Lecture Problems Week #2b

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May 17, 2016

Question

If f is continuous which of the following is false?

1. *If S is closed then $f^{-1}(S)$ is closed.*
2. *If S is open then $f^{-1}(S)$ is open.*
3. *If S is both then $f^{-1}(S)$ is both.*
4. *None of the above.*

Question

If $f : \mathbb{R}^2 \to \mathbb{R}$ *is continuous what is necessarily true about* $S = f^{-1}([0, 1))$*?*

1. S *is open.*
2. S *is closed.*
3. *None of the above.*

Question

Which function/range pair has

$$S = \{(x, y) : y \leq \cos(x)\}$$

as its pre-image?

1. $f(x, y) = y - \cos(x)$ *and* $(-\infty, 0]$.
2. $g(x, y) = \cos(x) - y$ *and* $(-\infty, 0]$.
3. $h(x, y) = y - \cos(x)$ *and* $[0, \infty)$.
4. *None of the above.*

Question

Which function/range pair has

$$S = B_{\mathbf{0}}(10) \setminus B_{\mathbf{0}}(5) \subset \mathbb{R}^2$$

as its pre-image?

1. $f(x, y) = x^2 + y^2 - 1$ *and* $R = [4, 9)$.
2. $f(x, y) = x^2 + y^2$ *and* $R = (5, 10)$.
3. $f(x, y) = x^2 + y^2$ *and* $R = (5, 10]$.
4. $f(x, y) = x^2 + y^2 - 1$ *and* $R = [6, 11)$.

Question

Suppose $f : \mathbb{R}^n \to \mathbb{R}$. *For values of* C *is it true that:*
If $\lim_{\mathbf{x}\to\mathbf{a}} |f| = C$ *then* $\lim_{\mathbf{x}\to\mathbf{a}} f = C$*?*

1. $C = 0$.
2. *All values of* C.
3. *No value of* C.
4. $C = \infty$.

Question

If $||(x, y, z)|| \leq 1$ *then how big can* $f(x, y, z) = x + y + z$ *be?*

1. $f(x, y, z) < 2$ *for all points in* $B_1(\mathbf{0})$.
2. $f(x, y, z) = 10$ *for some point in* $B_1(\mathbf{0})$.
3. $f(x, y, z) < 1$ *for all points in* $B_1(\mathbf{0})$.
4. $f(x, y, z) > 5$ *for some point in* $B_1(\mathbf{0})$.

Question

Suppose $f : \mathbb{R}^n \to \mathbb{R}$ *is continuous.*
Which of the following statements can be false?

1. f *maps convergent sequences to convergent sequences.*
2. f^{-1} *maps convergent sequences to convergent sequences.*
3. f^{-1} *maps open sets to open sets.*

Question

Suppose that $||(x_1, \ldots, x_n)|| \leq C \max\{|x_i|\}$ *for all points in* $\mathbb{R}^n$. *What is the smallest possible value of* C*?*

1. $C = 1$
2. $C = \sqrt{n}$
3. $C = n$
4. *None of the above.*

Question

Consider the sequence of points $\mathbf{x}_n = (\cos(\pi n/2), \sin(\pi n/2))$.
Does this sequence converge?

1. *Yes.*
2. *No.*

Question

Consider the sequence of points $\mathbf{x}_n = (\cos(2\pi n), \sin(2\pi n))$.
Does this sequence converge?

1. *Yes.*
2. *No.*