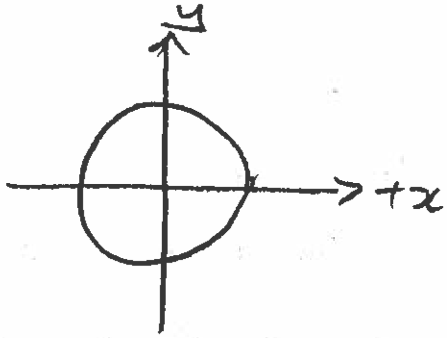

# Compute

$$\theta = \arctan\left(\frac{695\,700}{149.6\times10^6}\right)$$

$$= 0.266 \text{ degrees}.$$

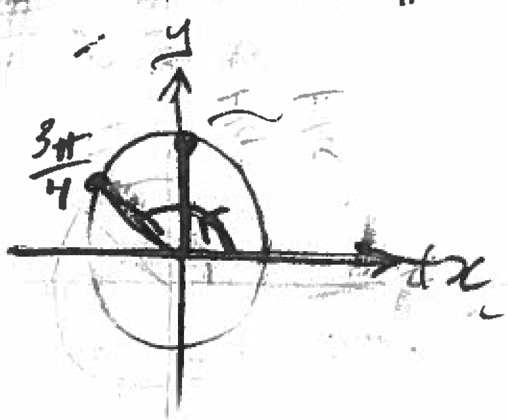
Ex: Find $\cos\left(\frac{3\pi}{4}\right)$ by hand.

# Draw the unit circle (!)



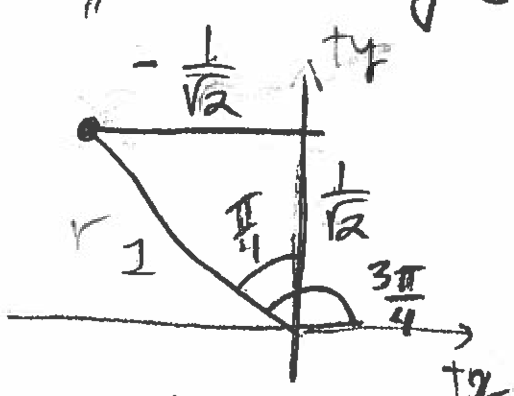

counter-clockwise

Trace from +x to $\frac{3\pi}{4}$




NB: +y is $\frac{\pi}{2}$ away from +x

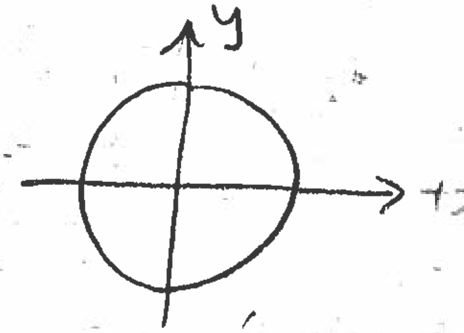
# Note the special triangle



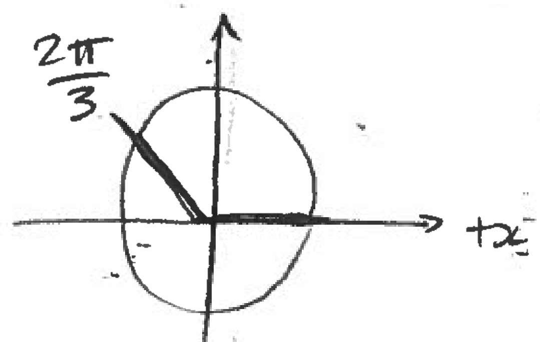

$$\cos\left(\frac{3\pi}{4}\right) = \frac{-\frac{1}{\sqrt{2}}}{1} = -\frac{1}{\sqrt{2}}.$$

Ex: Find $\sin\left(\frac{2\pi}{3}\right)$ by hand.

# Draw the unit circle

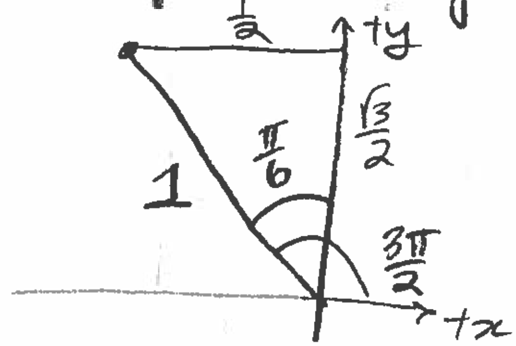



# Trace from +x to $\frac{2\pi}{3}$




NB: $\frac{2\pi}{3} - \frac{\pi}{2} = \frac{4\pi - 3\pi}{6} = \frac{\pi}{6}.$

# Note the special triangle.




$$\sin\left(\frac{2\pi}{3}\right) = \frac{-\frac{1}{2}}{1} = -\frac{1}{2}.$$

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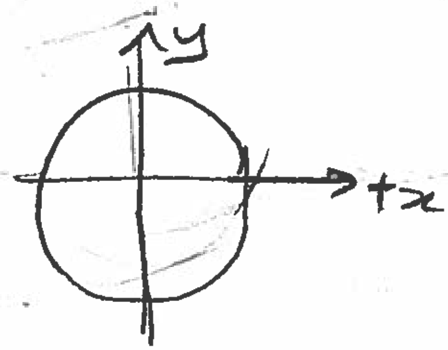
Ex: Find all the solutions of

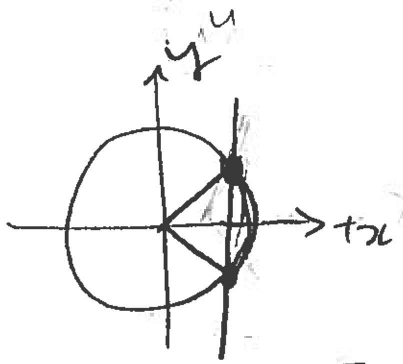
$$\cos 3t = \frac{\sqrt{3}}{2}.$$

\# Draw the unit circle



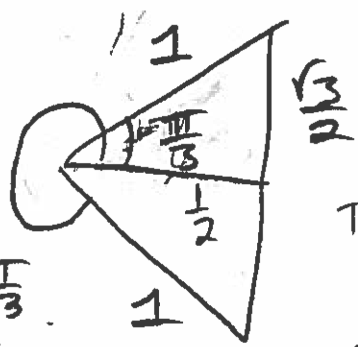

\# Draw the condition $\cos(\cdot) = \frac{\sqrt{3}}{2}$

$(x,y) = (\cos(\cdot), \sin(\cdot))$




$x = \frac{\sqrt{3}}{2}.$

\# Solve $\cos(x) = \frac{\sqrt{3}}{2}$




$2\pi - \frac{\pi}{3}$

$= 5\pi$

Thus,

$x = \frac{\pi}{3} + 2\pi k$

$x = 5\pi - 2\pi k$

\# Translate

$$\cos(x) = \frac{\sqrt{3}}{2}$$

to $\cos(3t) = \frac{\sqrt{3}}{2}.$

Any solution of

$$\cos(3t) = \frac{\sqrt{3}}{2}$$

can be obtained from a solution of

$$\cos(x) = \frac{\sqrt{3}}{2}$$

by dividing by three.

Thus,

$$t = \frac{\frac{\pi}{3} + 2\pi k}{3}$$

$$t = \frac{\frac{5\pi}{6} + 2\pi k}{3}.$$