Discussion Questions – Week #1a

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Please do the following questions as a group. Make sure that everyone in your group understands how each questions works. These questions are open ended and admit several approaches each. If you need help, please ask.

Question 1. *Let $A, B \subseteq \mathbb{R}^n$. Write each of the following in terms of A, B, and their complements:*

1. $(A \cup B)^c$
2. $(A \cap B)^c$

Be sure to prove your results. [Hint: The dimension does not matter. Try playing with examples for A and B to determine what the answer should be, then prove your result.]

Question 2. *Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a function and $A, B \subseteq \mathbb{R}^m$.*

1. *Show that $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$*
2. *Show that $f^{-1}(A \cap B) \subseteq f^{-1}(A) \cup f^{-1}(B)$. Find a counter example where the converse does not hold.*

Question 3. *Let $f(x) = (\cos(x), \sin(x))$. What is the image of f?*
Compute $f^{-1}((x, y))$ where $(x, y) = (0, 1)$.

Question 4. *Let $f, g, h : \mathbb{R} \to \mathbb{R}$ and set $h = f \circ g$. If h is injective, show that g is also injective.*

Question 5. *Show that*

$$|x_i| \leq \sqrt{x_1^2 + \cdots + x_n^2} = ||x||$$

for any $(x_1, \ldots, x_n) \in \mathbb{R}^n$. Challenge: Try to reverse the inequality to get

$$||x|| \leq (???) \max_{i=1,\ldots,n} \{|x_i|\}.$$

The expression in brackets should be constant with respect to the x_i.

Question 6. *If $\vec{a} \times \vec{b} = \vec{0}$ then what else is true about $\vec{a}$ and $\vec{b}$? [Hint: It's easy to come up with sufficient conditions like $\vec{a} = 0$, but come up with a necessary condition!]*

Puzzle 1. *Suppose you have a drawer containing an equal number of red and blue socks. Suppose the number that you need to draw out to guarantee a matched pair is the same as the number you need to guarantee an unmatched pair. How many socks do you have?*