# Discussion Questions – Week #4a

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Please do the following questions as a group. Make sure that everyone in your group understands how each questions works. These questions are open ended and admit several approaches each. If you need help, please ask.

**Question 1.** *(i) Let* $f(x, y) = x^2 + y^2$*. Compute the equation of the tangent plane at* $(x, y) = (1, 1)$*. Sketch the parabola and plane.*

*(ii) Let* $f : \mathbb{R}^n \to \mathbb{R}$ *be differentiable everywhere. Write down a general formula for the tangent plane to* $f$ *at* $\mathbf{x} = \mathbf{a}$*.*

**Question 2.** *Show that if* $\gamma : \mathbb{R} \to \mathbb{R}^n$ *is differentiable everywhere and satisfies* $\gamma \cdot \gamma' = 0$ *then* $||\gamma||$ *is constant. In particular, this forces* $\gamma$ *to live on a sphere of constant radius.*

**Question 3.** *Construct* $f : \mathbb{R}^2 \to \mathbb{R}$ *such that* $\partial_{\mathbf{u}} f(0)$ *exists for all unit vectors* $\mathbf{u}$ *but* $f$ *is not differentiable. (Hint: The situation is analogous to when a limit exists along lines, but fails to exist along parabolas.)*

**Puzzle 1.** *Let* $O(2) = \{M \in M_{2\times 2}(R) : MM^T = I\}$ *be the set of orthogonal* $2\times 2$ *real matrices. A path through* $I$ *is a differentiable map* $\gamma : [-1, 1] \to O(2)$ *such that* $\gamma(0) = I$*. We define*

$$\mathfrak{o}(2) = \{\gamma'(0) : \ \gamma \text{ is a path through } I\}$$

*Write down all the elements of* $\mathfrak{o}(2)$*.* Hint: $M_{2\times 2}(\mathbb{R}) \simeq \mathbb{R}^4$.