Discussion Questions – Week #2a

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Please do the following questions as a group. Make sure that everyone in your group understands how each questions works. These questions are open ended and admit several approaches each. If you need help, please ask.

Question 1. *Recall that a sequence* $(\mathbf{x}_n)$ *in* $\mathbb{R}^k$ *is a* Cauchy sequence *if for every* $\epsilon > 0$ *there exists* $N \in \mathbb{N}$ *such that whenever* $m, n \geq N$ *then* $\|\mathbf{x}_n - \mathbf{x}_m\| < \epsilon$.

1. *Show that the sequence* $x_n = (-1)^n$ *is not a Cauchy sequence.*
2. *Show that* $x_n = 1/n$ *is a Cauchy sequence.*
3. *Find a sequence* x_n *in* $\mathbb{R}$ *such that* $\|\mathbf{x}_{n+1} - \mathbf{x}_n\| \to 0$ *but* (x_n) *is not Cauchy.*

Question 2. *Come up with proofs. Don't consult the notes too much.*

1. *Show that all Cauchy sequences are bounded.*
2. *Show that every subsequence of a Cauchy sequence is a Cauchy sequence.*
3. *Show that every convergent sequence is Cauchy.*

Question 3. *Construct each of the described sequences.*

1. *A sequence of* rational *numbers* (x_n) *such that* $(x_n) \to 0$.
2. *A sequence of* negative *rational numbers* (x_n) *such that* $(x_n) \to 0$.
3. *A sequence of* irrational *numbers* (x_n) *such that* $(x_n) \to 0$.
4. *A sequence of* real *numbers* (x_n) *such that* $(x_n) \to 0$ ***and*** *no other group will choose the same* (x_n).
5. *A sequence of* rational *numbers which converge to* π.

Question 4. *Suppose that* $(x_n), (y_n)$ *are two sequences in* $\mathbb{R}^n$. *Find a sequence* (z_n) *such that both* (x_n) *and* (y_n) *are subsequences of* (z_n).

Puzzle 1. *What is the least number* $N = N(10cm, 1cm)$ *such that: If a big cubical box of side length* $10cm$ *contains* N *cubes of side length* $1cm$ *then two of the small cubes overlap? Generalize this puzzle to balls of radius* r *and* R *then estimate* $N(R, r)$ *by proving an upper bound.*