Discussion Questions – Week #4b

Parker Glynn-Adey and Tyler Holden

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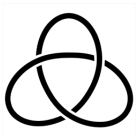
Please do the following questions as a group. Make sure that everyone in your group understands how each questions works. These questions are open ended and admit several approaches each. If you need help, please ask.

Question 1. *Suppose that $F : \mathbb{R}^2 \to \mathbb{R}$ is a C^1-function and $S = \{\mathbf{x} \in \mathbb{R}^2 : F(\mathbf{x}) = 0\}$. Show the following: If $\gamma : \mathbb{R} \to S \subset \mathbb{R}^2$ is a path in S then $\nabla F(\gamma(t))$ is always perpendicular to $\gamma'(t)$.* Hint: Recall, if $\vec{a} \cdot \vec{b} = 0$ then $\vec{a}$ and $\vec{b}$ are perpendicular by definition.

Question 2. *Suppose $F : \mathbb{R}^n \to \mathbb{R}^n$ is linear and continuous. Show that $F(\mathbf{x}) = A\mathbf{x}$ for some matrix $A \in M_{n\times n}(\mathbb{R})$. Moreover, show that the Jacobian matrix of F is <u>also</u> equal to A.*

Hint: Recall, $F(\mathbf{x} + \mathbf{y}) = F(\mathbf{x}) + F(\mathbf{y})$ and $F(c\mathbf{x}) = cF(\mathbf{y})$.

Puzzle 1. *Parametrize the curve pictured below.*



Hint: Build your parametrization as a sum of simpler paths.
Check all your guesses by plotting them online.