

# Discussion Questions – Week #5a

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Please do the following questions as a group. Make sure that everyone in your group understands how each questions works. These questions are open ended and admit several approaches each. If you need help, please ask.

**Question 1.** *A set  $U \subset \mathbb{R}^n$  is convex if: For any  $\mathbf{x}, \mathbf{y} \in U$  the line segment with endpoints  $\mathbf{x}$  and  $\mathbf{y}$  is contained in  $U$ .*

*Prove or disprove (by counter-example) the following propositions:*

- *A finite union of convex sets is convex.*
- *All convex sets are path connected.*
- *A finite intersection of convex sets is convex<sup>1</sup>.*
- *All path connected sets are convex.*

**Question 2.** *Show, by example, that the mean value theorem does not hold if  $U$  is not path connected.*

**Question 3.** *Construct two  $C^1$ -functions  $f, g : \mathbb{R}^2 \rightarrow \mathbb{R}$  such that there is no  $C^2$ -function  $F$  such that  $\nabla F = (f, g)$ . You will need to prove that there is no such  $F$ .*

**Question 4.** *Write out  $\sum_{|\alpha|=2} \frac{x^\alpha \partial^\alpha f(0)}{\alpha!}$  for  $f(x, y) = e^{xy}$ .*

**Puzzle 1.** *Show the following: If  $C_i \subset \mathbb{R}^2$  is compact and convex for each  $i \in I$  and  $C_i \cap C_j \neq \emptyset$  for each  $i, j \in I$  then  $\bigcap_{i \in I} C_i \neq \emptyset$ .*

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<sup>1</sup>The empty set is convex. If you do not like this idea then solve Puzzle 1.