Lecture Problems Week #2a

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Question

Which of the following is not a subsequence of $x_n = 2n$?

1. $z_n = 2^n$
2. p_n *the n-th prime*
3. $w_n = (n+2)!$
4. $y_n = 2n$

Question

Is the sequence $x_n = \frac{1}{n^2+1}$ *bounded and decreasing?*

1. Bounded and not decreasing.
2. Both.
3. Neither.
4. Decreasing and not bounded.

Question

Is the sequence $x_n = \frac{n}{n+1}$ *bounded and increasing?*

1. Bounded and not increasing.
2. Both.
3. Neither.
4. Increasing and not bounded.

Question

Let $\mathbf{x}_n = \left(n^2, \frac{1}{n+1}\right)$.

Which of the following is not a subsequence of $\mathbf{x}_n$*?*

1. $a_n = (4, 1/3), (16, 1/5), (100, 1/11), \ldots$
2. $b_n = \left(4n^2, \frac{1}{2n+1}\right)$.
3. $c_n = (1, 1/3), (9, 1/5), (25, 1/7), \ldots$
4. $d_n = \left(4^n, \frac{1}{2^n+1}\right)$

Question

Suppose $x \leq \epsilon + 1$ *for all* $\epsilon > 0$*. What is true about* x*?*

1. $x < 1$
2. $0 < x$
3. $x \leq 4$
4. $x = 0$

Question

Does the sequence $x_n = \sin(n)/n$ converge?

1. No – Diverges to infinity.
2. Yes – It is bounded and increasing.
3. Yes – Squeeze theorem.
4. No – It is bounded but oscillates.

Question

You have x_n a sequence in $(1,2) \cup (2,3)$.
Which of the following points can x_n not converge to?

1. π
2. 2
3. $\sqrt{2}$
4. $\pi/2$.

Question

You have a sequence $\mathbf{x}_n$ *in* $S \subset \mathbb{R}^3$ *which coverges to a point* $\mathbf{p} \in \mathbb{R}^3$*. Which of the following is necessarily false?*

1. $\mathbf{p} \in \overline{S}^c$
2. $\mathbf{p} \in S$
3. $\mathbf{p} \in \mathrm{Int}(S)$
4. $\mathbf{p} \in \partial S$

Question

Assume that $\mathbf{x}_n \to \mathbf{x}$*? Which of the following statements must also be true?*

1. $\forall \epsilon > 0, \exists N \in \mathbb{N}$, such that if $n \geq N$ then $\|\mathbf{x}_n - \mathbf{x}\| \leq 2\epsilon$
2. $\forall \epsilon > 0, \exists N \in \mathbb{N}$, such that if $n \geq N$ then $\|\mathbf{x}_n + \mathbf{x}\| < \epsilon$
3. $\exists \epsilon > 0, \forall N \in \mathbb{N}$, such that if $n \geq N$ then $\|\mathbf{x}_n - \mathbf{x}\| < \epsilon$
4. $\forall \epsilon > 0, \exists N \in \mathbb{R}$, such that if $n \leq N$ then $\|\mathbf{x}_n - \mathbf{x}\| < \epsilon$

Question

Suppose $\mathbf{p}_n = (x_n, y_n, z_n)$ *is a convergent sequence in* $\mathbb{R}^3$*. How many of the sequences* $\{x_n\}, \{y_n\}, \{z_n\}$ *must converge?*

1. All.
2. None.
3. At least one.

Question

Suppose $\mathbf{p}_n = (x_n, y_n, z_n)$ *is a divergence sequence in* $\mathbb{R}^3$*. How many of the sequences* $\{x_n\}, \{y_n\}, \{z_n\}$ *must diverge?*

1. All.
2. None.
3. At least one.

Question

Let $S = [0, 1)$.
Does every sequence in S *have a subsequence converging to a point in* S*?*

1. No
2. Yes
3. It is impossible to tell.

Question

Let $\mathbf{x}_n$ be a sequence and $\mathbf{p}$ a point. Which of the following is equivalent to the statement that $\mathbf{x}_n \to \mathbf{p}$?

1. Every open ball around p contains infinitely many points of the sequence.
2. Every open ball around p contains all but finitely many points of the sequence.
3. There exists $N \in \mathbb{N}$ such that $\mathbf{x}_k = \mathbf{p}$ for all $k \geq N$.
4. $\mathbf{x}_k = \mathbf{p}$ for some $k \in \mathbb{N}$.

Question

Assume that $\mathbf{x}_n$ *is a sequence in* $\mathbb{R}^n$ *and* $\mathbf{p} \in \mathbb{R}^n$ *is a point such that every open ball around* $\mathbf{p}$ *contains infinitely many points of the sequence.*
Which of the following is necessarily true?

1. $\mathbf{x}_n \to \mathbf{p}$
2. There exists a subsequence of $\mathbf{x}_n$ which converges to $\mathbf{p}$
3. $\mathbf{x}_k = \mathbf{p}$ for some k
4. There exists an $N \in \mathbb{N}$ such that $\mathbf{x}_k = \mathbf{p}$ for all $k \geq N$.