

Lecture Problems Week #1a

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May 10, 2016

Question

What is $\bigcup_{n \in \mathbb{N}} \left[0, 1 - \frac{1}{n}\right]$?

- ① $[0, 1]$
- ② $[0, 1)$
- ③ $(0, 1)$
- ④ \emptyset

Question

What is $\bigcap_{n \in \mathbb{N}} \left(-\frac{1}{n}, \frac{1}{n}\right)$?

- 1 $\{0\}$
- 2 \mathbb{R}
- 3 $[-1, 1]$
- 4 \emptyset

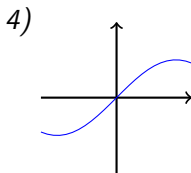
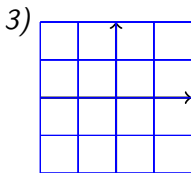
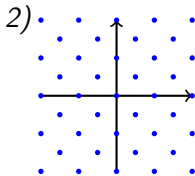
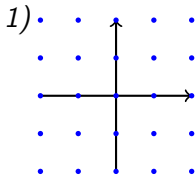
Question

Which of the following is not an element of $\{1, 2, 3\} \times \{A, B, C\}$?

- ① (2, C)
- ② (1, C)
- ③ (2, A)
- ④ (1, 1)

Question

Which of the following correctly represents the subset $\mathbb{Z} \times \mathbb{Z} \subset \mathbb{R} \times \mathbb{R}$.



Question

Let $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be $f(x, y, z) = (x, y)$.

Let $S^2 = \{(x, y, z) : x^2 + y^2 + z^2 = 1\}$ be the unit sphere in \mathbb{R}^3 .

What is $f(S^2)$?

- ① $f(S^2) = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$
- ② $f(S^2) = \{(x, y) \in \mathbb{R}^2 : x = \pm y\}$
- ③ $f(S^2) = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$
- ④ $f(S^2) = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$

Question

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be $f(x) = x^2$. What is $f^{-1}(-10)$?

- ① $\{-10, 10\}$
- ② $\{-\sqrt{10}, \sqrt{10}\}$
- ③ $\{-100, 100\}$
- ④ \emptyset

Question

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be $f(x) = x^2$. What is $f^{-1}(\{y : -1 < y < 1\})$?

- ① $(-1, 1)$
- ② $[-1, 1]$
- ③ $[0, 1]$
- ④ $(0, 1]$
- ⑤ \emptyset

Question

Let $p : \mathbb{R}^2 \rightarrow \mathbb{R}$ be $p(x, y) = x$. What is $p^{-1}(10)$?

- 1 $\{(x, y) \in \mathbb{R}^2 : x \in \mathbb{R}, y = 10\}$
- 2 $\{(10, 0)\}$
- 3 $\{10\} \times \mathbb{R}$
- 4 \emptyset

Question

Let f be a function. What is $f^{-1}(U \cup V)$?

- ① $f^{-1}(U) \cup f^{-1}(V)$
- ② $f^{-1}(U) \cap f^{-1}(V)$
- ③ $f(U \cap V)$
- ④ $f^{-1}(U \cup V) \cap f^{-1}(U \cap V)$

Question

Which of the following functions is injective? [Select all that apply]

- ① $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = e^x$
- ② $g : \mathbb{R} \rightarrow \mathbb{R}$ given by $g(x) = \sin(x)$
- ③ $h : (0, \infty) \rightarrow \mathbb{R}$ given by $h(x) = \log(x)$
- ④ $k : [-1, 1] \rightarrow \mathbb{R}$ given by $k(x) = e^{x^3}$

Question

Which of the following functions is injective? [Select all that apply]

- ① $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined as $f(x, y) = x$
- ② $g : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined as $g(x, y) = (x^2, y^3)$
- ③ $h : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined as $h(x, y) = (-3y, 2x)$
- ④ $k : [0, 1] \rightarrow \mathbb{R}^2, k(x) = (\cos(\pi x), \sin(\pi x))$

Question

Suppose f and g are injective. Is $f \circ g$ necessarily injective?

- 1 Always
- 2 Never
- 3 Only if $f = g^{-1}$
- 4 It is impossible to tell

Question

Let $f : [0, 1] \rightarrow \mathbb{R}$ be given by $f(x) = (g(x), h(x))$, and assume g is injective on $[0, 1]$.

Is f necessarily injective?

- 1 Always
- 2 Never
- 3 Only if h is also injective on $[0, 1]$.
- 4 It is impossible to tell

Question

Consider the set of all $\begin{bmatrix} x \\ y \end{bmatrix}$ which satisfy $\begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 0$. Which of the following lines describes this set?

- ① $y = -\frac{2}{3}x$
- ② $y = \frac{1}{2}x + 3$
- ③ $3y = 2x$
- ④ $2y + 3x = 0$

Question

Consider the set of $\begin{bmatrix} x \\ y \end{bmatrix}$ that satisfy: $\begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 6$. Which of the following lines describes this set?

- ① $3y = 2x + 3$
- ② $y = -\frac{2}{3}x + 6$
- ③ $y = -\frac{2}{3}x + 2$
- ④ $y = -\frac{3}{2}x + 3$