University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

Teaching Dossier

Parker Adey
Assistant Professor Teaching Stream

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Contents

Chapter 1

Introduction

Welcome to my dossier! We are all busy people, and I appreciate the time that you are taking to review this dossier. In this dossier, I have a couple of notational conventions. There are a few footnotes which serve as asides[1]. Due to the formatting of the LaTeX book document class, there are a lot of blank pages (such as the next page). Whenever I quote from course materials, student communications, or other documents, I will include the material in a gray box to distinguish it from the main text of the dossier. For example, a student e-mail will appear like this.

```
 Hello, I'm writing this just to thank you for everything you've done
throughout the semester. MAT A22 has been a really fun course for
me and I have learnt a lot of new concepts. Apart from being a great
teacher I also want to thank you for being so approachable whenever I
needed any sort of advice or help. It's been a great semester and I
hope to take more courses with you in the future and even become a TA
perhaps :) Thanks a lot once again. Regards, STUDENT.
```

Throughout this dossier, I include material that is both positive and negative. That is to say, there is some honest criticism of my teaching in here. I think that it is important to face criticism directly, and learn from it. I am still learning and developing as an educator. The process of writing this dossier has been deeply reflective. I have learned a lot from preparing it.

Acknowledgements I want to thank David Chan and Nancy Johnston for giving me advice early on about how to prepare this dossier. Megan Shaw, my first reader, deserves many thanks, not least of all for her careful line-editing of an early draft of this work. Paco Estrada provided the template on which this dossier is based, and insightful comments on an early draft. Many thanks to all of you!

[1] They're not essential to the text, but writing them brought a smile to my face.

1.1 What Is New?

This dossier is an enlargement and revision of my previous dossier submitted for probationary review in July 2024. The major changes are itemized below.

CTL Best Practices Course I participated in the year-long Best Practices in Teaching and Learning course run by CTL[2]. I wrote a series of reflections for that course which expand upon my Teaching Philosophy. They adress topics such as universal design for learning, indigenous pedagogy, and articial intelligence.

Teaching Evaluations The Summary of Course Evaluation Data has been updated to include recent courses. A theme that emerges from this data is a career long commitment to creating a classroom atmosphere that students find conducive to learning.

The Magic of Numbers: Active and Inclusive In Winter 2026, I re-wrote MAT A02 The Magic of Numbers, to be more active and inclusive. A write-up of that class, as well as some amazing facts about the incredible diversity present at UTSC, are included in MAT A02: The Magic of Numbers.

Student Mentorship In 2025-2026, I mentored three incredible students: Samira Goder, Emily Forbrigger, and Vanessa Schattman. They all went on to publish articles, win awards, and present at major conferences based on our work. I've included my joint article with Vanessa Shattman as an appendix: Of Loops, Braids, and String Figures.

[2]Center for Teaching and Learning. Many thanks to everyone at CTL!

Chapter 2

Statement of Teaching Philosophy

Good teaching is kind, interactive, and accessible.

The most important principle that guides my teaching is “be kind”. I first got introduced to the pedagogy of kindness during a faculty-development reading group on *The Courage to Teach* by Parker Palmer [6]. The author is a Quaker, like myself, and writes extensively about the importance of being kind to both our students and ourselves. Kindness has a practical impact on the way that I teach.

There is a strong power dynamic in teaching. For better or worse, the students are compelled to do as we say. We can treat them harshly, or we can treat them as we would wish to be treated. In my experience, practicing kindness and radical honesty are more effective than harshness. I want to illustrate this by contrasting how I handled two academic integrity cases. Several years ago, I had a student who was caught with unauthorized aids during an exam. As an instructor for the course, I filed the relevant paperwork during the exam, noted down the student’s information, and said that I would have to bring them to meet with the course coordinator after the exam. I was stern: “This is a very serious offense. We will report this academic integrity offense. You must explain yourself to the course coordinator.” The student went in to a blind panic. There were crying hysterically as we walked across campus. Reflecting on this incident, I would say that what I did was fair, followed procedure, and had a negative impact on the student’s well-being.

Now, let’s compare this situation to something which happened last year. Again, a student was caught with an unauthorized aid during a final exam. And yet, when filing the paperwork and talking to the student about it, I adopted a light tone. “Thanks for being patient with me. We need to chat about this further. I’m sure this will all work out for the best.” The student was happy, calm, and went about their day. The difference in tone, the choice kindness over sternness, made a world of difference. The end result was the same, but the student’s experience was radically different.

This is an extreme example. Unauthorized aids during assessments must be handled appropriately. But I think that we must handle our students with care and kindness. The farther I go in teaching, the more I see opportunities to be kind and compassionate. A gentle word, a compliment, or an extra moment of attention, can make a world of difference.

Here is another example of kindness in action. As teaching faculty, students often come to me with big questions about their lives. “Should I drop this course? Should I pursue teaching, industry, or research?” These questions can only be answered in the context of their life goals. And I make a point of talking about life goals in class. For example, in my welcoming message to students in MAT A29, I will highlight that I admire their commitment to the life sciences. Students taking my classes know that I am sensitive to their post-university life goals. To help individual students answer these big questions, I setup meetings to specifically address their life goals. (For more details on this, see my article The Life Goals Exercise in this dossier.)

Another pillar of my teaching is interaction. Although teaching is not (yet) an evidence-based field, I strive to follow evidence-based practices in my teaching. All the evidence seems to indicate that interactive teaching is more effective than passive teaching. In the landmark meta-analysis [1], Freeman et al write:

> These results indicate that average examination scores improved by about 6% in active learning sections, and that students in classes with traditional lecturing were 1.5 times more likely to fail than were students in classes with active learning. ... The results raise questions about the continued use of traditional lecturing as a control in research studies, and support active learning as the preferred, empirically validated teaching practice in regular classrooms.

This last statement is a bold claim. In medicine, an established treatment will be abandoned and a new standard of care will be established if a new treatment is demonstrably safe and more effective. These findings about active learning have some education researchers asking to establish a new "standard of care" for calculus [3]. This led me to redesign MAT A29 to promote active in-class learning. (See the attached Teaching Enhancement Grant: MAT A29: Active and Applicable.)

Active learning, and interaction, however do not requite massive course redesigns. In 2021, I organized a workshop series on Inquiry Based Learning. I learned a number of easy to implement techniques for fostering active learning in any classroom at Dr. Su Dorée's talk *The Active Learning Pedagogy Sequence: A Model for Expanding the Use of Active Learning Structures in the College Mathematics Classrom*. This talk re-enforced concepts that I learned when I organized a reading group on the MAA Instructional Guide to Evidence-Based Instructional Practices in Undergraduate Mathematics [4].

One teaching technique that I use in every lecture is guided problem solving through discussion. I always ask students to discuss the content of lecture in small groups. It is a core practice of my teaching.

There are a number of good reasons to encourage discussion. These opportunities for discussion provide a natural break in the lecture. They allow students to verbalize their understanding of the material, and they also provide an opportunity for me to interact with individual students. This personalized guidance during problem solving and discussion is another means of formative assessment; I can see where my students are stuck.

To begin a discussion period, I model a problem for the students by way of an example. I highlight the pivotal steps in the solution and check that the class is comfortable with the process. A typical example from MAT A29, Calculus I for Life Sciences, might be computing the derivative of $\sin(x)$ from first principles. While explaining the example, I would give particular attention to: the limit definition of derivatives and the use of trigonometric limits. Once everyone is comfortable with the example, I write a topic for discussion on the board: *Discuss: Find the derivative of* $\cos(x)$ *from first principles.*

As a discussion period proceeds, I circulate around the room talking with individual groups. If one group is stuck, I may refer them to the previous example and ask: "Did you write out the definition of a derivative? Which trigonometric limit do you need this time?" These questions scaffold the discussion and keep things moving. As I walk around the room, I also talk with individual students, creating a space for them to ask about the lecture material. The student might not have followed the model example and may want an opportunity to check an idea before discussing it with others.

Group discussion can also be used to highlight subtleties and nuances in material that are difficult to convey in lecture, and which are best explored by students on their own. Often, I ask a tricky question in order to provoke discussion. For example, in MAT A22: Linear Algebra I for Mathematical Sciences, I might ask students to discuss the question: "Is $\mathbb{R}$ in $\mathbb{R}^2$"? This is a contentious question that is not precise enough to admit a definitive answer. By asking students to discuss the question with their peers, I give them space to build their own individual mental frameworks of the material and engage in their own mathematical explorations. Guided problem solving and group discussion keep the class interactive and engaging. It breaks up the lecture into more manageable chunks; it gives students time to reflect, and allows them to help each other process the material in the lecture.

My desire for an interaction is supplemented by enthusiasm for mathematics. And that enthusiasm in the classroom comes from a deep love of mathematics. I present talks to high school students and math clubs as a form of outreach on behalf of the university (for more details, see: Outreach to the

Community). I organize extra-curricular mathematical activities such as: Board Game Night and CMS Seminar. To put it bluntly, I am a huge geek. I really love mathematics. When an aperiodic monotile was announced by Smith et al [7], in March 2023, I got excited about it and collaborated with the UTSC MakerSpace to print a number of tiles for the department. I told all my classes about the discovery. I tell this story here because I want to point out the way that my passion for mathematics comes through in ways that promote inclusion and access. Students can see that mathematics is happening in the real world, and that its findings are accessible to them.

I strive to teach accessibly. When I selected an Open Educational Resource for MAT A29, I made sure to pick an online book that was fully AIRA compliant. Similarly, I follow best-practices when lecturing such as wearing a microphone, and using multiple modes of delivery. All my lecture notes and videos are available online immediately following class. I post hand-written lecture notes, for every class, to model effective note taking strategies. When posting homework sets, I even go so far as to post the original LaTeX source for students who use screen-readers or want to typeset their solutions. This open format makes my teaching ore transparent and accessible.

In addition to posting lecture notes, I use technology to supplement my teaching. I use my website to invite students to submit anonymous feedback at any time. This anonymous feedback has been useful in modifying my teaching to suit students needs. It allows students to report sensitive issues that they might not want to address in class or by e-mail. Every week, I remind students of this tool for interacting with me and giving feedback. (For more details about feedback, see: Anonymous Feedback and Exit Surveys.)

In conclusion, I am a kind, interactive, and accessible teacher. In every teaching choice that I make, I strive to be fair, honest, and kind. My students interact with each other and with me in a dynamic classroom environment. I use student feedback to make informed and evidence-based decisions about my teaching. My work is fueled by a deep love of teaching, and mathematics; a love that I strive to share with our students.

Additional Information on My Teaching Philosophy

In addition to this Teaching Philosophy, I've included my Advice for New Teachers in this dossier. This piece was written for a friend who was just about to start teaching. It is interesting to compare and contrast the two documents. In particular, I think that these documents give a clear picture of the day-to-day choices that I make while teaching. There is also an Advice for Students document, which gives encouraging advice to students about a variety of topics. I think that it showcases my deep concern for student well-being.

In the 2025-2026 academic year, I participated in the year-long Best Practices in Teaching and Learning course run by CTL[1]. I wrote a series of reflections for that course which expand upon my Teaching Philosophy. The reflections are more narrowly focussed than my general teaching philosophy. They adress topics such as universal design for learning, indigenous pedagogy, and articial intelligence.

[1] Center for Teaching and Learning. Many thanks to everyone at CTL!

Chapter 3

Teaching Responsibilities

3.1 List of Courses Taught

Scarborough 2025/2026
- **Coordinator** (2×) **MAT A29** – Calculus I for the Life Sciences
- **MAT A02** – The Magic of Numbers
- **Coordinator MAT B41** – Techniques of the Calculus of Several Variables I
- **MAT B42** – Techniques of the Calculus of Several Variables II
- **MAT D93** – Braid Theory and Applications
  ✯ This supervised study course resulted in the student presenting at a conference.
- **MAT D93** – Loop Braid Groups
  ✯ This supervised study course resulted in the student publishing an article.

Scarborough 2024/2025
- **Coordinator** (3×) **MAT A29** – Calculus I for the Life Sciences
- **Coordinator MAT A22** – Linear Algebra I for Mathematical Sciences
- **MAT B41** – Techniques of the Calculus of Several Variables I
- **MAT B42** – Techniques of the Calculus of Several Variables II

Scarborough 2023/2024
- **MAT D92** – Algorithmic Knot Theory
  ✯ This supervised study course resulted in the student submitting an article for publication.

Scarborough 2022/2023
- **Coordinator** (2×) **MAT A29** – Calculus I for the Life Sciences
- **Coordinator** (2×) **MAT A22** – Linear Algebra I for Mathematical Sciences
- **MAT B41** – Techniques of the Calculus of Several Variables I
- **MAT B42** – Techniques of the Calculus of Several Variables II

Scarborough 2021/2022
- **Coordinator MAT A29** – Calculus I for the Life Sciences
- **Coordinator MAT A22** – Linear Algebra I for Mathematical Sciences
- **MAT B42** – Techniques of the Calculus of Several Variables II

Mississauga 2020/2021
- **MAT 402** – Classical Geometry
- **MAT 133** – Calculus and Linear Algebra for Commerce
- **MAT 223** – Linear Algebra I
- **MAT 388** – Topics in Combinatorics
  ✯ This supervised study course resulted in the student publishing an article.

Mississauga 2018/2019
- **CSC 493** – Game Theory and the Probabilistic Method
- **MAT 133** – Calculus and Linear Algebra for Commerce
- **MAT 135** – Calculus
- **MAT 223** – Linear Algebra I
- **Coordinator MAT 232** – Calculus of Several Variables

Scarborough 2017/2018
- **Coordinator MAT B41** – Techniques of the Calculus of Several Variables I

Mississauga 2017/2018
- **MAT 133** – Calculus and Linear Algebra for Commerce
- **MAT 134** – Calculus for Life Sciences
- **MAT 223** – Linear Algebra I

Scarborough 2016/2017
- **MAT A31** – Calculus for the Mathematical Sciences
- **MAT A29** – Calculus for Life Science
- **MAT A33** – Calculus for Management II

Toronto 2016/2017
- **MAT 246** – Concepts in Abstract Mathematics

Scarborough 2015/2016
- **MAT A33** – Calculus for Management II

3.2 Course Development

In this section, I will discuss four course development projects that I've completed at UTSC. The first two concern projects undertaken in my first two years at UTSC. The third project is general and spans all my courses. The fourth and final project was undertaken in Winter of 2026. It is a recent example to meant highlight my current teaching philosophy.

MAT A29: Adopting an OER.

MAT A22: Communicating the essence of linear algebra.

Document Camera Notes: A style of preparing lectures notes to foster inclusion.

MAT A02: An active and embodied course design.

First, I'm going to highlight two courses: MAT A29, and MAT A22. These are courses that I undertook in my first few years at UTSC. These courses have a large role in our department. The former is a high enrollment service course, and the latter is critical for POSt[1] decisions. I begin this section by looking at the adoption of an OER in MAT A29, then move on to my re-write of MAT A22. The material about document camera notes at the end of this section is applicable to all the courses that I teach, but I illustrate it with course notes from MAT A22.

Before we move on to a description of each course development initiative, I want to highlight how these developments align with my teaching philosophy. The adoption of an OER in MAT A29 was intended to ease the financial cost of textbooks on students, and provide more accessible course materials. This is a kind and inclusive gesture. The re-write of MAT A22 focused on conveying the intuition and heuristics used in abstract algebra. This was an enthusiastic move meant to share my love of mathematics with students. Lastly, the re-write of MAT A02, The Magic of Numbers, is intended to show my commitment to teaching that highlights students' lived experience and action.

[1] "POSt" is an acronym for "Program of Study." Students need to achieve a certain average across a set of list of courses to be admitted to the program of their choice.

3.2.1 Adoption of OER in MAT A29

Textbooks are expensive, cumbersome, and rarely read. Looking at the introductory calculus textbooks in my office reveals that a typical book costs about $180 and weighs around 4lbs. After asking around informally, I got the sense that students in MAT A29 rarely, if ever, consulted the textbook. If they consult a textbook, they usually download a pirated copy of it. When I started to develop my version of MAT A29, I decided that I wanted to adopt an accessible, free, and online text.

There are many open educational resources (OER) for calculus available online. I decided to adopt the OpenStax Calculus Volume I for MAT A29. Later, in another iteration of the course, feedback from another faculty member led me to adopt parts of OpenStax Calculus II as well. (See Weeks 11 & 12 in the attached reading guide: MAT A29 Fall 2022 Course Schedule.) The book was written by Gilbert Strang (MIT) and Edwin Herman (University of Wisconsin-Stevens Point). Both authors are mathematical heavyweights and have won awards for their expository writing. OpenStax Calculus I has been used in thousands of introductory calculus classes. A major advantage of the text is that it is fully compliant with Accessible Rich Internet Applications (ARIA) standard. That is to say, it is a really accessible text.

And so, I re-wrote the course to align with the material in OpenStax Calculus I. To do this, I selected an appropriate sequence of exercises and wrote slides that aligned closely with the text. Because the book is licensed under a Creative Commons Attribution-NonCommercial-Sharealike 4.0 International License, I could freely use the graphics, text, and exercises in my course notes.

There is an interesting lesson about academic integrity here. How often do we model good citation and attribution practices for our students? If we're really being honest with ourselves, we often borrow course material from preexisting resources rather than creating everything *ex nihilo*. Adopting an OER gave me a chance to be frank about the source of the course material. This lesson about academic integrity and citation adds value to the course.

I think that students enjoyed being able to access the course book on their devices. It was easy to link directly from the syllabus to portions of the text. Each week's Quercus announcements are linked directly to the relevant section of the text. This made it simple to navigate the course structure. One can see a week-by-week schedule of readings for the course here: MAT A29 Fall 2022 Course Schedule

3.2.2 Re-Writing MAT A22: Linear Algebra I

When I started teaching MAT A22, I inherited the course from Dr. Sophie Chrysostomou, who had taught the course for a number of years. She shared her course resources with me, and I used them to begin re-writing the course. The aim of the re-write was two fold: I wanted to adopt a particular text Little and Damiano (1989), and to create a proof-centric introduction to linear algebra which leads to abstract algebra.

As I prepared the course, I wanted a complete set of notes which would put across my ideas about the subject. There is interesting educational research which suggests that students don't pick up on what instructors perceive as the "main idea" of a lecture, unless that idea is explicitly written down [2]. And so, throughout the course notes, I feature small remarks which talk about the intuition behind certain things. Here is a concrete example from MAT A22 Winter 2022.

Remark: The Importance of Uniqueness

Uniqueness is a major theme in algebra. It is customary to define "an" object with some property and then prove that it is "the" only object with that property. We use the axioms to show that there are unique objects with particular properties.

Example: A Simple Uniqueness Proof

A3. There exists an element $\mathbf{0} \in V$ such that:

$$\mathbf{0} \boxplus \mathbf{x} = \mathbf{x}$$

for all $\mathbf{x} \in V$. We call $\mathbf{0}$ the **zero vector** of V.

Show that the zero vector $\mathbf{0} \in V$ is unique.

During class, I would then show students the proof that $\mathbf{0} \in V$ is unique. What I want to point out using this example is that I explicitly write out the major ideas of a course. I want to evoke, for the students, the real ideas of algebra. If I just said this idea out loud, it would not land well with the students. They would memorize the proof and miss the message. I take the time to put intuition, ideas, and richness, into my lecture materials.

There is another message in this example. Building on my work in Investigating Mathematical Reading Comprehension, I designed the course to highly prioritize student reading. These remarks, interspersed throughout the course content, aim to build up student reading comprehension. One can read a lot of mathematics, and follow every line of the proofs, without understanding the author's intent. Explicitly emphasizing the purpose, or intent, of the proofs helps to build up a foundation of mathematical literacy.

3.2.3 Writing Document Camera Notes

When I prepare a set of course notes, I create what I call "document camera notes." I was first introduced to this style of teaching when I inherited MAT A22 from Dr. Sophie Chrysostomou. A set of document camera notes enables me to sit at the podium, annotate a set of notes, and distribute the annotated notes to the students after class.

There are several things that I like about working with document camera notes. First, there are (essentially) no surprises. The students have a copy of the notes in advance. They can look through them, track down the relevant material, and get ready for class. I know, in advance, that all the examples work out and fit in the allocated spaces. Second, I get to face the class while teaching. This isn't something that I consciously thought much about before I tried teaching in this style. When we're teaching at the blackboard, giving a traditional "chalk and talk" lecture, there is the unavoidable fact that we have to spend a lot of time facing away from our students. Of course, we can pivot back and forth, but we can't maintain a face-to-face connection. I like the constancy of connection that document camera notes provides. Third, the document camera style of teaching creates a permanent written record of what was said in class. Elsewhere in this dossier, I've noted that educational research [2] has found that students primarily retain what is written down, neglecting what we say out loud. This is in stark contrast to many instructors' pre-theoretical belief that the oral component of lecture is primary and the written component is secondary. I find that the document camera notes allow for capturing all the minutiae of both the written and spoken components of lecture. Fourth, document camera notes are more accessible than chalk-and-talk notes. I have a complete written record that I can share with students. The scanned PDFs of the completed notes can be zoomed, colour-inverted, or fed through text-to-speech software. This is much better than what we get from hastily hand-written notes taken during a blackboard lecture.

I want to point out that not everyone loves this style of teaching. I have, on occasion, gotten negative feedback about it. The extensive quote given in the Anonymous Feedback section contains the following insightful observation:

> `Also the whole printing out the notes from before and doing questions doesn't fit calculus. You can't expect to us to read 10 definitions and theorems every lecture and be able to solve complex questions regarding those in the assignments. ... So please get up and start teaching on the black board cause your lecture feels like I am practicing my writing skills.`

Reading this charitably, we can see that some students believe that typesetting notes crams too much material in to lecture. There is also an equity issue here: students who don't have access to a printer, or the tools to annotate a PDF in class, will have to hand write a huge amount of content. They feel that handwriting is taking too much time from lecture. It is true that document camera notes, or lectures given from slides more generally, do contain more material than could reasonably be handwritten on a chalkboard. With this feedback in mind, I've pared down my document camera notes somewhat. I hope it helps.

For examples of document camera notes, see these appendices:

- MAT A22 Winter 2023 Week 1 Blank
- MAT A22 Winter 2023 Week 1 Filled

3.2.4 Re-Writing MAT A02: The Magic of Numbers

MAT A02, The Magic of Numbers, is a non-calculus mathematics course meant to satify students' quantitative reasoning breadth requirement. As such, it is a course exclusion for all the first year calculus courses. The description in the Academic Calendar is rather broad.

> `A selection from the following topics: the number sense (neuroscience of numbers); numerical notation in different cultures; what is a number; Zeno's paradox; divisibility, the fascination of prime numbers; prime numbers and encryption; perspective in art and geometry; Kepler and platonic solids; golden mean, Fibonacci sequence; elementary probability.`

In January 2026, I taught this course for the first time. I decided to re-write the course with a heavy emphasis on active learning, in-class activities, and student's lived experience. These choices led to a very *magical* semester. On the first day of class, moments after class began, we had the following class in-class activity.

Activity: The Birthday Circle

Everyone arrange yourselves in a big circle according to your date of birth.

All hundred and fifty students shuffled out of the room and tried to make sense of the activity together. Notice how brief the instructions are. There is no scaffolding there. The students, with the help of the TAs, had to make some choices. Where should the calendar year begin? Should it go clockwise or counter-clockwise? Everyone had to talk to each other to find some order.

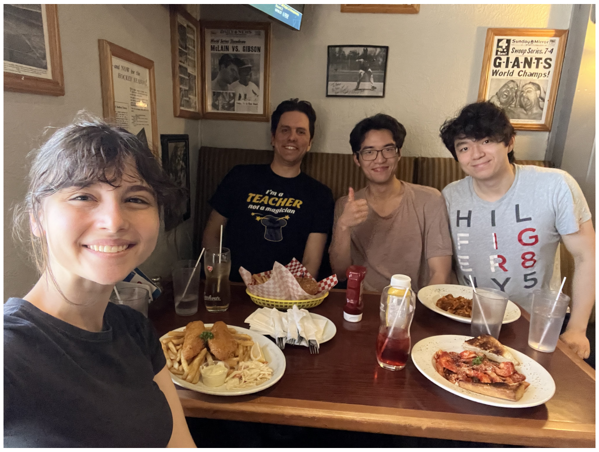
After about twenty minutes, once we formed a circle, I asked everyone to introduce themselves and share their birthday. In such a large group, there were bound to be some people with a common birthday. This observation is known as the birthday paradox, a topic we explored in the unit on probability later in the semester. Every time a pair of students shared a birthday, the whole class clapped. A pair of twins taking the class got an especially strong ovation. We started the class chatting with each other, making something together, and cheering for one another.

This classrom shot shows an early activity from the class. The students are responding to the following activity. There were a total of 21 languages! Knowing a bit about the students, and their language backgrounds, helped foster a good classroom community. I particular like this activity because it demonstrated that mathematics is a universal cultural phenomena. Every culture develops a method of counting. And, with the linguistic diversity in the room, we could talk about how different culture approach counting. In languages with isolating grammars, such as Vietnamese, counting is done by listing digits. In languages with more agglutinative grammars, such as Italian, there are special words for multiples of ten.

Activity: Your Language(s) Count!

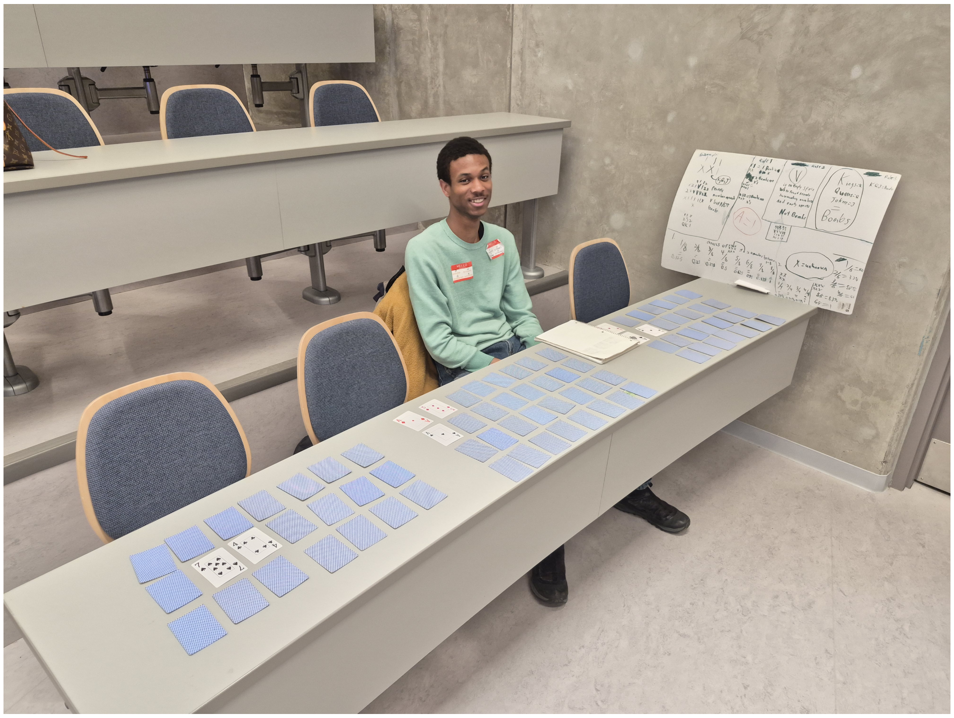
How do you count to twenty in your language? If you speak a language other than English, teach your neighbour. Is there a little counting rhyme? How many languages do we speak in this class?

In order to facilitate these full-scale in-class activities, I needed help from the TAs. This meant that I re-directed almost all TA grading hours to in-class activities. The TAs were present every class. This made the active learning more approachable for students because they could call on a peer for help. I also got very lucky with my TAs. They formed a little social clique and would even hang out together outside of the class to watch movies. The photo below shows our end-of-term lunch. (One the TAs, Leon Lee, wrote a letter for an earlier version of this dossier.)



The majority of the TA's grading work was replaced by multiple choice assessments that could be graded instantly. In general, I think that multiple choice assessments are suitable for calculation-heavy courses. I wouldn't use multiple choice exclusively in MAT A22 but it is suitable for a course like MAT A29 or A02. The non-multiple choice component of MAT A02 which was quite innovative, and focussed on students lived experiences, was the Math Festival. This was an end-of-term project where students would give a real-world activity connected, however tangentially, to mathematics. A few examples of student projects are included below.

Fibonacci poetry.



Minesweeper with playing cards.

Plant physiology.

Chapter 4

Evidence of Teaching Effectiveness

4.1 Narrative Contextualization of Course Evaluation Data

The following list of courses taught includes courses from all three campuses of the University of Toronto. There are several reasons for this. First, when I asked Academic HR for a spreadsheet of my course evaluation data, they gave me this list of courses. Second, there was a period of time when I was working at both UTM and UTSC. For a while, I alternated between all three campus working as a sessional. This lists includes courses taught during that time. For ease of reading, I placed a thick horizontal line separating the UTSC data from the UTM and UTSG data.

4.2 Summary of Course Evaluation Data

The following table is broken up in to two pages for ease of reading and typesetting. The first page lists the responses to Q1-Q5. The second pages lists the Institutional Composite Mean (average of Q1-Q6) and the response to Q6. Throughout the course evaluation date, cells are highlighted in yellow if they: were taught at UTSC, and my instructor score meets or exceeds the departmental average. The questions Q1-Q6 mean the following.

Q1. I found the course intellectually stimulating (Scale = 1 to 5)

Q2. The course provided me with a deeper understanding of the subject matter (Scale = 1 to 5)

Q3. The instructor created an atmosphere that was conducive to my learning (Scale = 1 to 5)

Q4. Course projects, assignments, tests, and/or exams improved my understanding of the course material (Scale = 1 to 5)

Q5. Course projects, assignments, tests and/or exams provided opportunity for me to demonstrate an understanding of the course material (Scale = 1 to 5)

Q6. Overall, the quality of my learning experience this course was (Scale = 1 to 5)

I want to draw your attention to **Q3** “The instructor created an atmosphere that was conducive to my learning.” Across all my courses, in almost all the terms, I’ve managed to create this atmosphere. I think it is a strong demonstration of my teaching philosophy that the atmosphere is consistently above average.

Course Code	Term	Invited	Response	Q 1		Q 2		Q 3		Q 4		Q 5	
				Instr	Dept	Instr	Dept	Instr	Dept	Instr	Dept	Instr	Dept
MAT133Y5	20181	124.0	37%	3.6	3.8	3.6	3.9	3.8	4.2	3.3	3.6	3.4	3.6
	20191	109.0	38%	3.6	3.8	3.4	3.8	4.0	4.0	3.4	3.6	3.4	3.6
	20201	56.0	25%	3.1	3.7	3.2	3.9	3.8	4.0	3.0	3.7	3.1	3.7
MAT134Y5	20181	158.0	64%	3.4	3.8	3.5	3.9	4.2	4.2	3.1	3.6	3.1	3.6
	20185	47.0	40%	2.9	3.8	3.3	4.2	3.8	4.5	3.3	4.0	3.3	3.9
MAT135Y5	20191	226.0	33%	4.2	3.8	4.2	3.8	4.7	4.0	4.1	3.6	4.2	3.6
MAT223H5	20179	89.0	47%	3.6	3.6	3.7	3.8	4.1	3.9	3.3	3.6	3.2	3.6
	20201	101.0	17%	3.6	3.7	3.7	3.8	3.8	3.7	3.6	3.6	3.5	3.6
	20209	109.0	21%	3.7	3.6	4.0	3.7	4.4	3.8	4.0	3.5	4.1	3.5
MAT232H5	20189	93.0	52%	3.9	3.7	4.0	3.8	4.6	3.9	4.1	3.7	4.1	3.7
	20195	82.0	33%	4.1	3.7	4.4	3.9	4.7	4.0	4.3	3.8	4.2	3.7
	20199	101.0	50%	3.6	3.5	3.8	3.6	4.3	3.6	3.5	3.5	3.4	3.5
	20201	102.0	25%	4.1	3.7	4.0	3.8	4.7	3.7	3.7	3.6	3.6	3.6
MAT402H5	20201	74.0	16%	4.1	4.5	4.3	4.5	4.8	4.6	4.1	4.4	4.2	4.3
	20209	55.0	38%	3.9	4.4	3.7	4.2	4.1	4.4	3.6	4.2	3.4	4.1
MATA02H3	20261*	139	28%	3.5	4.0	3.4	4.1	3.7	4.1	3.3	4.0	3.5	4.0
MATA22H3	20221	376	27%	3.8	3.9	3.5	4.0	4.0	3.9	3.5	3.9	3.4	3.8
	20231	424	20%	4.0	3.9	3.8	4.0	4.0	4.0	3.7	3.9	3.5	3.9
	20251	277	35%	3.9	4.0	3.7	4.0	3.9	4.0	3.7	4.0	3.7	4.0
MATA29H3	20219	514	34%	4.1	3.8	4.2	4.0	4.7	4.0	4.4	3.9	4.4	3.9
	20229	470	27%	3.8	3.9	4.0	4.0	4.5	4.1	4.1	4.0	4.3	4.0
	20249	439	43%	3.7	4.0	3.7	4.0	3.8	4.1	3.7	4.0	3.7	4.0
	20251	180	18%	3.9	4.0	4.0	4.0	4.3	4.0	3.8	4.0	3.8	4.0
	20259	457	37%	3.8	4.0	3.8	4.0	4.3	4.1	3.8	4.0	3.8	4.0
MATB41H3	20229	220	23%	4.3	3.9	4.3	4.0	4.6	4.1	4.4	4.0	4.2	4.0
	20249	225	27%	4.3	4.0	4.3	4.0	4.6	4.1	4.5	4.0	4.4	4.0
	20259	219	21%	4.0	4.0	4.0	4.0	4.1	4.1	3.9	4.0	3.8	4.0
MATB42H3	20221	224	27%	4.0	3.9	4.0	4.0	4.2	3.9	3.9	3.9	3.8	3.8
	20231	262	15%	3.7	3.9	3.7	4.0	4.0	4.0	3.9	3.9	3.9	3.9
	20251	281	36%	3.4	4.0	3.2	4.0	3.3	4.0	3.2	4.0	3.2	4.0
	20261*	287	20%	4.1	4.0	3.9	4.1	4.1	4.1	3.8	4.0	3.8	4.0

(Table continued on next page.)

Course Code	Term	Institutional Composite Mean (Avg of Q1-5)		Q6	
		Instr	Dept	Instr	Dept
MAT133Y5	20181	3.5	3.8	3.3	3.4
	20191	3.5	3.9	3.0	3.4
	20201	3.2	3.8	2.5	3.4
MAT134Y5	20181	3.5	3.8	2.9	3.4
	20185	3.3	4.0	2.8	3.8
MAT135Y5	20191	4.3	3.9	4.0	3.4
MAT223H5	20179	3.6	3.8	3.1	3.3
	20201	3.6	3.8	3.1	3.2
	20209	4.0	3.7	3.6	3.2
MAT232H5	20189	4.1	3.8	4.1	3.4
	20195	4.3	3.9	4.3	3.5
	20199	3.7	3.8	3.5	3.1
	20201	4.0	3.8	3.8	3.2
MAT402H5	20201	4.3	3.8	4.0	4.4
	20209	3.8	3.7	3.4	4.0
MATA02H3	20261*	3.5	4.0	3.3	3.7
MATA22H3	20221	3.6	3.9	2.8	3.5
	20231	3.8	3.9	3.2	3.5
	20251	3.8	4.0	3.3	3.6
MATA29H3	20219	4.4	3.9	4.2	3.6
	20229	4.1	4.0	3.8	3.6
	20249	3.7	4.0	3.3	3.7
	20251	4.0	4.0	3.5	3.6
	20259	3.9	4.0	3.5	3.6
MATB41H3	20229	4.4	4.0	4.1	3.6
	20249	4.4	4.0	4.4	3.7
	20259	4.0	4.0	3.6	3.6
MATB42H3	20221	4.0	3.9	3.7	3.5
	20231	3.8	3.9	3.5	3.5
	20251	3.3	4.0	2.7	3.6
	20261*	3.9	4.0	3.5	3.7

(End of table.)

Chapter 5

Leadership in And Professional Contributions to Teaching

In my experience, Leadership and Professional Development go hand-in-hand. Below, I described the two of the major ways in which my leadership in professional organizations has directly led to professional development. Two major roles, detailed under Professional Development Undertaken to Enhance Teaching were:

- Serving as the Chair of the IBL SIGMAA
- Serving on the Steering Committee of the Fields Insitute MathEd Forum

5.1 Publications/Presentations on Teaching and Learning

I love writing[1]. I do a tonne of writing, both formal and informal, about my teaching practice with an eye towards the scholarship of teaching and learning (SoTL). I think that it is important for teaching faculty to remain informed about SoTL. The only way to deeply pay attention to something is to participate in it. And so, I tend to do a fair bit of writing about my teaching practice. I've attached several representative pieces of scholarship to this dossier. I want to say a little bit about each one, to help contextualize them. There is a story behind each of these publications.

- Using Departmental Publications to Foster Student Creativity in Mathematics
  This paper, co-authored with Dr. Shahbazi, got me started on writing about my teaching practice. Prior to writing this paper, I thought that my life would be about writing research papers. I thought that it was my duty to write about minimal surfaces in seven-dimensional manifolds. And so, from 2017 to 2020, I was stuck in a terrible writer's block. This paper came about because The Journal of Humanistic Mathematics (JHM) put out a call asking for papers on creativity. Zohreh and I realized that, at a deep level, the $U(T)$-Mathazine (at the time it was called MSLC Magazine) was about fostering student creativity. However, very few departments are doing this sort of in-house publication. We decided to answer the call about creativity, document our work, and write this paper. I like this paper, because I have a lot of happy memories of that collaboration. Zohreh is an amazing colleague to work with.
- Investigating Mathematical Reading Comprehension This presentation got me interested in more statistical or quantitative work in the scholarship of teaching and learning. My natural inclination is to write very qualitative (or "soft") papers about education. Luckily, the co-authors on this presentation, my former colleagues Micheal Pawliuk, Alex Rennet, and Jaimal Thind of UTM,

[1] It helps that my wife, Megan Shaw, writes speculative fiction.

pushed for a full statistical treatment. We developed a lot of carefully written materials which scaffolded student reading in mathematics. It was a nice interplay of writing curriculum and assessing student gains. There is another paper or two yet to be written about this topic; the collaboration with UTM continues.

- The Life Goals Exercise: Context for Students' Big Questions This is the only single author paper that I'm including here. It is a very soft educational paper. I think that it speaks to my deep concern for students. As teaching faculty, we often are asked big questions for which we lack sufficient context. For example, a student might ask: "Should I drop out of school to pursue my interest in crypto-currency trading?" This paper is about a concrete exercise which helps provide context for these big questions. I've found the exercise helpful for myself and for students.

- Tangible Experiences that Broaden the Mathematical Horizon: Exploring the Dihedral Calculator This is the most "theoretical" education paper of the bunch. It is a deep dive in to educational philosophy. It discusses a manipulative (education-ese for a physical object) that I had students build in a geometry course to help explore symmetry groups. The paper then launches in to an exploration of various types of knowledge that students have of mathematics.

  I include this article because Ami Mamolo (OntarioTech) and I have been written a few papers about this topic. We also gave a presentation at the 2023 Winter CMS about this work. It shows that I'm multi-faceted. I write about things other than my year-to-year course assignments, and I participate in a broader educational context than the University of Toronto.

And lastly, I want to note that I like collaborating in writing with undergraduates. I think that it's important for undergraduate students to have experience publishing and attending conferences. Some of my work with students has resulted publications and attendance at conferences. In 2021, I co-authored a paper with a student, Brian Zhengyu Li: *In Tetracycles: a SET Deck Magic Trick*. This paper landed in Math Horizons, and Brian went to two conferences: The Mathematics of Various Entertaining Subjects (MOVES), and Gathering for Gardner. These are the two foremost conferences dedicated to recreational mathematics. The program of MOVES 2022 is attached as an appendix.

Finally, I've added a recent paper with an undergraduate: Of Loops, Braids, and String Figures (2026). A letter by Vanessa is also included in this dossier. This paper was an absolute whirl wind to write. Vanessa approached me about the possibility of doing a project course in Winter 2026. We started the course with the intent of writing about loop braid groups. We got the suggestion to submit a paper to Bridges. We got this suggestion in the Week 2 of the semester. The deadline for submissions was in Week 7 of the semester. We decided to go ahead and put together the complete paper in five weeks.

There are also a several less formal pieces of writing attached to this dossier. The following documents of advice originated as posts on my website. I think that they pair together well, and give a comprehensive view of my teaching.

- Advice for New Teachers: My response to a friend who reached out asking for advice before he began teaching for the first time. This has the clearest statement of the core of my teaching philosophy: "Be kind."

- Advice for Students: A short write-up of advice that I give to students across a whole range of courses. It features advice about reading, solving problem, and maintaining a healthy school-life balance. I think it shows the practical, student-focused, aspect of my work.

5.2 Innovations in Teaching and Learning

In this section, I describe various innovations in teaching and learning that I've implemented during my time at UTSC. I going to start at the very beginning with a sub-section about Kind and Welcoming Syllabi. The reason I'm starting here is that it exemplifies my core teaching philosophy "Be kind."

Later, I move on to Anonymous Feedback, which ties in to my commitment to accountability and listening to student feedback.

5.2.1 Kind and Welcoming Syllabi

Syllabi are incredibly important. A syllabus is the agreement that we make with our students before a course begins. It is also the first point of contact that we have with our students, and remains an important reference document through out a course. And so, I am very careful about preparing my syllabi. I do several things in my syllabi to make my courses more welcoming on first contact. One can find full course outlines with reading guides as appendices to this dossier:

- MAT A29 Fall 2022 Syllabus
- MAT A29 Fall 2022 Course Schedule
- MAT A22 Winter 2023 Syllabus
- MAT A22 Winter 2023 Course Schedule.

To save a lot of page turning or hyperlink navigating, I am going to walk you through the syllabi in this dossier and highlight specific features of them. As I do so, I'll extensively quote from the example syllabi in question. The features that I'm about to discuss are:

- Pronouns and preferred name
- Professor's Message
- FAQs
- Communication Policy with Example E-Mail
- Graphic Syllabi
- The Course Content Egg.

Pronouns and Preferred Name UTSC is a campus built on inclusive excellence. We value and support students with all gender identities and from all ethnic backgrounds. In CMS, many of our students are international. By listing my pronouns and preferred English name, I normalize these practices for my students. I also have my TAs list their pronouns and preferred name on our Quercus page.

> ✯ Parker Glynn-Adey (he/him)
> *Lectures*: LEC 02 and LEC 03
> *Preferred Names*: "Parker" or "Professor Parker"
> *E-Mail*: `parker.glynn.adey@utoronto.ca`
> *Website*: `https://pgadey.ca/`
> *Office*: IC 344

Professor's Message The syllabus is the first point of contact that students have with my courses. And so, I make an effort to be welcoming and encouraging from the very first moment that students interact with the course. All of my syllabi include a brief "Professor's Message" which position the course as friendly and welcoming.

> Hi! I'm Parker Glynn-Adey, the professor for MAT A29. This is one of my favourite courses at the University of Toronto. It was the first course that I ever taught, and I'm glad to be teaching it again.
>
> I like it so much because the students in this course are awesome. You want to be doctors, pharmacists, nurses, mental health workers, and all sorts of people in the life sciences. And that's awesome! I want to help you succeed in that. If I can get you started doing a bit of math, and you can use it on your mission in the life sciences, then I'll be tremendously happy.
>
> I've tried to design this course so that you can succeed. I'm hoping that there are no surprises in the course, and that it does not stress you out too much. If you're feeling unsure about your ability to succeed, or you need someone to talk to about the course, then please come to me. I'd be glad to help. My goal is to help you succeed, to go on to finish your program in life science, and to support you on your journey.
>
> —MAT A29 Syllabus Winter 2022

There are a few things that I want to point out in this message. First, it establishes my position on the course: I like teaching it. Second, describes what I think about the students: they are awesome. Notice that it frames the their awesomeness in terms of their life goals: they want to be doctors, nurses, and mental health works and therefore they are awesome. This is another place where my teaching involves an explicit link to life goals. (For more details on this, see my article The Life Goals Exercise in this dossier.) Third, the final paragraph of the message establish how I want to relate to my students: I want to help them succeed. I want to send the message that I am absolutely and unambiguously in support of my students passing and succeeding.

FAQs Syllabi are complicated documents. One way to think of a syllabus is as the terms and conditions which apply to the course. Alternatively, if a course is a board game, then the syllabi is its rule set. (Some students from Board Game Night really like this metaphor.) To help navigate the rules, terms, and conditions, of my courses, I provide lengthy FAQs (Frequently Asked Questions) with my syllabi.

In a syllabus, one could state "All work must be submitted through Crowdmark". However, that statement admits a number of corner-cases: What happens if it is uploaded, but not submitted? What happens if the work is uploaded and submitted, but illegible? What happens if the work is upload and submitted, but in the wrong order? The purpose of the FAQ section is to answer all these questions in a definitive manner. These FAQs are useful to students, because they often answer students' questions. They're also useful to me, as they provide a place that I can direct students. Here is a sample FAQ from MAT A22 Winter 2022.

FAQ: Errors While Submitting Homeworks

What happens if I submit my work late?
You will receive a mark of zero. Crowdmark will not show an error message, or notification.

What happens if I upload all my work and forget to hit submit?
Your work will not be graded. You will receive a zero for the homework.

Can I send you a screenshot to show that I completed the work on time?
Screenshots sent to us to prove that it was completed before the zero date will not be accepted.

What happens if I e-mail my homework instead of submitting it to Crowdmark?
The instructional team of professors and TAs will not accept work sent by e-mail. All work must be submitted through Crowdmark.

What happens if I slip my paper under the professor's door?
Your work will not be accepted. All work must be submitted through Crowdmark.

What happens if I upload the files in the wrong order?
The instructional team will not correct your file order. You must check that you uploaded everything in the correct order.

What happens if I upload all the questions to one question's slot?
Your work will not be graded. The TAs will not search Crowdmark for your work.

What happens if the TAs cannot read my work?
The grader will flag your work as illegible, it will not be graded, and you can request a regrade.

What happens if I don't submit some of my work by accident?
The instructional team will not accept additional work, unless it is entered via Crowdmark before the due date.

Communication Policy Communication with students is essential to my teaching. I want to be responsive and approachable, even in my online communication. However, e-mail is a difficult medium for students. Some students worry about appearing overly informal, and write long and profusely apologetic e-mails. Other students have the opposite problem, and write e-mails which are so informal as to be unreadable. In my syllabi, I set the tone for e-mail communication in a communication policy, which serves as a model for my students. Here is the policy from MAT A29 Fall 2022.

Communication Policy

All e-mail must be from an official University of Toronto account. You must include [MAT A29] in the subject line, or your e-mail might get lost. Please include your name and student number in every e-mail that you send. Be sure to include the precise question, and the problem or difficulty.

```
To: parker.glynn.adey@utoronto.ca
From: leonhard.euler@utoronto.ca
Subject: [MAT A29] What is a derivative?

Hi! I am Leonhard Euler (12932188) from MAT A29.
I need help with this question: Find the derivative
of f(x)=x^2. My problem is this: I don't know what the
word ``derivative'' means.

Thanks!
```

Parker checks his e-mail between 09:00 and 16:00 during the work week. Parker will respond to all email inquiries within two business days. He will respond to emails sent after Friday at 16:00 by 16:00 on the following Monday. If you do not get a quick response, please follow up with another e-mail. Don't worry about contacting Parker. He is happy to help!

I want to point out a couple things about this example. First, it sets a particular tone of communication. I'm fine with students greeting me casually, writing briefly, and e-mailing me anything. Second, it subtly models a form of question posing. Notice the format of the example e-mail.

```
 I need help with this question: ___________.
My problem is this: ___________.
```

I've found that students adopt this model, and it leads to much clearer communication in my courses. And lastly, I end on a positive note: "Don't worry about contacting Parker. He is happy to help!" My personal website has a similar encouraging note on it about e-mails.

Feel free to contact me!
I am always glad to receive comments, questions, or suggestions via e-mail.

parker dot glynn dot adey at utoronto dot ca

Advice for Students in My Courses

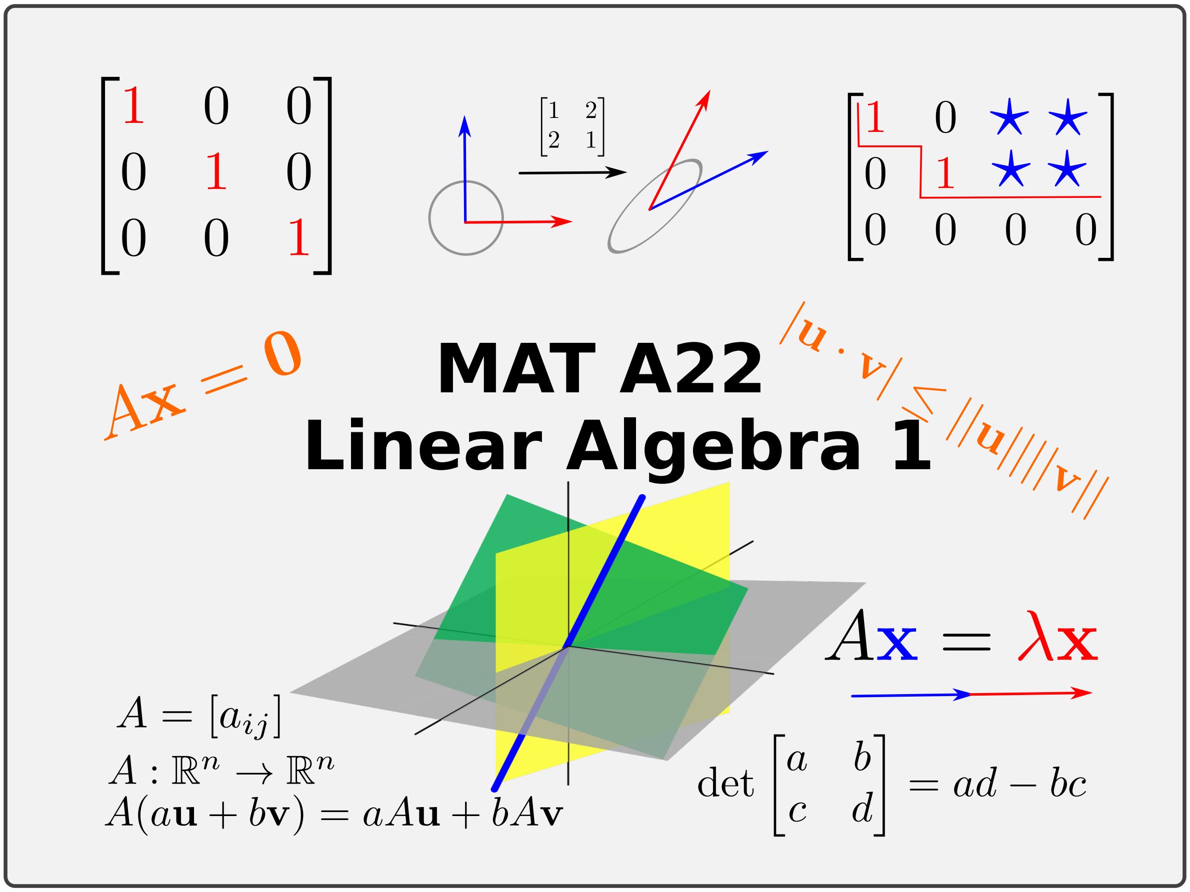
- Don't worry about e-mailing me. I'm not an evil bear. I won't get angry if you e-mail me. Answering student e-mails is a part of my job. In fact, I enjoy getting e-mails from students.
- When writing to me about a course, please include:
  - Your name and student number
  - The course code e.g. MAT A29 or MAT B42
- When asking about a specific question from a textbook, or assignment, please write the question in the body of your e-mail. If you're not able to write out the question, take a photo or attach a PDF.

Graphic Syllabus During the COVID-19 Pandemic, when we all transitioned to online teaching, I started to teach with slide decks. I found it helpful to have an initial slide that represented the course graphically; I called this slide the graphic syllabus[2] and would start every slide deck with it. As students arrived in the virtual class, it would be screen-shared as I played welcoming music.

Throughout the course, I would refer to various things on the graphic syllabus to a narrative about the course. Ideally, at the beginning of the semester the graphic would be visually appealing but meaningless. And, at the end of the semester, it would be a rich and meaningful mosaic of ideas relevant to the course.

Take a look at the following graphic syllabus from MAT A22 in Winter of 2023. What do you notice? Are there any major themes of linear algebra that are missing?

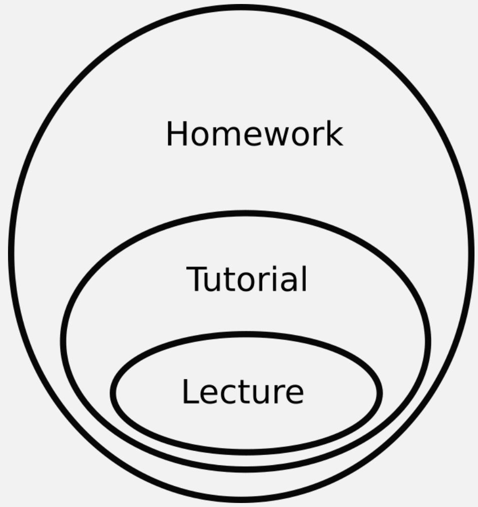
[2]I am using the term "graphic syllabus" in a somewhat unusual way here. It turns out that it has a different meaning in the education literature. Usually, the term means a syllabus in the traditional sense but transposed in to the medium of infographics.



The Course Content Egg Another graphic which I include in all my syllabi arose from student feedback. During MAT B42 of Winter 2022, I got the following anonymous feedback from a student:

```
During tutorials, TAs wonder as why material on assignment has not been
thought. My TA for instance starts smiling and mentions that he has to start
teaching us instead of solving questions. We are not even given tutorial
solutions after so long to help us solve the assignment. For instance, Ta
checked as if the material has been thought. After checking , half of the
lecture note pages were covered with a yellow sticker and smiley face. I do
not have enough lecture notes material to be able to meet expectations on
assignment and term tests.
```

After reflecting on this feedback, I realized that I had a different mental model of the course structure than my students. In my mental model of teaching, the lecture is an introduction to the material which is then elaborated upon in tutorial. The material on the assignments then expands upon the material from both lecture and tutorial, and can assess students on their ability to solve novel problems relevant to the course. To summarize this, I've added what I call the "Course Content Egg" to my syllabi. Here is an example from the MAT A22 Winter 2023 syllabus.



In this course, we will have the following structure to our lectures, tutorials, and homeworks.

$$\text{Lecture} \subsetneq \text{Tutorial} \subsetneq \text{Homework}$$

That is to say, there will be material covered in the tutorials is not covered in lecture. Similarly, there will be material covered in the homework which is not covered in tutorial or lecture. This is an intentional design choice. We want to give you a broad exposure to linear algebra.

After sharing this Course Content Egg with a colleague, they said: "That's evil!" This allocation of material is controversial. And it is rightly so; one could go too far with this distribution of material and really abandon their students to learn a lot of material on their own without support. But that would not be kind. I think that it would be helpful for readers of this dossier to see a worked example, illustrating where various tasks would be placed in this framework.

Let's think about MAT A22. The course notes for the first week are included in this dossier. In the first week, we introduce the abstract axioms for a real vector space $(V, \boxplus, \boxdot)$. An example from lecture is the following.

Definition: The Vector Space of Functions $\mathbf{F}(\mathbb{R})$

The vector space of **real valued functions** is given as follows:

$$\mathbf{F}(\mathbb{R}) = \{f : \mathbb{R} \to \mathbb{R}\}$$

The addition on $\mathbf{F}(\mathbb{R})$ is:

$$(f + g)(x) = f(x) + g(x)$$

The scalar multiplication on $\mathbf{F}(\mathbb{R})$ is:

$$(cf)(x) = cf(x)$$

After introducing this definition, I go on to prove that it is indeed a vector space. This is introduces the concept of proving that something is a vector space. One checks that the given set and given operations satisfy the axioms of a vector space. Now, let's look at how tutorial builds on this lecture material. In the first tutorial, students were then faced with the following task.

> In abstract algebra, we study ways of combining algebraic structures together. One operation common operation is taking "products" of algebraic structures. In the next question, you'll define a product for real vector spaces.

Q3. Suppose that $(V, \boxplus, \boxdot)$ and $(W, \oplus, \odot)$ are real vector spaces.

1. Define addition $+$ and scaling $\cdot$ on the following set:

$$V \times W = \{(\mathbf{v}, \mathbf{w}) : \mathbf{v} \in V \text{ and } \mathbf{w} \in W\}$$

2. Prove that $(V \times W, +, \cdot)$ is a vector space.

This tasks builds on the material from lecture. However, it feels very different from lecture. In lecture, we only though about one vector space at a time. This tutorial problem introduces the possibility that the class of vector spaces itself supports algebraic operations. Notice, however, that while this might feel novel or new, it is just another instance of applying the definition of a vector space. To solve Q3.2, one carefully checks the axioms of a vector space using the two structures $(V, \boxplus, \boxdot)$ and $(W, \oplus, \odot)$. Now we, turn our attention to a homework question from the first homework.

Q2. Consider $\mathbb{R}^+ = \{x : x > 0\}$. We define the following operations:

$$x \boxplus y = xy \qquad c \boxdot x = x^c \text{ for } c \in \mathbb{R}$$

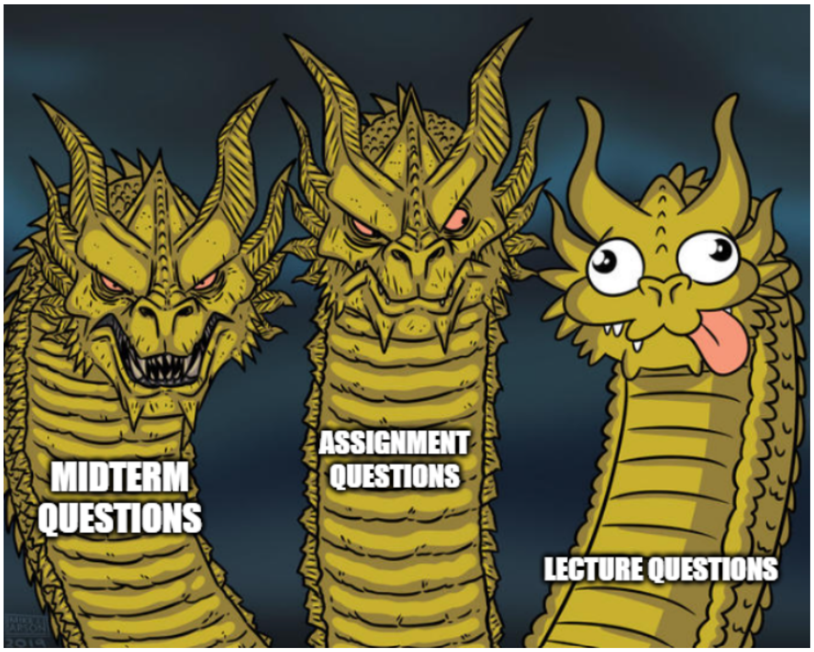
Show that $V = (\mathbb{R}^+, \boxplus, \boxdot)$ is a real vector space.

This exercise feels rather distinct from the material in lecture and tutorial. In linear algebra, it is very rare that we get to multiply things and take powers. One does not think about the set $\mathbb{R}^+$. So, this is decidedly outside the lecture and tutorial parts of the egg. But is it? This is just another instance of checking whether the given operations satisfy the definition of a vector space. If one calmly checks the axioms of a vector space, then this $(\mathbb{R}^+, \boxplus, \boxdot)$ too turns out to be a vector space.

I include this worked example of material from lecture, tutorial, and homework, to highlight that the distribution of material between lecture, tutorial, and homework, is cumulative but not evil. I am not blindly throwing novel material at students. When I design my courses, I build in common threads. These tasks all circle around a common core: checking that something is a vector space. One could say that the layers of the egg are distinct, but quite thin. The steps that I'm asking the students to take from one layer to the next are small.

Now, I want to consider things from a student perspective. A student created the following image showing three dragons labelled "midterm questions, assignment questions, and lecture questions". It was posted to the MAT A22 Piazza anonymously, and gives a sense of the student perception of the Course Content Egg. I think that it rightly pokes fun at the way of lecture questions feel compared to midterms and homeworks from a student perspective.

Of course, lecture questions would feel comically easier than homework and midterm questions. The contexts are totally different. In the former, an expert is carefully explaining a solution to the student. In the latter, time pressure and nerves are hampering the student's ability to think through something independently. I like to share this image with students, to help explain this dissimilarity.



5.2.2 Exit Surveys

One tool that I use to ensure my teaching is accountable to students' needs is an exit survey. A ticket-out-the-door survey is anonymous exit survey that students write before they leave lecture. These anonymous hand-written surveys take only a few minutes of class time, but they provide a wealth of information about the students' experience of a course.

As a term progresses, I survey students about a variety of topics. On the first day of a first year course, I always ask students two questions: "Where did you grow up?" and "What do you want to do when you're finished university?" These soft questions habituate students to providing anonymous feedback. I am always amazed to learn who my students are and what they want from their education. Through these surveys, my students begin to understand that I care about them at a personal level.

As the weeks go by, I use exit surveys in class to ask additional questions: "How are your courses progressing?" or "How are you managing the work load?" These questions help them reflect on their study habits and provide an opportunity to talk about time management and learning outcomes. Eventually, I ask my students about their experience of my course: "How do you like this course? What should change?" These questions provide the most valuable student feedback. The answers that I get help me to take action and adapt my teaching respond to student needs. Often, the details I learn are totally unexpected: the audio system makes a buzzing noise and needs to be turned off, for example.

Once students are thoroughly familiar with ticket-out-the-door surveys, and trust that I will take their responses seriously, I begin to use these surveys as a tool for formative assessment. For example, I might announce that the weekly survey is going to be an exercise or short calculation: *Find the derivative of* $\sin^{-1}(x^2)$. Sometimes the responses are sometimes short and direct. A response such as"I have no idea" is very informative; because it tells me that I need to re-iterate techniques for finding the derivatives of the inverse trigonometric functions immediately and in a new way.

Ticket-out-the-door surveys help me to: build familiarity and trust with my students, stay accountable to their needs, assess their grasp of the course content, and welcome them to university.

Back in 2018, I gave a talk about exit surveys at Scholarship of Teaching and Learning in Higher Education (STLHE). *Start, Stop, Continue and Ticket Out of the Door: Collecting and using student feedback to improve teaching in a large first-year math class.*

5.2.3 Anonymous Feedback

I think that student feedback is essential to delivering high quality instruction. This is why I use exit surveys and constantly talk with my students. But there are certain things that students don't want to share for fear of repercussions. For example, negative feedback about teaching assistants, suspicions of academic integrity violations, and other private concerns. Even though exit surveys are anonymous, students still hesitate to put serious concerns on them.

And so, I implemented an anonymous feedback form for my teaching. This was inspired by our colleague Andrew Petersen at UTM, who has a simple feedback form here. My anonymous feedback form is listed on my website. The following in an extract from a syllabus explaining the purpose of anonymous feedback.

> All feedback is welcome in this course. You can submit anonymous feedback here:
>
> `https://pgadey.ca/feedback/`
>
> You may use the form to comment on lecture, ask questions about the course, or give me tips. You do not need to enter your name or email address unless you want a private response from Parker. Note that your anonymous feedback may be discussed (and answered) in lecture, or on Quercus.

Let's have a look at how I use this concretely to enhance my teaching. The following comment about MAT A22 came in via Anonymous Feedback:

```
I wish there were sample answers for the recommended exercises (particularly
proofs), I feel that sometimes when I'm stuck I need to look at a solution and
work backwards to better understand the reasoning behind it. I have never done
proofs before this course, and I feel that the answers to some of my questions
on Piazza are not helpful. If you could release sample solutions/templates for
the recommended exercises that would be great.
```

This resulted in the following Quercus announcement. I think it addresses the student's concerns.

> **Proof Practice**
>
> The student solution manual for LAWA (which is linked on the Syllabus) contains some sample proofs. Check the answers for "true/false" questions. These are the only solutions that we can post for the recommended exercises. Another source of examples of proofs is the textbook itself. You can think of every proof in the textbook as an exercise. Read over each chapter, see how the book proves statements, and look for patterns among different proofs. Finally, if you feel that the answers on Piazza are not helpful, then say so! If you ask post a follow-up or ask for further clarification, then other students and the instructional team will help you and post more details.
>
> Summary:
>
> - Check out the student solution manual Download student solution manual .
> - Think of the proofs in the book as practice questions with solutions.
> - Ask follow-up questions on Piazza.

One thing I would like to make clear about Anonymous Feedback is that it attracts negative feedback. It is impossible to please everyone all the time. Some of my courses are quite difficult for students. And everyone is much more vicious when they're anonymous. Having an anonymous feedback form is somewhat like inviting all the negative comments from course evaluations to trickle in constantly. Consider the following example of anonymous feedback from MAT B41.

```
 Your teaching style deters me from lectures. I am sorry but since A22, I just
don't understand they way you "teach" and I know I am not the only one cause
I talked to a lot of people who stopped going to lectures (both my classmates
this year and upper years). Before you assume we are "average" students, we
aren't. We have pretty good grades. Anyways my point here is that all you do
is just load a bunch of information on us and expect us to digest it. There's
no actual understanding going on, just a bunch of memorization of definitions
and theorems. Also the whole printing out the notes from before and doing
questions doesn't fit calculus. You can't expect to us to read 10 definitions
and theorems every lecture and be able to solve complex questions regarding
those in the assignments. 99% of the time, your tutorial questions are wayyyy
ahead of the lectures. Even the TAs said so. So please get up and start
teaching on the black board cause your lecture feels like I am practicing my
writing skills.
```

Sometimes, a really cutting comment like the one above will arrive anonymously and throw off my day. I think that opening up my ego to this risk is worth it, as long as someone benefits from reporting anonymously. And so, I keep the Anonymous Feedback form online.

I'll close out this sub-section with a nice bit of feedback. Sometimes, students really appreciate what we do for them. For example, I also use the Anonymous Feedback form to solicit honest feedback about midterms and assessments. Here is another bit of feedback from that same MAT B41.

```
The test went way better than expected because it was almost exactly like
the practice test we were given and I put a lot of focus on that. Although
I may not have done that great because of some possible small mistakes on my
questions I think I was able to write something down for every question at
least because I had the general idea of how to answer these questions. THANK
YOU TO THE PROFESSOR FOR HELPING ME BE THIS PREPARED FOR THE MIDTERM.
```

5.2.4 The Office Camera

In the 2022-2023 academic year, students returned to campus and we finally had students coming in to our offices to do mathematics in person. To keep an accessible record of my conversations with students during office hours, I mounted a camera in my office to take photos of the whiteboard and wrote some code to post the photos online.

- For more details about how the camera works and the code which runs it: https://pgadey.ca/office-camera/about/
- The photos are available here: https://pgadey.ca/office-camera/

5.3 Extracurricular Innovations

Learning happens inside and *outside* the classroom. I think that creating interesting and engaging extra-curricular experiences for students is just as important as creating welcoming classroom experiences. There are various reasons for this. First, our students are people. They want to meet socially, engage with each other, and have a good time. If we're meeting them as whole people, and engaging them fully, then we need to create extra-curricular spaces. Second, real mathematics is a communal project, and it tends to happen in seminar rooms, hallways, and coffee shops, between people engaging as equals. To give students an experience of mathematics per se, we have to (occasionally) get out of classrooms. Third, assessments and the classroom experience change the game of mathematics. Just as Poker is a different game when it is played for money in a casino instead of casually at home, learning is a different experience when it is done in a classroom with the aim of maximizing a grade point average. For specific projects related to extra-curricular student engagement see:

- $U(T)$-Mathazine
- Board Game Night
- CMS Seminar

5.3.1 $U(t)$-Mathazine

My work on creating spaces for extra-curricular mathematics started when I began to help edit the $U(t)$-Mathazine (formerly, MSLC Magazine) in 2016. The Mathazine is an interesting venue. It is place for fostering student creativity in mathematics, statistics, and computer science (Shahbazi, Z., & Glynn-Adey, P. (2020)). The Mathazine give students an opportunity to experience the full peer-review cycle: submission, feedback from reviewers, revisions, and print. This is a rare opportunity for undergraduates to experience something that prefigures scholarship in mathematics.

As the magazine has grown, I've taken on more of a mentorship role. Now, I advise students through the process of writing their first articles. I meet with students regularly to discuss their contributions, helping them narrow their focus, and fine-tune their writing. I also have a few articles in the $U(T)$-Mathazine. The article marked $\star$ was co-authored with a student.

- Glynn-Adey, P., (2023). Random Permutations and the 100 Lockers Puzzle. *$U(t)$-Mathazine*, 6(1), 19-22.
- Glynn-Adey, P., (2022). Ringing the Changes: A Dance of Algebra and Geometry. *$U(t)$-Mathazine*, 5(1), 5-9.
- Glynn-Adey, P., (2021). The Mathematics of Juggling. *$U(t)$-Mathazine*, 4(1), 9-11.
- Glynn-Adey, P., (2020). The Rochester Mathematics Olympiad. *MSLC Magazine*, 5(1), 8.
- Glynn-Adey, P., & ⋆Tsui, H. (2019). String Figures and Thomas F. Storer's Calculus. *MSLC Magazine*, 5(1), 14-17.

5.3.2 Undergraduate Seminar

Another space for extra-curricular mathematics is the Undergraduate Seminar. I have hosted this Seminar in various forms since I was a sessional at UTSC back in 2017. In the last few years, Drs. Lisa Jeffrey and Albert Lai have helped co-host the Seminar. It is a place where students give talks to other students. Initially, I founded Seminar to give NSERC URSA students a place to prepare final presentations, and get speakers ready to present at the Canadian Undergraduate Mathematics Conference (CUMC). After several years of running it exclusively in the summer, we moved to running it year round.

The Pandemic led to a novel shift in the Seminar. We started to recruit speakers and audience members from outside UTSC. It became possible to invite speakers from all over the world. By listing the Seminar on ResearchSeminars.org, we were able to attract an audience from all over the world. This led to hosting a hybrid seminar when we returned to in-person instruction on campus.

There are two reasons why the seminar is important. First, it promoted student engagement with mathematics outside of the classroom. It serves as a hub for people who want to know more about mathematics. The regular attenders at Seminar develop close relationships with me, and each other. We're a little core of math geeks who like to meet and listen to interesting mathematics. And second, the Seminar exposes students to material that they wouldn't otherwise see in the undergraduate curriculum.

To give a sense of the scope of the Seminar, two schedules are attached to this dossier as appendices.

- CMS Undergraduate Seminar Fall 2022
- CMS Undergraduate Seminar Winter 2023

A full list of all past seminars, together with slides and recordings, is available on my website.

`https://pgadey.ca/seminar/`

In this updated dossier, I've included some additional material about Seminar. The following article from the FYMSiC newsletter expresses a new felt sense that Seminar is an academic "third space" for students.

> Adey, P. (2026) Undergraduate Seminar: A Place to Sow Seeds. FYMSiC Newsletter: Issue 18, First Year Math and Stats in Canada, February 2026, (link)

5.3.3 Problem Solving Group

Another extra-curricular initiative that I started at UTSC is the Problem Solving Group. A number of students in MAT A22 during Winter 2023 asked if we could start a Putnam Training Group. And so, by popular demand (!), I set about organizing a Problem Solving Group for Summer 2023.

`https://pgadey.ca/problem-solving/`

The group was somewhat short-lived, but I look forward to revitalizing it in the future.

5.3.4 Board Game Night

And lastly, I host a monthly board game night. To position this as relevant to teaching, I will tell a little anecdote. Usually, at Board Game Night, I spend most of my time directing students to games. There are small groups forming to play particular games, and they need players. So, I walk around, introducing people to each other and getting games going. This means that I rarely actually sit down and play games during board game night. However, when I do get a chance to sit down, I usually sit with students who are waiting for a game. One time, I sat down with some students and we got chatting about math. The conversation became more and more lively. People talked with emphatic hand gestures and asked things like: “What are finite dimensional vector spaces *really* about?!” As this was going on, a student came up and asked me, “Professor Parker, what game are you playing here?” And I said: “I’m playing mathematics. My favourite game.”

Here is another anecdote that conveys the feel of Board Game Night. A student from a country where playing cards are quite rare asked if I could help with their MAT A67 assignment which involved counting hands of cards. I ran up to my office, grabbed a deck of cards, and taught them how to play Poker. We then solved their problems and they learned something both mathematical and social.

And this is the purpose of Board Game Night. It’s about creating an atmosphere where people feel free to let loose a bit. It’s a third space beyond lecture and office hours, where professors and students can mingle. During board game night, students meet professors as normal(ish) people. We play games together, talk about mathematics, and be casual. It is a magical space. A poster for this event is included as an appendix: CMS Board Game Night.

5.4 Outreach to The Community

5.4.1 Visits to Scarborough Highschools

Our campus’s mission is to promote inclusive excellence. A foundational imperative of that mission is to “strengthen, grow, and sustain local networks”. I think that it’s important to know the local community. Who comes to UTSC? Where did they do highschool? What’s it like in Scarborough?

Drs. Anya Tafliovich and Francisco Estrada established a formal partnership with the Toronto District Schoolboard. This resulted in a memorandum of understanding which allowed faculty to visit highschools, and schools to visit UTSC. During my research leave 2023/24, I visited highschools using this partnership and acted on our mission’s imperative. I visited several local highschools and got to meet their students and teachers. It was a wonderful experience for everyone involved.

A Presentation at Albert Campbell Collegiate February 21st 2024
(The student, center in black hoodie, holding the three erasers is about to juggle them.)

Feedback from School Visits

As a result of this effort, there are now some highschools which enthusiastically want us to visit again. We established a connection. Here is some feedback from schools that I visited. They want CMS to visit them again.

```
 Dear Dr. Glynn-Adey
Thank you so much for your amazing presentation about the mathematics of
juggling. The grade 10 math class at David and Mary Thomson C. I. thoroughly
enjoyed seeing your impressive juggling skills and learning about different
juggling patterns. It was an interesting and engaging experience and we hope
you will come back again!
Ms. C. Mok
(Mathematics teacher, David and Mary Thomson C. I. )
```

```
 Hi Dr. Glynn-Adey,
Thank you so much for coming through! It was personally very fun for me and
I asked the students to offer some feedback, and the words I heard were:
Engaging, interesting, entertaining, friendly, passionate,simple to use,
taught in a digestible way and easy to understand. For a Calculus Professor
to do all that is extremely impressive, kudos! Our classroom feedback is:
It was such a thrill hosting a talented and communicative professor like Dr.
Glynn who took a math lesson and made it so fun that it didn't feel like a
math lesson until the last 10 minutes. His progressive approach, combined with
his juggling skills, engaged the students in both numerical and theoretical
learning. Albert Campbell hopes to host Dr. Glynn again for another guest
speaker lesson.
I hope we can keep in touch regarding a future guest speaker opportunity or
even a UTSC visit. Have a wonderful day!
Kind wishes,
Mr. Shayan Mantegh
```

List of School Visits

Camera Workshop: 2025/11/18 (2 hours)
- School: Birchmount Collegiate Institute
- Contact: Francisco Estrada (francisco.estrada@utoronto.ca)

Braiding on Computers: 2025/11/04 (2 hours)
- School: Webtree Academy
- Contact: danielle.andriantsiferana (danielle.andriantsiferana@utoronto.ca)
- Link: https://pgadey.ca/notes/talk-braiding-on-computers/

The Mathematics of Juggling: 2024/03/27 (2 hours)
- TDSB Outreach to David and Mary Thomson
- Contact: Christine Mok (christine.mok@tdsb.on.ca)

The $\sin(x)$ Button: 2024/03/27 (2 hours)
- TDSB Outreach to David and Mary Thomson
- Contact: Grace Richards

The Mathematics of Juggling: 2024/02/21 (4 hours)

- TDSB Outreach to Albert Campbell Collegiate Institute
- Contact: Shayan Mantegh (shayan.mantegh@tdsb.on.ca)

Infinite Limits in Python: 2024/02/12 (2 hours)

- TDSB Outreach to Lester B. Pearson
- Contact: Jeganathan Nanthivarman (jeganathan.nanthivarman@tdsb.on.ca)

The Mathematics of Juggling: 2023/07/04 (4 hours)

- TDSB Outreach to Lester B. Pearson
- Outreach to The Russian School of Mathematics
- Contact: Francisco Estrada (francisco.estrada@utoronto.ca)

Green Path Student Orientation: 2023/06/23 (3 hours)

- Contact: Gwen Wang (gwen.wang@utoronto.ca)

5.4.2 Green Path

CMS has a lot of international students. The majority of these students come from China. And so, I felt that it would be important to specifically welcome this group to campus. To get to know them. To understand their motives in coming to UTSC. To let them know that their presence is acknowledged by the faculty. And so, I co-organized a welcome session for students attending UTSC via the Green Path program. This welcome session was held 2023/06/23. Although this is not exactly outreach to the broader community, I think that it is important to reach out to strong communities within UTSC.

Talking at the Green Path Student Orientation

5.5 Service to Professional Organizations

In my experience, Service and Professional Development go hand-in-hand. Below, I described the two of the major ways in which I've been of service to Professional Organizations.

- Serving as the Chair of the IBL SIGMAA
- Serving on the Steering Committee of the Fields Insitute MathEd Forum

In addition to these two leadership positions, I do review work for mathematics education journals and publishers.

- Reviewer for International Journal of Mathematical Education in Science (JMEST). 2020-Present
- Reviewer for CRC Press. 2018-Present
- Reviewer for Problems, Resources, and Issues in Mathematics Undergraduate Studies (PRIMUS). 2018-Present

While at UTSC, I have contributed to a number of initiatives as service within the UTSC community.

- Math Kangaroo: In 2022-2023, I coordinated Math Kangaroo at UTSC. This was the first in-person instance of Math Kangaroo after the Pandemic. We hosted over two hundred students, grades one to twelve, and their parents. Sixty volunteers, all CMS undergraduates, invigilated and helped coordinate the event. The event was significant in that brought in a diverse group of students to write the contest, and helped formed social bonds between students in the department.
- Science Literacy Week: This event was co-organized with our liaison librarian, Elizabeth O'Brien. I organized a model making activity, where students built models of the platonic solids. Other faculty contributed origami activities, and various math games.
- New Faculty and Librarian Orientation (NFLO): I've been panelist at the NFLO several times. It is always a pleasure to introduce new faculty to UTSC, and share my experience as a new hire. One panel discussion led me to write a guide to PTR Document Management which now gets shared with new hires.
- Women and STEM: Making New Paths: I participated in a panel at the Graduate Professional Day Conference. It was nice to talk with Dr. Fiona Rawle, a former colleague from UTM. We compared and contrasted our pathways through STEM.

Chapter 6

Professional Development Undertaken to Enhance Teaching

6.1 Professional Coaching

One thing that has consistently amazed me at UTSC is the incredible supports our institution offers to new faculty. I am very grateful for the grant of $3685 that I received from the UTSC Professional Academic Coaching Fund in March 2022. This grant has enabled me to hire Dr. Rich Furman, MFA, MSW, PhD (University of Washington Tacoma) as a coach.

Rich is an incredible mentor. He brings a life time of experience in academia and social work to his coaching. It is striking to have an outsider perspective on our teaching. Someone at arm's length sees things differently. Rich has talked me down in the face of "emergencies" and boosted me up when I've despaired. The main thing that Rich and I have worked on is scholarly productivity. We've developed routines for preparing course materials efficiently, and creating meaningful learning experiences for students.

Rich's coaching has enabled me to achieve a harmonious work-life balance, publish articles, and create incredible courses for our students. (I wrote a small piece about working with Rich for the Teaching & Learning Collaboration Newsletter here: Growing a Writing Practice with an Academic Coach.)

6.2 Faculty Writing Group

As I mentioned above, I love writing. This led me to participate in the UTSC Faculty Writing Group. It has been a pleasure to meet other writers across campus, and learn from their experience. See the attached posted for more details.

6.3 IBL Workshop Series

I was first introduced to Inquiry Based Learning (IBL) by the late Alfonso Gracia-Saz while I was a graduate students learning how to teach for the first time. These formative experiences set my trajectory through teaching. When I started as a CLTA at UTM, I participated in a week long training session on IBL organized by the MAA in Washington, DC. This got me up to speed on evidence-based teaching practices that promote inclusive excellence.

For the first two years that I was at UTSC, I served as the Chair of the Inquiry Based Learning (IBL) Special Interest Group of the Mathematics Association of America (SIGMAA). During that time, I organized a series of workshops on IBL. The flyers for those workshops are attached as appendices.

- IBL SIGMAA Workshop Series Fall 2021

- IBL SIGMAA Workshop Series Winter 2021

One workshop that impacted my approach to teaching, beyond all the other workshops in the series, was Dr. Su Dorée's *The Active Learning Pedagogy Sequence: A Model for Expanding the Use of Active Learning Structures in the College Mathematics Classrom*. This talk introduced a series of active learning techniques that work at scale. I immediately started to adopt them in to my teaching practice. There is so much more than just think-pair-share. In Dorée's framework, think-pair-share is the first step in a long journey. As I write this dossier, I'm preparing an active version of MAT A29 inspired by this talk, and funded by a Teaching Development Grant (TDG): MAT A29: Active and Applicable. The summary of the proposal is as follows:

> We propose to develop teaching materials for MAT A29 that are: (1) directly relevant to the life sciences, and (2) support active learning in the classroom. We will hire three students with experience in the life sciences to assist us in developing these materials. These teaching materials will be released as an Open Educational Resource (OER) by September 2025.

6.4 MathEd Forum

The Fields Institute MathEd Forum is a unique mathematics education venue, which brings together mathematics educators from primary to tertiary education. We meet once a month, on the last Saturday of the month, during the academic year. The participants are mathematics educators, mathematics education researchers, or mathematicians[1]. This diverse group combines theory and practice to produce a mutually enriching dialogue.

I have been involved with the Fields Institute Math Ed Forum for several years. First, as a participant, and later as a member of the Steering Committee. As a member of the Steering Committee, I co-organize the sessions at the Forum. I've organized sessions in celebration of mathematics educators, on computational thinking, and on the role of mathematicians in mathematics education. The Steering Committee also nominates educators for the Margaret Sinclair award.

As a participant, I learn from more experienced educators' perspectives. It turns out, that there is more commonality than difference across the primary to tertiary spectrum. The bottom line is that we all strive to do our best for our students. A community of passionate mathematics education researches, who believe in evidence-based teaching, has been invaluable.

In the appendices, I've included a schedule for Forum that I co-organized with Dr. Dragana Martinovic (Windsor). Mathematicians Leading Mathematics Education The description of that Forum was as follows:

[1] Laypeople, outside of academia, always find this three-way split of the mathematics territory surprising. Parker: "Hi, I teach mathematics at UTSC." Layperson: "Wow! You're a mathematician?!" Parker: "No, I'm a mathematics educator." Layperson: "But you work at a university. So, you're a mathematician?"

The objective of this month's Forum is to discuss the role mathematicians play in mathematics education. There are many famous mathematicians who contributed to mathematics education; Felix Klein, Hans Freudenthal, Erich Christian Wittmann, Leonhard Euler, György (George) Pólya, and Nikolai Lobachevsky come to mind, as well as Hung-Hsi Wu. However, "mathematics is not mathematics education" (Bass, 2005), so, what might have sparked their interest in the study of teaching, and how were they able to affect it? Wittmann (2020) argues that "Connecting mathematics and mathematics education requires looking at mathematics from the point of education and also looking at mathematics education with a broad understanding of elementary mathematics," and cites Heintel (1978) who wrote that "taking subject matter fundamentally into account in building didactical models means breaking up the narrow boundaries of special disciplines, reconstructing 'deep-frozen' learning processes, and elaborating the social use of knowledge and also its limitations." Our speakers, who are mathematicians, also have interest in mathematics education and will present their views and involvement in it.

6.5 CTL's Exploring Best Practices in Teaching and Learning

6.5.1 Reflection #1: Universal Design for Learning

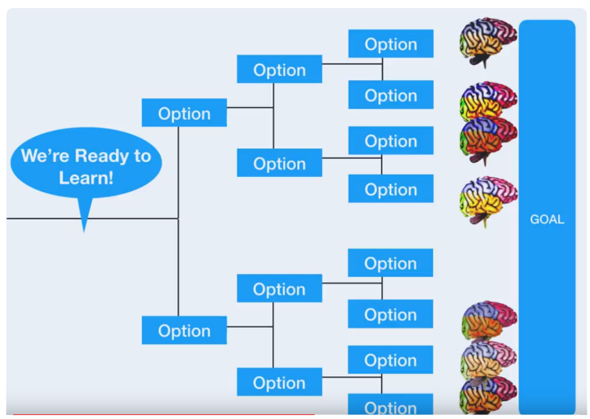
- How does learner variability present in the courses you teach?
- Was there something about the concept of learner variability that surprised you?

As I watched the videos and did the readings, I had the sinking feeling that my courses are designed in such as way as to teach "to the average" in the language of the UDL IRN video. This is framed as a bad or inappropriate pedagogy in the UDL materials we read and watched this week. Take a moment to consider this screenshot from the first video.

Here is my reflection which I wrote as I watched the first video: "I'm feeling a bit attacked right now. This is basically a picture of me in twenty years or so. I fully acknowledge that I'm a boring white guy teaching people math in the worst way possible. Math is the terrible for being all lecturing, assigning written homework, and in-person on-paper tests. And I like this feeling of being attacked, because it's signalling a possible growth opportunity. This is why I signed up for the CTL Best Practices course; to learn about pedagogy."

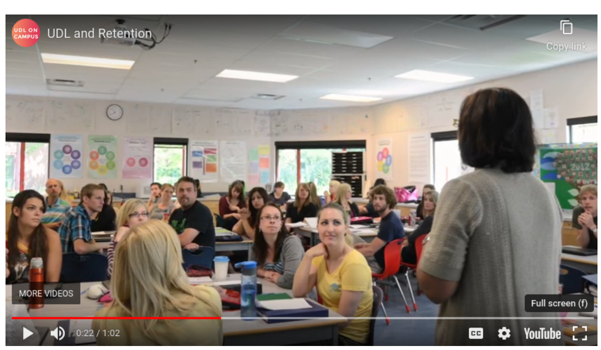
Putting aside the negative connotations of "teaching to the average", I want to acknowledge that the students in my classroom are highly variable. UTSC is a campus of inclusive excellence, and this manifests in my classes. There is a wide range of learners with high variability. I'm certain that "variability is the rule, not the exception." As Dr. Pape wrote: "Learner variability is the young person who lives in poverty, or is learning to speak English and may not yet have the background knowledge to enable comprehension of a reading passage. Or, the student who already has the skills to excel at a pace beyond the curriculum and is bored because traditional methods of instruction do not engage her or meet her needs." This is a description of my students. So, I want to find points of growth in my teaching, and learn how to teach in a way that celebrates this diversity.

Below I've included some more material which I wrote while watching the videos. It is pretty defeatist, but I think it summarizes why I believe it is hard to implement UDL best practices around learner variability at scale.



This picture is wonderful, but it does not scale especially well. If you're teaching a handful of learners, then they can all have their own meaningful option-full pathways. Give me a class with seven students (as in the infographic), and I'll teach this way. From my current perspective, this is not feasible with +700 students in a class. Following this diagram, we'd need a bit more than nine layers of choices. There would be around 500 nodes in the tree of choices. My classes usually only have ten assessments. I would love to promote the UDL goal of student autonomy, and embedding authentic choices in assessment, but I don't see how to do it at this scale.

My colleague, Mike Pawliuk (UTM), teaches a third year combinatorics course in a totally ungraded manner. Students are free-range and can create their own pathways through the course. He has about a hundred students in the class. His timetable for reading week is nine hours of interviews per day, as he meets with every student in the course followed by a similar round of meetings during the exam period. This is, I think, an extreme work regime and points to the upper end of what's feasible with ungrading.



I really wish that my classes looked like this. It would be amazing! Look at all that space to walk around and chat with individuals. Look at all the cool visuals. Instead, I've got AA 112. There are architectural barriers in place which make addressing student variability, and meeting students where they are, almost impossible. As they said in the second set of UDL videos: "If you can't reach them, then you can't teach them."

One big surprise for me was that "learner variability" is a cutting edge concept. This seems so basic and fundamental that I'm surprised it has only become a concept in mainstream education circles within the 21st century. In my church, we often talk about "answering that of God" in others. It is a basic belief that everyone has an intrinsic and sacred self, which we must acknowledge. Parker Palmer, an American author, educator, and activist, writes at length about incorporating this way of life in to teaching in his book *To Know as We Are Known* [5]. He writes that we all have a deep need "to understand, and to be understood." I think that mainstream higher education focuses on the students' need to understand and does almost nothing to address their need "to be understood". Learner variability, and UDL more generally, seek to address this second part.

6.5.2 Reflection #2: Online Learning

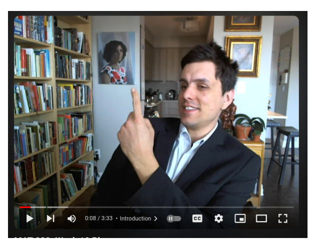
- What are one or two lessons you've learned about your own teaching as a result of the transition to online teaching during the pandemic and now back to in-person/blended teaching?

The Pandemic shocked us all. It was incredibly surprising to suddenly be thrust full-time into online teaching. I distinctly remember March 13th 2020 as the day it all changed. We were writing a midterm, and got the announcement that the university would be closing at the end of the day. That week, I posted two videos on YouTube:

- MAT 232 Midterm #2
- MAT 232: Week 10 Plan

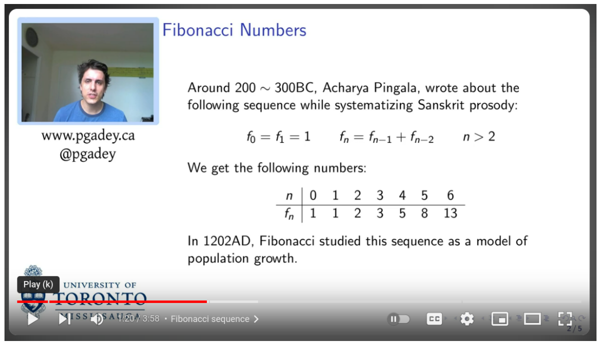
I've included a couple stills from these videos with this response. One is clearly shot in my university office with a blackboard behind me. The other is shot at home with all the stuff in my living room behind me. In it, I'm pointing at a painting on the wall and saying that it's of a friend of mine from graduate school named Lucy.

This juxtaposition reveals a deep lesson that I learned during the pandemic: we can be fully human, and messy, and present with our students. They enjoy it when we share the mundane and day-to-day aspects of our lives with them.



As the Pandemic wore on, I shared more and more of my life with my students. I stopped wearing blazers to sit at my computer. I had a new baby. The baby would come to class sometimes and "cheer" the students on. After the Pandemic, when I asked a colleague if he shared his new baby with his class, he said: "Oh – Of course not. My kids were born before the Pandemic. I was teaching in-person." And this made me wonder, "Why not share your new kid with the class whether it's online or in-person?" And so, the Pandemic was deeply humanizing. It took meeting virtually, and being deprived of so much human contact, to remind me that we can share our humanity with our students.

On a much more technical note, one thing that I took away from the transition to online learning was teaching with videos and slide decks. I remember spending hours and hours learning to edit videos. I reached out to a bunch of popular math YouTubers to ask how they did it.



(A still from my video on Linear Algebra and the Fibonacci Numbers. Notice: no blazer!)

I got used to expressing myself in short, succinct, highly edited videos. I learned about the educational best practices surrounding video creation. My colleagues and I experimented with producing extremely dense mini-lectures and encouraging students to pause, rewind, and rewatch them. (This turned out to be really tedious for the students, and we went back to producing more reasonably paced full lectures.)

Now that we're back in-person, I present myself in a much more relaxed and easy going manner to my students. I try to come across as more human and humane. If need be, I produce a short video from time to time for my students. Recently, I posted a walk-in video to show students how to get to a classroom for a midterm. Often, I'll post little encouraging videos when the grades are released and explain the policies around re-grades. I'm glad that I learned these lessons during the Pandemic. I think my students have benefitted from them.

6.5.3 Reflection #3: Assessment Design

- Thinking about your own past as a student/learner, what was one assignment/assessment that really stood out to you? Why was it so memorable? What made it really effective in helping you in the learning process (or really ineffective in the learning process).

The first thing that I want to note about my experience as a student is that I remember almost nothing at all about any of the assessments. When I first started this reflection, I felt a bit put out by this fact. Certainly, I spent thousands and thousands of hours doing homework. And almost none of it comes to mind as I reflect on this prompt. Is that so strange though? I've eat thousands of meals and there are only a few that really come to mind.

One of the most outstanding assessments I ever had was an exam for an advanced algebra course that I took in Moscow. I studied at the Independent University of Moscow for my final semester of undergraduate, via the Math in Moscow program. The course was structured as follows: On the first day of class, we were presented with an uchyebnik (little booklet) of problems. There were something like two hundred problems, which ranged from routine to extraordinarily difficult. The booklet was our "homework" for the class. It would be well-nigh impossible to do all the problems in the allotted semester. Instead, we were encouraged to work with others, and share solutions. The professor for the course, Katsylo, would occasionally solve problems in class. He didn't make a grand announcement that he was about to solve a problem from the list, but would simply teach us algebra and work through problems.

The final exam was structured as follows: Katsylo would interview each student based on their answers to the problems from the list. He allowed students to declare a list of questions that they did not want to answer. There was some penalty associated with declaring a problem as "not on the exam". The penalty was something like 5they didn't want to talk about four particular problems, then their highest possible grade would be 80% assuming they answered the remaining exam problems perfectly. If you failed to answer a question during the interview, your grade would drop by a similar amount. And so, a student who failed a question and declared four questions not on the exam would score 75%.

This meant that the final exam went something like the following: You walked in to our small classroom, sat down across from Katsylo, and declared a few questions. He then asked a handful of questions of escalating difficulty, but which you knew essentially knew the answers of. He would encourage you if you stumbled, and tell you when things weren't sounding quite right. The whole process took about five minutes. You got a grade, and left.

As I write about it now, I'm struck by how the whole course was really designed around that list of problems. There was no other homework, no midterms, no superfluous quizzes, or attendance marks. It was all in that booklet.

What made that exam so effective? A couple things come to mind. First, it was built to encourage collaboration. All the students in the course were constantly talking about the list. If someone "cracked" a hard problem, they would run around the dorm telling everyone. Late night conversations ensued. Second, the list kept us attentive in lecture. We had a reason to keep track of what was going on. Katsylo wouldn't announce that he was doing something on the list, unless it was exceedingly difficult.

Reflecting on it now, after more than a decade, I think Katsylo's advanced algebra exam was the most humane exam I ever "wrote". It was a conversation between two people. You had the opportunity to say what you did and did not want to talk about. It was simple, effective, and relational.

As part of writing this reflection, I tracked down the blog that I wrote one during my stay in Moscow back in 2010. One of the problems from Katsylo's list included constructing a very strange an exotic object called an "outer automorphism of S_6". I was so excited about it, that I posted it online. It was the second post on that blog.

6.5.4 Reflection #5: Acknowleding Holistic Learners

- What are the benefits for all students, both Indigenous and non-Indigenous, of integrating Indigenous approaches into the curriculum?
- How can reciprocity be experienced in the educator-student relationship through the use of holistic models in learning and assessment design?

Everyone has a soul. We are more than our material bodies and our immaterial minds; we have souls. We are, whether we acknowledge it or not, spiritual beings. The benefit of adopting the Medicine Wheel as a framework for teaching and learning in higher education is that it will benefit everyone, both Indigenous and non-Indigenous people, by honouring their spiritual reality.

I'm going to be honest here and say that I think much of the mental health epidemic affecting youth in Canada is really a spiritual or religious crisis. Until quite recently, Western settlers were a religious people. We knew what our souls were and how to take care of them. Now that we've alienated ourselves from our souls, we're seeing a rise in mental health issues. Adopting the Medicine Wheel as a framework for education could help to restore the balance of mind, body, and spirit, and hopefully move everyone back towards a place of wellness.

There were a couple of things in the reading which stood out to me about how adopting Indigenous perspectives could benefit all learners. The first was the sense of connection. I see a lot of students on social media expressing their frustration about the lack of social connection at UTSC. If our classrooms create a sense of connection, or a sense of being in a community, then I think that there would be a massive benefit to students.

Another aspect of the Medicine Wheel teaching which spoke to me was Self-Actualization. I think it's important to have a sense of one's life goals. If a person knows what their unique purpose is then they're much better equipped to handle the day-to-day struggles of living. Victor Frankl, an Austrian psychologist, wrote that Westerners are suffering from a vacuum of meaning. We don't know what we're here for. Inspired by this problem, I developed a small coaching practice working with students on their life goals. I wrote about this practice here: Glynn-Adey, P. (2023) *The Life Goals Exercise: Context for Students' Big Questions.* The Teaching Professor, 7 Sept. 2023, (link). This exercise is something that I do with students one-on-one, as they need it.

Regarding reciprocity, I want to first note that the university as it is currently setup cannot foster student-teacher reciprocity. There are logistical and even architectural barriers which prevent it. If you look at a lecture hall of three hundred people taking notes while a single person drones on at length about a slide deck, then you'll see that there just isn't room for reciprocity in a lecture hall. Too many people. Too narrow a focus.

In order to foster reciprocity, students and teachers need to acknowledge each other as people first. We need to be together informally. We need to chat while walking the same paths. In my current teaching context, with vast seas of students, I can't see a way to make this work with every student simultaneously. And that's why I do the life goal work on an individual basis.

Working with our students spiritual lives requires that we are rooted in our own spiritual journey. We need to know in order to teach. We need to be healed before we can heal. If faculty in higher education are willing to take their own spiritual lives seriously, then I think they can move beyond Bloom's Taxonomy towards the Medicine Wheel. My own religion is Quakerism, a form of primitive Christianity which arose in seventeenth century England. By being rooted in my own tradition helps me to teach more holistically.

I think that the task of fostering reciprocity begins with the individual. During a lecture, I can walk up to an individual and ask them how they're doing. I can create a personal connection. We can meet each other holistically. In Quakerism, we call this "answering that of God" in them. Just as one tries to look for "spirit moving" in Quakerism, I can try to look for spirit moving in my classes. That's where reciprocity begins.

6.5.5 Reflection #6: Experiential Learning

- Describe and reflect on an experiential learning (EL) course that you have taken or have taught? If you have not taught or taken an EL course, could you discuss how you might add an EL component to a course are teaching or planning to teach?

- What do you feel is valuable about EL? What has influenced you to feel this way? If you don't think that EL is valuable, what has influenced you to feel this way?

- What are your thoughts about the assigned article on GenAi and EL?

As far as I can recall, I've never taken a course with a significant component of experiential learning. Perhaps my generation of students was too early for that sort of thing, or my department wasn't active in that field of pedagogy. Anyway, I wish that I had taken such a course. In response to the first prompt, I'm going to describe an experiential learning component that I would like to add to my courses.

I teach massive first year calculus courses. Every semester, thousands of students take Calculus I for the Life Sciences. Sometimes, I am haunted by the feeling that this course is just wasted time and effort. The students learn a bit about calculus, enough to get by in their degrees, but don't produce anything of lasting value or beneficial to the community. When I have these thoughts, I think that the following experiential component might remedy the situation.

I would like to pair all the students taking Calculus I for the Life Sciences with local highschools, and have the students work as peer-support or peer-mentors to highschool students. Perhaps the UTSC student become mentors and teachers for the last half of the course, once we've cleared our first term test and review of highschool material. This would directly benefit the highschoolers, and support UTSC's strategic mission of inclusive excellent by being rooted in our local community.

Let's look at this proposal through Kolb's model of experiential learning. What's the new concrete experience? Teaching. The UTSC students would have a novel experience of being mentors and tutors. How would reflective observation work? The students could keep a record of the material that they've covered and which ideas resonated with their mentees. How could they abstract their teaching? They could prepare a collection of problems, or a portfolio of the work that they've done with their mentees.

What's the experiment? The UTSC students bring their teaching experience back to the university, and teach others. Alternatively, they could switch mentees and see how their teaching improves.

To address the second point, I think that experiential learning is really interesting. If I reflect on the things that I've learned after my formal schooling, some of the most valuable things I've ever learned, really, then I can see that the learning all occurred experientially. For example, I got a solid education in administration by working on the clerking team at my Quaker meeting. It wasn't a course, or a formal process, I just started working with the clerks. It was a concrete experience that taught me. I think that university students could similarly benefit from concrete experiences.

To address the last prompt, I found the article tough going. It was pretty heavy academic pedagogy research with lots of abbreviations. An example taken at random: "They all pointed to the effective alignment of ILOs, TLAs, and ATs." This is mostly a sign that I didn't spend enough time reading the paper.

A couple things which stood out for me. I liked the emphasis on GenAI as agents-to-support in place of "the traditional use of agents-to-write or agents-to-answer questions". My personal exploration of the GenAI space, and the people who I've seen doing good math with them, generally suggests that we work with them as agents-to-support.

I also noticed the thing ethnography stance of the paper. To quote the paper at length:

> Accordingly, thing ethnography refers to collecting and interpreting things' perspectives from everyday data and trajectories that things provide access to and the theory-based analysis that humans undertake to identify patterns and gain novel insights into their socio-material interactions [21]. Thing ethnography involves stepping into things' shoes to explore the acting out of things' attributes, as in role-playing, to portray and empathize with the elusive "inner life of things" [48].

This is a neat lens to look at GenAI, and might help me better understand some of my own work with string figures. Perhaps, I should look at string from a thing ethnography.

6.5.6 Reflection #7: Reading and Writing

- Why do we ask students to write in our classes?
- What are the challenges for us in delivering writing assignments in the age of AI?

Reading and writing are the first technologies of any technological society. They are the base and foundation on which everything else is built. Compare the 300,000 years of human history before the invention of writing with the more recent 6000 years of documented history. Culture has evolved considerably since the invention of writing. We ask our students to write so that they can come to wield the most powerful meta-tool humanity has ever created.

This is, of course, a bit too dramatic. It is too profound for standard education. We don't, in our day to day lives, think much of the immense power of writing. As far as education is concerned, it is closer to the truth to say that reading and writing are in high demand. Industry highly values communication skills. We want people who can understand the written word and make themselves understood through it.

In my own teaching, I make an unusually big deal out of explanation in writing. I think that in order to really understand something, one needs to write about it. Writing, whether or not anyone else will ever read it, can help clarify our thinking and make it more precise. There's a great quote by an American computer scientist, Leslie Lamport, about this: "If you're thinking without writing, then you only think you're thinking." So, that's why we teach students to write in our classes: it's powerful, in high demand, and nutures the ability to think.

And now, let's turn to second prompt. As I mentioned above, writing is the primal technology. It is so powerful, and so expansive, that we almost forget about its existence. This makes it especially hard to draw make analogies about the role of writing in our culture. For a moment, though, let's pretend like cooking is analogous to writing. It's a surprisingly versatile skill, applicable in a broad range of contexts, and you need to practice to do it well.

The challenge of assigning writing tasks in the present bubble of Generative AI is that you get an overwhelming amount of terribly cooked food. Or rather, surreal-y cooked food. You ask for a salad, and you get the ingredients put through a food processor and served as a smoothie. You hope to teach people about a simple preparation of baked potatoes and instead they serve you french fries.

What I want to highlight about these examples is that the product you ask for is rather simple, a salad or a baked potato, but what you get relies on the existence of some high technology. In these cases, blenders and deep friers. If that technology goes away, then the cook is unable to prepare the meal. They can't whip up a salad, or even something as simple as a baked potato. They can't feed themselves.

What I'm trying to say is this: the challenge of assigning writing tasks in this bubble of generative AI is that students unknowingly impoverish their own learning experience and run the risk of not learning how to write. If people learn to write, for the very first time, immersed in generative AI then they are giving up their power and agency to technologists who don't have any incentive to serve their best interests. They're giving up their ability to think.

6.5.7 Reflection #8: Anti-Racist Pedagogy

- How do I show my students a commitment to inclusive and anti-racist teaching?
- What does it look like to diversify my curriculum and course material (i.e. authors, knowledges, perspectives, frameworks, perspectives)?

I want to begin by acknowledging that people working in computer science, mathematics, and statistics, generally have pretty shoddy notions of what anti-racist and anti-colonial pedagogy ought to be in our field. I've heard numerous colleagues remark in earnest: "Is my course racist? Of course not, we don't talk about race at all." And so, I think that we in STEM have a lot to learn about racism. It's from this place of learning and curiosity that I want to talk about my own attempts to in more inclusive and anti-racist ways.

We should acknowledge that UTSC is well-positioned to benefit from anti-racist pedagogy. The student body is, to put it lightly, extremely diverse. With this many cultures in one place, there is always something culturally important going on. Whenever a major cultural celebration occurs, I make a point of mentioning it in my class together with an acknowledgement that: I'm not a member of that group but that I like to cheer for others. Recently, we celebrated the Chinese Lunar New Year. As many of my students are from China, I made a point of including a cute snake graphic in a course announcement. I also shared a picture of my daughter, Mira, marvelling at a huge snake sculpture on display for the New Year at the Scarborough Town Center. I do similar moments of cultural appreciation around other major events such as Ramadan, Diwali, and Black History Month.

Another way in which I try to enact anti-racist teaching is that I structure my classes in such a way that I'm not just some young white man delivering truth as a "sage on the stage". I try to mix-and-mingle with my students a great deal. During in-class activities, I'll leave the stage and approach students individually. Not every room is equally accommodating of this maneuver, but I think it is worth the effort wherever we find ourselves. By approaching students individually, as a person talking with a person, I hope to create a sense of inclusion and participation in my class which transcends race and works to level the student-teacher dynamic.

To address the second prompt, about including diverse perspectives in course materials, I really must admit that this is a work in progress. I think that mainstream STEM materials are so thoroughly WASP-y that we don't even see them as culturally conditioned. This blindness is the reason why my colleagues can, in earnest, believe that their courses are not racialized.

There is one example, in my MAT A22 course, that I really like to use a talking point about race. I think it is also philosophically very rich. To be very concrete, consider the following (obviously contrived) problem:

> "Suppose that you want to produce one hundred loaves in total of two kinds of bread and have 240 cups of cups of flour. If one recipe requires two cups of flour per loaf and the other recipe requires three cups of flour, how many loaves of each type of bread should you produce?"

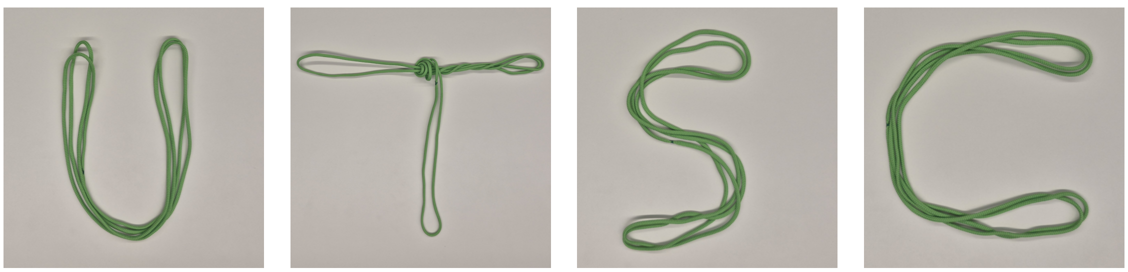
This situation is modelled by the following pair of equations:

$$\begin{cases} x + y & = & 100 \\ 2x + 3y & = & 240. \end{cases}$$

This is a "system of linear equations". The interesting point is this: every culture which develops mathematics eventually encounters the problem of solving systems of linear equations. That is to say, systems of linear equations are an inevitable part of the development of mathematics and, more broadly, market economies. And, even more astoundingly, every culture comes up with the same algorithm for solving them, a process known as "Gauss-Jordan Eliminiation". I think that calling this process by the name of two white European mathematicians is racist. When I teach this algorithm, I acknowledge that was independently discovered in India and China. I highlight the philosophical mystery that a particular algorithm is so inevitable.

Appendix A

Appendices



This figure was in an early draft of the paper Of Loops, Braids, and String Figures (2026). However, it got cut out of that paper during the peer review process. It is included here as a bonus to readers diligent enough to peruse the whole dossier. I hope that it puts a smile on your face.

A.1 Curriculum Vitae

CURRICULUM VITÆ Date of Revision: 5 May 2026

A. BIOGRAPHICAL INFORMATION

1. **Personal**

Parker Adey
35-14 Heron Park Place, Toronto
Ontario, CANADA M1E 0B8
Phone: 647-706-5740

Department of Computer & Mathematical Sciences
University of Toronto Scarborough
1050 Military Trail, Toronto
Ontario, CANADA M1C 1A4
Phone: 416-208-4794

2. **Degrees**

- Ph.D. Mathematics, University of Toronto, 2017
- M.Sc. Mathematics, University of Toronto, 2012
- B.Sc. Mathematics, Trent University, 2010

3. **Employment**

- **Assistant Professor, Teaching Stream** University of Toronto Scarborough Campus. 2021-Present
- **Assistant Professor (CLTA)** University of Toronto Mississauga Campus. 2017-2021
- **Instructor** University of Toronto Scarborough Campus. 2016-2017
- **Teaching Assistant** University of Toronto St. George Campus. 2011-2016
- **Teaching Assistant** Trent University 2007-2010

4. **Honours**

5. **Professional Affiliations and Activities**

- Steering committee of Fields Institute Math Education Forum. 2018-2025
- Chair of the IBL Special Interest Group of the MAA. 2019-2023
- Editor of *Math in Action Journal*. 2018-Present
- Reviewer for CRC Press. 2018-Present
- Reviewer for Problems, Resources, and Issues in Mathematics Undergraduate Studies. 2018-Present

B. ACADEMIC HISTORY

6. A. **Research Endeavours**

   B. **Research Awards** during preceding 5 years

   C. **Patents** awarded during past 5 years

C. SCHOLARLY AND PROFESSIONAL WORK

In the sub-sections below, a star (⋆) indicates an undergraduate co-author.

7. Refereed Publications

A. Articles

- Adey, P. (2025) Jayne in Brief: A Personal Formalization of Jayne's String Figures Using Storer's Calculus. *BISFA*, Vol. 32.
- Adey, P. (2025) Two Perspectives on Belonging. *The Canadian Friend*, 120(1), 27-28.
- Mamolo, A., Glynn-Adey, P. (2023) Tangible connections within the mathematical horizon: Exploring the Dihedral Calculator. *ZDM Mathematics Education* 55, 793–805 (2023). `https://doi.org/10.1007/s11858-023-01503-5`
- Glynn-Adey, P. (2023) The Life Goals Exercise: Context for Students' Big Questions. *The Teaching Professor*, 7 Sept. 2023, `www.teachingprofessor.com/topics/teaching-strategies/the-life-goals-exercise-context-for-students-big-questions/`.
- Glynn-Adey, P. & Mamolo, A. (2022) Modelling Beauty: Hands-on Experiences in Group Theory. Building on the Past to Prepare for the Future, Proceedings of the 16th International Conference of The Mathematics Education for the Future Project, King's College, Cambridge, Aug 8-13, 2022 (A. Rogerson, Ed.). WTM-Verlag. `https://doi.org/10.37626/ga9783959872188.0`
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- Mamolo, A., Yan, K., Lovric, M., Glynn-Adey, P., Ruttenberg-Rozen, R., Bohun, S., & Rodney, S. (2022). Teaching with Technology in Undergraduate Mathematics. *eCampus Ontario*. `https://openlibrary-repo.ecampusontario.ca/jspui/handle/123456789/1235`
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- Glynn-Adey, P. (2021). Using a wiki to collect student work in vector calculus. *International Journal of Mathematical Education in Science and Technology*, 52:7, 1131-1137, DOI: 10.1080/0020739X.2020.1827174.
- Shahbazi, Z., & Glynn-Adey, P. (2020). Using departmental publications to foster student creativity in mathematics. *Journal of Humanistic Mathematics*, 10:2, 445-464.
- Glynn-Adey, P., & Liokumovich, Y. (2017). Width, ricci curvature, and minimal hypersurfaces. *Journal of Differential Geometry*, 105:1, 33-54. [arXiv:1408.3656]
- Glynn-Adey, P., & Zhu, Z. (2017). Subdividing three-dimensional riemannian disks. *Journal of Topology and Analysis*, 9:03, 533-550. [arXiv:1508.03746]

B. Books and/or Chapters

C. Books edited

8. **Non-Refereed Publications**

- Adey, P. (2026) Undergraduate Seminar: A Place to Sow Seeds. *FYMSiC Newsletter*: Issue 18, First Year Math and Stats in Canada, February 2026, (link)
- Adey, P. (2025) Teach The Fundamental Theorem First! *FYMSiC Newsletter*: Issue 17, First Year Math and Stats in Canada, July 2025, (link)
- Glynn-Adey, P., (2024). Advice for Students. *Self-published*, 1-16.
- Glynn-Adey, P., (2023). Random Permutations and the 100 Lockers Puzzle. $U(t)$-*Mathazine*, 6(1), 19-22.
- Glynn-Adey, P. (2023) Writing for Large Class with `docstrip`. *FYMSiC Newsletter*: Issue 13, *First Year Math and Stats in Canada*, 17 July 2023, (online).
- Parker Glynn-Adey. (2022) Modern String Figures: Winged Heart. *BISFA*, Vol. 29.
- Glynn-Adey, P., (2022). Ringing the Changes: A Dance of Algebra and Geometry. $U(t)$-*Mathazine*, 5(1), 5-9.
- Glynn-Adey, P., Pawliuk, M., Rennet, A., & Thind, J. (2021). MAT223: Active, Flipped, and Pandemic Strong! *FYMSiC Newsletter*, 8.
- Glynn-Adey, P., (2021). The Mathematics of Juggling. $U(t)$-*Mathazine*, 4(1), 9-11. `https://mailchi.mp/eb4bc9725`
- Derry, K., Glynn-Adey, P., & Parke, E. (2020). Un-Office Office Hours. *Teaching & Learning Collaboration Newsletter*. `https://www.utm.utoronto.ca/tlc/un-office-office-hours`
- Glynn-Adey, P., (2020). The Rochester Mathematics Olympiad. *MSLC Magazine*, 5(1), 8.
- Glynn-Adey, P., Arden, A., & Chernoff, E. J. (2019). Report: Canadian Mathematics Education Study Group 2019 annual meeting. *Ontario Association for Mathematics Gazette*, 58(1), 11-14.
- Glynn-Adey, P., & ⋆Tsui, H. (2019). String Figures and Thomas F. Storer's Calculus. *MSLC Magazine*, 5(1), 14-17.

9. **Manuscripts/Publications in Preparation**

- Adey, P. & Schattman, V. (in press) Of Loops, Braids, and String Figures. Bridges. 2026.
- Adey, P. & Hart, H. Fitch Cheney for Set: A Card Trick with 4.44 Cards. Math Horizons. 2026.

10. **Papers Presented at Meetings and Symposia**

11. **Invited Lectures**
    - Adey, P. & Goder, S. *String Stars: A Joyful Ending for a Class*. CMS Winter Meeting 2025. (link)
    - *Supporting A Departmental Culture of Undergraduate Research*. CMS Winter Meeting. 2025. (link)
    - *Calculus for Life Sciences: Active and Applicable*. Building Capacity in the Scholarship of Mathematics Education 2025.
    - Adey, P. & Vandendriessche, E. *(Ethno-)mathématiques des jeux de ficelle inuit*. Espace Mathématique Francophone (EMF) 2025. (link)
    - *Geometry, Models, and Inquiry*. CMS Winter Meeting. 2019.
    - *A Community of Mathematicians: Using a Wiki in a Large Calculus Class*. PCMI Workshop on Equity and Mathematics Education. 2019.
    - *Start, Stop, Continue and Ticket Out of the Door: Collecting and using student feedback to improve teaching in a large first-year math class*. Scholarship of Teaching and Learning in Higher Education 2018.
    - *Calculus Readiness Assessment - Are Students Ready?* UTM MCS High School CS/Math Teacher Workshop 2018.
    - *Simplicial homology for beginners*. MSLC Summer Seminar 2019
    - *Euler's equation: some personal reflections*. Fields Institute Math Education Forum 2018
    - *Storer calculus for unknot designs*. UTM Math Club 2018
    - *The infinitude of primes and variations*. March Break Math Academy 2018
    - *Triangulating the hyperbolic plane*. March Break Math Academy 2017
    - *Unknot designs*. Appleby College 2017
    - *Ideal hyperbolic polyhedra*. Geometry learning seminar 2014
    - *A tour of recent work in geometric geometry*. Math Graduate Student Seminar 2013
    - *Asymptotic cycles and ergodicity on flat surfaces*. Flat Surfaces Learning Seminar 2013
    - *From expander graphs to super-expanders*. Probability, Geometry, and Groups Learning Seminar 2013
    - *Rotationally distinct ways of labelling a die.* Canada Math Camp 2013
    - *Goemans-Linial approximation to the sparsest cut problem*. Probability, Geometry, and Groups Learning Seminar 2012
    - *Modern integer factorization techniques*. AARMS Graduate Summer School 2009
    - *Integer partition identities*. Canadian Undergraduate Math Conference 2009
    - *Numeration systems*. Canadian Undergraduate Math Conference 2008
    - *(Mathematical) Typesetting with LaTeX*. Trent Undergraduate Seminar Series 2008
12. **<u>LIST OF COURSES</u>** (in preceding 5 years)
13. **Undergraduate courses taught**

    Scarborough 2025/2026
    - **Coordinator** (2×) **MAT A29 –** Calculus I for the Life Sciences
    - **MAT A02 –** The Magic of Numbers
    - **Coordinator MAT B41 –** Techniques of the Calculus of Several Variables I
    - **MAT B42 –** Techniques of the Calculus of Several Variables II
    - **MAT D93 –** Braid Theory and Applications
    - **MAT D93 –** Loop Braid Groups

    Scarborough 2024/2025
    - **Coordinator** (3×) **MAT A29 –** Calculus I for the Life Sciences

- **Coordinator MAT A22 –** Linear Algebra I for Mathematical Sciences
- **MAT B41** – Techniques of the Calculus of Several Variables I
- **MAT B42** – Techniques of the Calculus of Several Variables II

Scarborough 2023/2024

- **MAT D92** – Algorithmic Knot Theory

Scarborough 2022/2023

- **Coordinator** $(2\times)$ **MAT A29 –** Calculus I for the Life Sciences
- **Coordinator** $(2\times)$ **MAT A22 –** Linear Algebra I for Mathematical Sciences
- **MAT B41** – Techniques of the Calculus of Several Variables I
- **MAT B42** – Techniques of the Calculus of Several Variables II

Scarborough 2021/2022

- **Coordinator MAT A29** – Calculus I for the Life Sciences
- **Coordinator MAT A22 –** Linear Algebra I for Mathematical Sciences
- **MAT B42** – Techniques of the Calculus of Several Variables II

Mississauga 2020/2021

- **MAT 402** – Classical Geometry
- **MAT 133** – Calculus and Linear Algebra for Commerce
- **MAT 223** – Linear Algebra I

Mississauga 2018/2019

- **CSC 493 –** Computer Science Expository Work: Game Theory and the Probabilistic Method
- **MAT 133** – Calculus and Linear Algebra for Commerce
- **MAT 135** – Calculus
- **MAT 223** – Linear Algebra I
- **Coordinator MAT 232 –** Calculus of Several Variables

Scarborough 2017/2018

- **Coordinator MAT B41** – Techniques of the Calculus of Several Variables I

Mississauga 2017/2018

- **MAT 133** – Calculus and Linear Algebra for Commerce
- **MAT 134** – Calculus for Life Sciences
- **MAT 223** – Linear Algebra I

Scarborough 2016/2017

- **MAT A31 –** Calculus for the Mathematical Sciences
- **MAT A29 –** Calculus for Life Science
- **MAT A33** – Calculus for Management II

Toronto 2016/2017

- **MAT 246** – Concepts in Abstract Mathematics

Scarborough 2015/2016

- **MAT A33 –** Calculus for Management II

14. **Graduate courses taught**

15. **Theses supervised**

16. **Other teaching and lectures given**

Camera Workshop: 2025/11/18 (2 hours)
School: Birchmount Collegiate Institute
Contact: Francisco Estrada (francisco.estrada@utoronto.ca)

Braiding on Computers: 2025/11/04 (2 hours)
School: Webtree Academy
Contact: danielle.andriantsiferana (danielle.andriantsiferana@utoronto.ca)
Link: `https://pgadey.ca/notes/talk-braiding-on-computers/`

Discover String Art: 2024/04/03 (2 hours)
Contact: Maggie Cummings (maggie.cummings@utoronto.ca)
`https://www.utsc.utoronto.ca/anthropology/discover-string-art`

The Mathematics of Juggling: 2024/03/27 (2 hour)
TDSB Outreach to David and Mary Thomson
Contact: Christine Mok

The $\sin(x)$ Button: 2024/03/27 (2 hour)
TDSB Outreach to David and Mary Thomson
Contact: Grace Richards

Stress Less Week: 2023/03/06 (1 hour)
Contact: Elizabeth O'Brien

The Mathematics of Juggling: 2024/02/21 (4 hours)
TDSB Outreach to Albert Campbell Collegiate Institute
Contact: Shayan Mantegh (shayan.mantegh@tdsb.on.ca)

Infinite Limits in Python: 2024/02/12 (2 hours)
TDSB Outreach to Lester B. Pearson
Contact: Jeganathan Nanthivarman (jeganathan.nanthivarman@tdsb.on.ca)

The Mathematics of Juggling: 2023/07/04 (4 hours)
Outreach to The Russian School of Mathematics
Contact: Francisco Estrada (francisco.estrada@utoronto.ca)

Green Path Student Orientation: 2023/06/23 (3 hours)
Contact: Gwen Wang (gwen.wang@utoronto.ca)

Undergraduate Projects Supervised

- Schattman, V. (2026) Loop Braid Groups.
  - Adey, P. & ⋆Schattman, V. (in press) Of Loops, Braids, and String Figures. Bridges. 2026.
  - Schattman, V. (in press) Reconstructing 'The Chief of The Two Trees' Using Dynamic Knot Software. BISFA. 2026.
- Goder, S. (2025) Braiding: Theory and Applications.
  - CMS Undergraduate Research Symposium: Best Oral Presentation
    Title: Braid Theory & Groups
  - Adey, P. & ⋆Goder, S. *String Stars: A Joyful Ending for a Class*. CMS Winter Meeting 2025. (link)
- Liu, Y. (2023) Algorithmic Knot Theory.
  - Liu, Y. A Computational Approach to String Figures. (unpublished manuscript)
- Li, Z. (2020) Combinatorics of the SET deck.

  - Glynn-Adey, P. & ⋆ Li, Z. (2021). A magic trick using the SET deck. *Math Horizons*, 28:4, 16-18, `https://doi.org/10.1080/10724117.2021.1881388`.
- Tahir, H. (2019) Hyperbolic geometry and Escher.
- Arnoldt-Smith, K. (2019) Game theory and the probabilistic method.
- Salwinski, D. (2018). Euler's sine product formula: an elementary proof. *The College Mathematics Journal*, 49(2), 126-135.
- Salwinski, D. (2018). The continuous binomial coefficient: an elementary approach. *The American Mathematical Monthly*, 125(3), 231-244.

D. ADMINISTRATIVE POSITIONS

17. A. **Positions held and service on committees and organizations within the University**
    - Tri-Campus Mathematics Teaching Stream Meet-Up Co-Organizers. June 2025–Present.
    - Mathematics Teaching Stream Hiring Committee. December 2025–April 2026.
    - $U(t)$-Mathezine Editor-in-Chief. September 2025–Present.

    B. **Positions held and service on committees and organizations outside the University** (of scholarly and academic significance)
    - Steering committee of Fields Institute Math Education Forum. 2018-Present
    - Chair of the IBL Special Interest Group of the MAA. 2019-2023
    - Editor of *Math in Action Journal*. 2018-Present
    - Reviewer for CRC Press. 2018-Present
    - Reviewer for Problems, Resources, and Issues in Mathematics Undergraduate Studies. 2018-Present

E. OTHER RELEVANT INFORMATION

GRANTS IN SUPPORT OF TEACHING OR RESEARCH:

- eCampus Ontario: Enhancing Online Post-Secondary Mathematics Instructions. $188,788
  Co-Applicant with: Miroslav Lovric, Sean Bohun, Robyn-Ruttenber-Rozen.
  Principal Investigator: Ami Mamolo, Ontario Tech University.
  Funded: March 2021

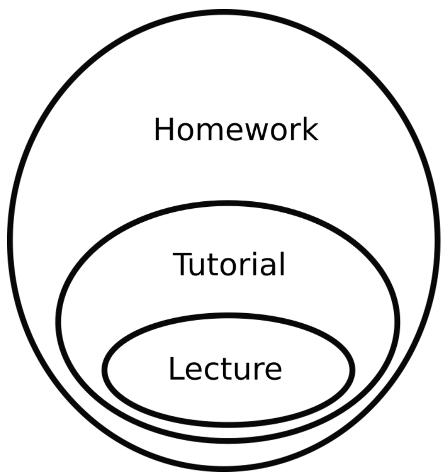
- UTSC: Professional Academic Coaching Fund. $3685
  Funded: March 2022

A.2 Lectures Notes

Week 1: A Quick Tour of Linear Algebra

Welcome to MAT A22
Linear Algebra I for Mathematical Sciences

Exam		40%
Test		30%
Assignments	$(11-1)\times 3\% =$	30%
Quizzes	$5\times 2\% =$	10%



Algebra is about definitions, theorems, proofs, and STRUCTURE.

On this quick tour, we're going to look at:

- The Surprisingly Common Vector Space Structure
- Vectors in $\mathbb{R}^n$.
- The Axioms of A Real Vector Space
- Uniqueness Proofs

Activity: What does Algebra Study?

Compare the following tables of numbers.

$x \boxplus y$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

$x \oplus y$	0	1	2	3	4
0	2	4	2	4	1
1	3	0	0	2	2
2	0	3	3	4	4
3	0	4	1	1	3
4	1	4	3	2	1

Do you notice any structure in the tables?

Remark: What is an Axiom?

An axiom is a definition. We use axioms to describe how mathematical objects work.
Axioms are the basic elements of mathematical structures.

Definition: Binary Operations

A **binary operation** is a function $\boxplus : X \times X \to X$ with two inputs.

- The usual addition on real numbers $+ : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a good example.

$$2 + 3 = 3 + 2 = 5$$

- Another common example is the usual multiplication $\cdot : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ of real numbers.

$$3 \cdot 2 = 3 \cdot 2 = 6$$

Definition: Commutative and Associative

An operation $\boxplus : X \times X \to X$ is **commutative** if:

$$x \boxplus y = y \boxplus x.$$

An operation $\boxplus : X \times X \to X$ is **associative** if:

$$x \boxplus (y \boxplus z) = (x \boxplus y) \boxplus z$$

Notation: We use the notation $\boxplus$ for a general operation with two inputs.

Activity: Non-Examples

1. Give an example of a binary operation $\boxplus_1 : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ that is NOT commutative.
2. Give an example of a binary operation $\boxplus_2 : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ that is NOT associative.

Definition: The Axioms of a Real Vector Space

A **real vector space** or **$\mathbb{R}$-vector space** is a set V together with two binary operations

$$\boxplus : V \times V \to V \qquad \boxdot : \mathbb{R} \times V \to V$$

called **addition** and **multiplication by a scalar** such that the following axioms hold:

A1. $\boxplus$ is associative.

A2. $\boxplus$ is commutative.

A3. There exists an element $\mathbf{0} \in V$ such that:

$$\mathbf{0} \boxplus \mathbf{x} = \mathbf{x}$$

for all $\mathbf{x} \in V$. We call $\mathbf{0}$ the **zero vector** of V. (Could there be more than one?)

A4. For each $\mathbf{x} \in V$ there is an **additive inverse** $\mathbf{x}' \in V$ such that:

$$\mathbf{x} \boxplus \mathbf{x}' = \mathbf{0}.$$

A5. For all $\mathbf{x}$ and $\mathbf{y} \in V$ and $c \in \mathbb{R}$, scaling by c **distributes over** addition:

$$c \boxdot (\mathbf{x} \boxplus \mathbf{y}) = (c \boxdot \mathbf{x}) \boxplus (c \boxdot \mathbf{y}).$$

A6. For all $\mathbf{x} \in V$ and c and $d \in \mathbb{R}$, the usual addition in $\mathbb{R}$ distributes over addition in V:

$$(c + d) \boxdot \mathbf{x} = (c \boxdot \mathbf{x}) \boxplus (d \boxdot \mathbf{x}).$$

A7. For all $\mathbf{x} \in V$ and c and $d \in \mathbb{R}$, $\boxdot$ **associates with** the usual multiplication on $\mathbb{R}$:

$$(cd) \boxdot \mathbf{x} = c \boxdot (d \boxdot \mathbf{x}).$$

A8. For all $\mathbf{x} \in V$, we have: $1 \boxdot \mathbf{x} = \mathbf{x}$.

Note: These axioms have been adapted from Little and Damiano p.7 to use $\boxplus$ and $\boxdot$.
Their order, numbering, and names are important.

Definition: The Vector Space $\mathbb{R}^n$

The vector space of **real n-tuples** is given as follows:

$$\mathbb{R}^n = \{(x_1, x_2, \ldots, x_n) : x_i \in \mathbb{R}\}$$

The addition on $\mathbb{R}^n$ is:

$$(x_1, x_2, \ldots, x_n) + (y_1, y_2, \ldots, y_n) = (x_1 + y_1, x_2 + y_2, \ldots, x_n + y_n)$$

The scalar multiplication on $\mathbb{R}^n$ is:

$$c(x_1, x_2, \ldots, x_n) = (cx_1, cx_2, \ldots, cx_n)$$

Note: Checking the details of Axioms 1-8 is a good (but long) exercise.
All the proofs essentially come down to "this property is true for real numbers".

Example: Check a Few Axioms!

Pick two axioms from A1 - A8 and Parker will check them.

Example: Calculating in $\mathbb{R}^3$

Calculate the following vectors in $\mathbb{R}^3$.

- $\mathbf{x} = (1, 2, 3) + 2\left((1, 1, 1) - (3, 2, 1)\right)$
- $\mathbf{y} = 1(1, 0, 0) + 2(0, 1, 0) + 3(0, 0, 1)$

Example: The Geometry of Vectors in $\mathbb{R}^2$

Sketch the vectors $(1, 0)$, $(0, 1)$, and $(1, 2)$ in the plane.

Example: An Algebra Question in $\mathbb{R}^2$

Can $(-1, 1)$ be written as sum of multiples of $(1, 2)$ and $(1, 1)$?

Definition: The Vector Space of Functions $\mathbf{F}(\mathbb{R})$

The vector space of **real valued functions** is given as follows:

$$\mathbf{F}(\mathbb{R}) = \{f : \mathbb{R} \to \mathbb{R}\}$$

The addition on $\mathbf{F}(\mathbb{R})$ is:

$$(f+g)(x) = f(x) + g(x)$$

The scalar multiplication on $\mathbf{F}(\mathbb{R})$ is:

$$(cf)(x) = cf(x)$$

Definition: The Vector Space of $n \times k$ Matrices $\mathbf{M}_{n\times k}(\mathbb{R})$

The vector space of $n \times k$ **("n by k") matrices** is given as follows:

$$\mathbf{M}_{n\times k}(\mathbb{R}) = \left\{ \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ \vdots & & \ddots & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{bmatrix} : a_{ij} \in \mathbb{R} \right\}$$

We write $[a_{ij}] \in \mathbf{M}_{n\times k}(\mathbb{R})$ where the bounds n and k are understood from context.
The addition on $\mathbf{M}_{n\times k}(\mathbb{R})$ is given by:

$$[a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$$

The scalar multiplication on $\mathbf{M}_{n\times k}(\mathbb{R})$ is given by:

$$c[a_{ij}] = [ca_{ij}]$$

Note: The entry a_{ij} is in row i and column j.

Amazingly, n-tuples, functions, and matrices are all examples of vectors!

Remark: The Importance of Uniqueness

Uniqueness is a major theme in algebra. It is customary to define "an" object with some property and then prove that it is "the" only object with that property. We use the axioms to show that there are unique object with particular properties.

Example: A Simple Uniqueness Proof

A3. There exists an element $\mathbf{0} \in V$ such that:

$$\mathbf{0} \boxplus \mathbf{x} = \mathbf{x}$$

for all $\mathbf{x} \in V$. We call $\mathbf{0}$ the **zero vector** of V.

Show that the zero vector $\mathbf{0} \in V$ is unique.

Example: A Obvious (?) Fact

If $\mathbf{z} \in V$ then $0\mathbf{z} = \mathbf{0}$. (This seems obvious, but does it follow from the axioms?)

Remark: Structures and Substructures

Another theme of algebra is that there are "structures" and "substructures".
We can learn a lot about a structure by understanding all of its substructures.

Definition: Subspaces

A subset $W \subseteq V$ that is a **subspace** of V if it is a vector space with the same operations as V.

Theorem: Characterization of Subspaces by Closure

Let V be a vector space and W be a non-empty subset of V.
W is a subspace if and only if $c\mathbf{x} + \mathbf{y} \in W$ for all $c \in \mathbb{R}$ and $\mathbf{x}, \mathbf{y} \in W$.

Example: Some Subspaces

Prove that the following are subspaces.

1. $W_1 = \{(x, 0) : x \in \mathbb{R}\} \subset \mathbb{R}^2$
2. $\mathbf{C}(\mathbb{R}) = \{f : \mathbb{R} \to \mathbb{R} \text{ continuous}\} \subset \mathbf{F}(\mathbb{R})$

Activity: Even and Odd Functions

Let $\mathbf{F}_{\text{Odd}}(\mathbb{R})$ be the set of odd functions in $\mathbf{F}(\mathbb{R})$ and $\mathbf{F}_{\text{Even}}(\mathbb{R})$ be the set of even functions in $\mathbf{F}(\mathbb{R})$. Prove that $\mathbf{F}_{\text{Odd}}(\mathbb{R})$ and $\mathbf{F}_{\text{Even}}(\mathbb{R})$ are subspaces of $\mathbf{F}(\mathbb{R})$.

Example: Subspaces of $\mathbf{M}_{2\times 2}(\mathbb{R})$

Prove that the following sets of matrices are subspaces of $\mathbf{M}_{2\times 2}(\mathbb{R})$.

1. The symmetric matrices.

$$\mathrm{Symm}_{2\times 2}(\mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ b & c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$$

2. The trace zero matrices.

$$W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R} \text{ and } a + d = 0 \right\}$$

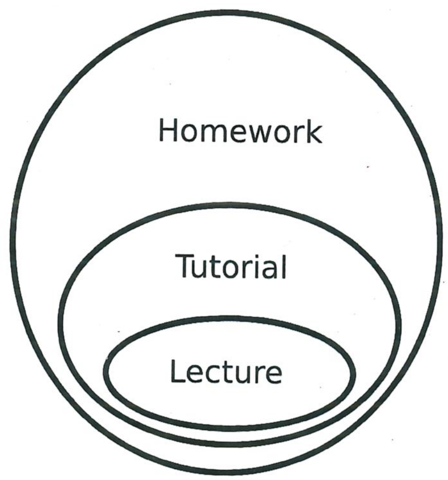
Theorem: Subspaces Closed under Intersection

If W_1 and W_2 are subspaces of V then $W_1 \cap W_2$ is also a subspace of V.

Week 1: A Quick Tour of Linear Algebra

Welcome to MAT A22
Linear Algebra I for Mathematical Sciences

Exam		40%
Test		20%
Assignments	$(11-1) \times 3\% =$	30%
Quizzes	$5 \times 2\% =$	10%



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On this quick tour, we're going to look at:

- The Surprisingly Common Vector Space Structure
- Vectors in $\mathbb{R}^n$.
- The Axioms of A Real Vector Space
- Uniqueness Proofs

Activity: What does Algebra Study?

Compare the following tables of numbers.

Remainder on division by five

$x \boxplus y$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

$x \oplus y$	0	1	2	3	4
0	2	4	2	4	1
1	3	0	0	2	2
2	0	3	3	4	4
3	0	4	1	1	3
4	1	4	3	2	1

Random numbers

Do you notice any structure in the tables?

This is a binary operation.

- $\boxplus$ is nicer or better than $\oplus$
- Lots of patterns "0 1 2 3 4" gets displaced by one place each row
- Along diagonals // the table is constant
- There are patterns about how many solutions to $= 0$ there are.
- Each column/row has {0 1 2 3 4} in some order.
- Symmetric across \ diagonal. $x \boxplus y = y \boxplus x$
- There is an identity $0 \boxplus x = x$

Remark: What is an Axiom?

An axiom is a definition. We use axioms to describe how mathematical objects work. Axioms are the basic elements of mathematical structures.

Cool ideas.

An operation that does not change things is an "identity" operation.

$f : \mathbb{R} \longrightarrow \mathbb{R}$

The identity function is

$$f(x) = x$$
$$= x + 0$$

Definition: Binary Operations

A **binary operation** is a function $\boxplus : X \times X \to X$ with two inputs.

- The usual addition on real numbers $+ : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a good example.

$$2 + 3 = 3 + 2 = 5$$

- Another common example is the usual multiplication $\cdot : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ of real numbers.

$$3 \cdot 2 = 3 \cdot 2 = 6$$

A binary operation does NOT need to be commutative associative

Definition: Commutative and Associative

An operation $\boxplus : X \times X \to X$ is **commutative** if:

$$x \boxplus y = y \boxplus x.$$

An operation $\boxplus : X \times X \to X$ is **associative** if:

$$x \boxplus (y \boxplus z) = (x \boxplus y) \boxplus z = x \boxplus y \boxplus z$$

Notation: We use the notation $\boxplus$ for a general operation with two inputs.
= "box plus"

Activity: Non-Examples

1. Give an example of a binary operation $\boxplus_1 : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ that is NOT commutative.
2. Give an example of a binary operation $\boxplus_2 : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ that is NOT associative.

(1) Division is not commutative $\frac{1}{2} \neq \frac{2}{1}$ (!) The domain is not $\mathbb{R} \times \mathbb{R}$.
Subtraction is not commutative: $1-2 \neq 2-1$
Exponents are not comm. $2^3 \neq 3^2$

(2) Subtraction is not associative $1-(2-3) \neq (1-2)-3$
Exponents are not assoc. $2^{(3^5)} \neq (2^3)^5$

Comm./Assoc. are rare or special properties.

We could define:

$a \boxplus b = 3a + b$ ← This is a binary operation.

Definition: The Axioms of a Real Vector Space

A **real vector space** or **$\mathbb{R}$-vector space** is a set V together with two binary operations

$$\boxplus : V \times V \to V \qquad \boxdot : \mathbb{R} \times V \to V$$ = "boxdot"

called **addition** and **multiplication by a scalar** such that the following axioms hold:

The elements of V are called vectors.

A1. $\boxplus$ is associative.

A2. $\boxplus$ is commutative.

A3. There exists an element $\mathbf{0} \in V$ such that:

$$\mathbf{0} \boxplus \mathbf{x} = \mathbf{x}$$

for all $\mathbf{x} \in V$. We call $\mathbf{0}$ the **zero vector** of V. (Could there be more than one?)

A4. For each $\mathbf{x} \in V$ there is an **additive inverse** $\mathbf{x}' \in V$ such that:

$$\mathbf{x} \boxplus \mathbf{x}' = \mathbf{0}.$$

A5. For all $\mathbf{x}$ and $\mathbf{y} \in V$ and $c \in \mathbb{R}$, scaling by c **distributes over** addition:

$$c \boxdot (\mathbf{x} \boxplus \mathbf{y}) = (c \boxdot \mathbf{x}) \boxplus (c \boxdot \mathbf{y}).$$

A6. For all $\mathbf{x} \in V$ and c and $d \in \mathbb{R}$, the usual addition in $\mathbb{R}$ distributes over addition in V:

Normal addition of real numbers →
$$(c + d) \boxdot \mathbf{x} = (c \boxdot \mathbf{x}) \boxplus (d \boxdot \mathbf{x}).$$

A7. For all $\mathbf{x} \in V$ and c and $d \in \mathbb{R}$, $\boxdot$ **associates with** the usual multiplication on $\mathbb{R}$:

Normal multiplication of real numbers →
$$(cd) \boxdot \mathbf{x} = c \boxdot (d \boxdot \mathbf{x}).$$

A8. For all $\mathbf{x} \in V$, we have: $1 \boxdot \mathbf{x} = \mathbf{x}$.

Note: These axioms have been adapted from Little and Damiano p.7 to use $\boxplus$ and $\boxdot$. Their order, numbering, and names are important.

Boxdot is "like multiplication."
Box plus is "like addition."

Definition: The Vector Space $\mathbb{R}^n$

The vector space of **real n-tuples** is given as follows:

$$\mathbb{R}^n = \{(x_1, x_2, \ldots, x_n) : x_i \in \mathbb{R}\}$$

Gr 12 vectors and Calculus

The addition on $\mathbb{R}^n$ is:

$$(x_1, x_2, \ldots, x_n) + (y_1, y_2, \ldots, y_n) = (x_1 + y_1, x_2 + y_2, \ldots, x_n + y_n)$$

The scalar multiplication on $\mathbb{R}^n$ is:

$$c(x_1, x_2, \ldots, x_n) = (cx_1, cx_2, \ldots, cx_n)$$

Note: Checking the details of Axioms 1-8 is a good (but long) exercise.
All the proofs essentially come down to "this property is true for real numbers".

Example: Check a Few Axioms!

Pick two axioms from A1 - A8 and Parker will check them.

(5) We check axiom A5. Pick $x, y \in \mathbb{R}^n$ and $c \in \mathbb{R}$.

$$c(x+y) = c((x_1 \cdots x_n) + (y_1 \cdots y_n))$$
$$= c(x_1+y_1 \quad x_2+y_2 \cdots x_n+y_n) \quad \text{\# Def}^n \text{ of } +.$$
$$= (c(x_1+y_1) \quad c(x_2+y_2) \cdots c(x_n+y_n)) \quad \text{\# Def}^n \text{ of } \cdot$$
$$= (cx_1+cy_1 \quad cx_2+cy_2 \cdots cx_n+cy_n) \quad \text{\# True in } \mathbb{R}.$$
$$= (cx_1 \quad cx_2 \cdots cx_n) + (cy_1 \quad cy_2 \cdots cy_n) \quad \text{\# Def of } +.$$
$$= c(x_1 \quad x_2 \cdots x_n) + c(y_1 \quad y_2 \cdots y_n) \quad \text{\# Def of } \cdot$$

(6) on the reverse.

(6) $(c+d)x$ We pick $x \in \mathbb{R}^n$ and $c, d \in \mathbb{R}$.

$= (c+d)(x_1 \; x_2 \cdots x_n)$

$= ((c+d)x_1 \; (c+d)x_2 \cdots (c+d)x_n)$ # Def of $\circ$

$= (cx_1 + dx_1 \; cx_2 + dx_2 \cdots cx_n + dx_n)$ # True in $\mathbb{R}$.

$= (cx_1 \; cx_2 \cdots cx_n) + (dx_1 \; dx_2 \cdots dx_n)$ # Def of +

$= c(x_1 \; x_2 \cdots x_n) + d(x_1 \cdots x_n)$ # Def of $\circ$

Example: Calculating in $\mathbb{R}^3$

Calculate the following vectors in $\mathbb{R}^3$.

- $\mathbf{x} = (1,2,3) + 2\left((1,1,1) - (3,2,1)\right)$
- $\mathbf{y} = 1(1,0,0) + 2(0,1,0) + 3(0,0,1)$ ←

$$x = (1,2,3) + 2(1-3, 1-2, 1-1)$$
$$= (1,2,3) + 2(-2,-1,0)$$
$$= (1,2,3) + (2(-2), 2(-1), 2(0))$$
$$= (1,2,3) + (-4,-2,0)$$
$$= (1-4, 2+(-2), 3+0) = (-3,0,3)$$

$$y = 1(1,0,0) + 2(0,1,0) + 3(0,0,1)$$
$$= (1,0,0) + (0,2,0) + (0,0,3)$$
$$= (1+0+0, 0+2+0, 0+0+3)$$
$$= (1,2,3)$$

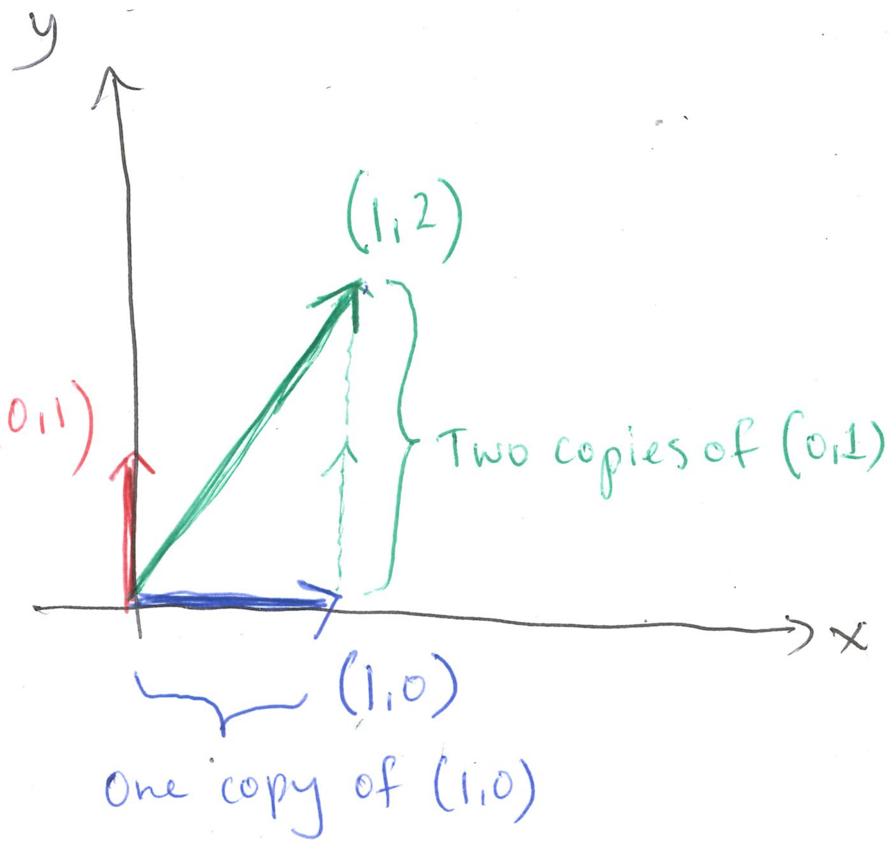
There is a very simple way to write any vector in $\mathbb{R}^3$ using $\{(1\,0\,0), (0\,1\,0), (0\,0\,1)\}$

$$(a,b,c) = a(1,0,0) + b(0,1,0) + c(0,0,1)$$

We will revisit this later on in the course.

Example: The Geometry of Vectors in $\mathbb{R}^2$

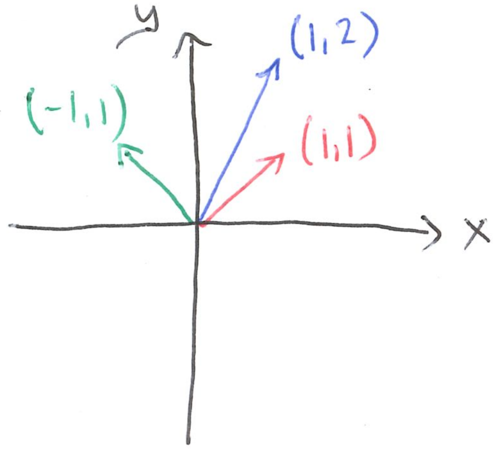
Sketch the vectors $(1,0)$, $(0,1)$, and $(1,2)$ in the plane.



$(1,2)$
$= 1(1,0) + 2(0,1)$

Example: An Algebra Question in $\mathbb{R}^2$

Can $(-1, 1)$ be written as sum of multiples of $(1, 2)$ and $(1, 1)$?



Geometry

We convert this geometry question in to an algebra question.

Re-phrase question as algebra.

$$(-1,1) = a(1,2) + b(1,1)$$
$$= (a, 2a) + (b, b)$$
$$= (a+b, 2a+b)$$

$+ \leftrightarrow$ sum

$a(1,2) \leftrightarrow$ multiples
$b(1,1)$

Algebra

Equivalently as a system of equations:

$$\begin{cases} -1 = a + b \\ 1 = 2a + b \end{cases}$$

Substitution + Elimination

We get: $(a, b) = (2, -3)$

This gives: $(-1,1) = 2(1,2) - 3(1,1)$

Yes it can be written as a sum of multiples of those vectors.

Definition: The Vector Space of Functions $\mathbf{F}(\mathbb{R})$

The vector space of **real valued functions** is given as follows:

$$\mathbf{F}(\mathbb{R}) = \{f : \mathbb{R} \to \mathbb{R}\} = \text{"all functions } f: \mathbb{R} \to \mathbb{R}\text{"}$$

The addition on $\mathbf{F}(\mathbb{R})$ is:

$$(f + g)(x) = f(x) + g(x)$$

The scalar multiplication on $\mathbf{F}(\mathbb{R})$ is:

$$(cf)(x) = cf(x)$$

Definition: The Vector Space of $n \times k$ Matrices $\mathbf{M}_{n\times k}(\mathbb{R})$

The vector space of $n \times k$ (**"n by k") matrices** is given as follows:

$$\mathbf{M}_{n\times k}(\mathbb{R}) = \left\{ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ \vdots & & \ddots & \\ a_{n1} & a_{n2} & \cdots & a_{nk} \end{bmatrix} : a_{ij} \in \mathbb{R} \right\}$$

row 1

column 2

a_{12} in row 1 and column 2

We write $[a_{ij}] \in \mathbf{M}_{n\times k}(\mathbb{R})$ where the bounds n and k are understood from context.
The addition on $\mathbf{M}_{n\times k}(\mathbb{R})$ is given by:

$$[a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$$

The scalar multiplication on $\mathbf{M}_{n\times k}(\mathbb{R})$ is given by:

$$c[a_{ij}] = [ca_{ij}]$$

Note: The entry a_{ij} is in row i and column j.

Examples in $\mathbf{F}(\mathbb{R})$

$$2(1+x) - 3(x) + \sin(x)$$
$$= 2 + 2x - 3x + \sin(x)$$
$$= 2 - x + \sin(x)$$

This is a vector in $\mathbf{F}(\mathbb{R})$

Examples in $M_{2\times 2}(\mathbb{R})$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + 2\left(\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix} \right)$$
$$= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + 2\begin{bmatrix} 1-0 & 0-0 \\ 1-1 & 0-2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 0 & -4 \end{bmatrix}$$

Amazingly, n-tuples, functions, and matrices are all examples of vectors!

Remark: The Importance of Uniqueness

Uniqueness is a major theme in algebra. It is customary to define "an" object with some property and then prove that it is "the" only object with that property. We use the axioms to show that there are unique object with particular properties.

Example: A Simple Uniqueness Proof

A3. There exists an element $\mathbf{0} \in V$ such that:

$$\mathbf{0} \boxplus \mathbf{x} = \mathbf{x}$$

for all $\mathbf{x} \in V$. We call $\mathbf{0}$ the **zero vector** of V.

Show that the zero vector $\mathbf{0} \in V$ is unique.

Suppose we had two zero vectors.

(1) $0 \boxplus x = x$ for all $x \in V$ and

(2) $0' \boxplus x = x$ for all $x \in V$.

Consider $0 \boxplus 0' = 0'$ by (1) letting $x = 0'$.

$= 0' \boxplus 0$ by commutivity of $\boxplus$

$= 0$ by (2)

Therefore $0' = 0$.

There is an alternative proof that begins:

"Consider $0' \boxplus 0 = 0$ by (2)"

Exercise: Write-out this alternative proof.

Example: An Obvious (?) Fact

If $\mathbf{z} \in V$ then $0\mathbf{z} = \mathbf{0}$. (This seems obvious, but does it follow from the axioms?)

We know that $0 = 0 + 0$ in the real numbers.

This gives:

$$\begin{aligned} 0\vec{z} &= (0+0)\vec{z} \\ &= 0\vec{z} \boxplus 0\vec{z} \quad \text{\# Distributive A6} \end{aligned}$$

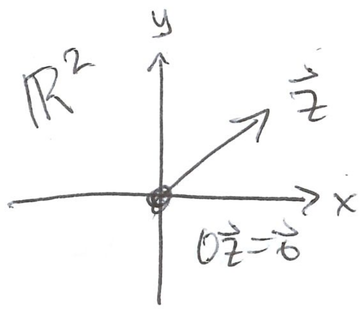
To cancel out a copy of $0\vec{z}$ we add the additive inverse of $0\vec{z}$ to both sides:

$$\begin{aligned} 0\vec{z} \boxplus (0\vec{z})' &= (0\vec{z} \boxplus 0\vec{z}) \boxplus (0\vec{z})' \\ \vec{0} &= (0\vec{z} \boxplus 0\vec{z}) \boxplus (0\vec{z})' \quad \text{\# Additive inverse A4.} \\ &= 0\vec{z} \boxplus \left(0\vec{z} \boxplus (0\vec{z})'\right) \quad \text{\# Associative A1} \\ &= 0\vec{z} \boxplus (\vec{0}) \quad \text{\# Additive inverse A4} \\ &= 0\vec{z} \quad \text{\# zero vector A3} \end{aligned}$$

Thus, $\vec{0} = 0\vec{z}$.

Rough Notes

$0\vec{z} = \vec{0}$



Assume
$0\vec{z} \neq \vec{0}$
then one of the "parts" is non-zero.
This works in $\mathbb{R}^n$.

$0 = 1 - 1$ in $\mathbb{R}$

$0\vec{z} = (1-1)\vec{z}$
$= (1)\vec{z} \boxplus (-1)\vec{z}$ # Distribution A6.
$= \vec{z} \boxplus (-1)\vec{z}$ # Identity A8.

Is the additive inverse of $\vec{z}$ equal to $(-1)\vec{z}$?

$0\vec{z} = (0 \cdot 0) \boxdot \vec{z} = 0 \boxdot (0 \boxdot \vec{z})$
$= 0 \boxdot (0\vec{z})$

$0 = 0 + 0$ $\qquad$ $0\vec{z} = (0+0)\vec{z} = 0\vec{z} \boxplus 0\vec{z}$
$\vec{0} = 0\vec{z}$

Remark: Structures and Substructures

Another theme of algebra is that there are "structures" and "substructures". We can learn a lot about a structure by understanding all of its substructures.

Definition: Subspaces

A subset $W \subseteq V$ ~~that~~ is a **subspace** of V if it is a vector space with the same operations as V.

Theorem: Characterization of Subspaces by Closure

Let V be a vector space and W be a non-empty subset of V.
(W is a subspace) if and only if ($c\mathbf{x} + \mathbf{y} \in W$ for all $c \in \mathbb{R}$ and $\mathbf{x}, \mathbf{y} \in W$.)

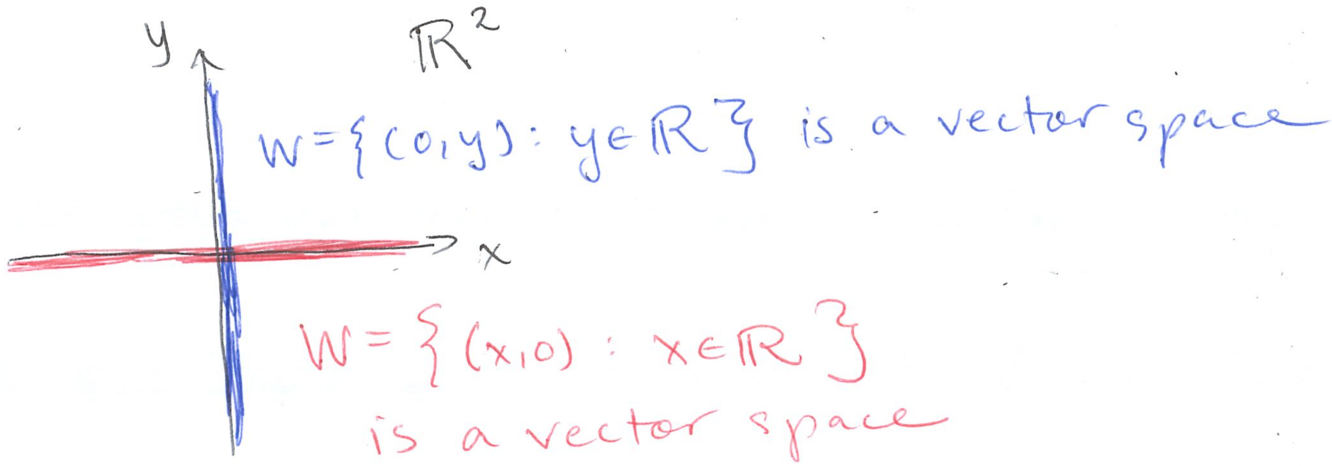
$\Rightarrow$ Suppose W is a subspace.
We pick $\vec{x}, \vec{y} \in W$ and $c \in \mathbb{R}$.
If $\vec{x} \in W$ and $c \in \mathbb{R}$ then $c\vec{x} \in W$ because W is a vector space.
If $c\vec{x} \in W$ and $\vec{y} \in W$ then $c\vec{x} + \vec{y} \in W$ because W is a vector space.

$\Leftarrow$ Notice that axioms A1, A2, A5, A6, A7, A8 are about the algebraic properties of $\boxplus$ and $\boxdot$. They hold on all of V and so they hold on W.
We need to check that A3 and A4 hold.

<u>Pf of A3</u>: Because W is non-empty we can pick $\vec{x} \in W$.

We have that: $(-1)\vec{x} + \vec{x} \in W$ taking $c = -1$, $\vec{y} = \vec{x}$.
$= (-1+1)\vec{x}$ # A6
$= 0\vec{x} = \vec{0} \in W$

Examples of Subspaces



$P_n(\mathbb{R})$ = polynomials of degree $\leq n$ with real coefficients

$\subset F(\mathbb{R})$

The polynomials are a subspace of all functions.

Pf of A4: Suppose $c\vec{x} + \vec{y} \in W$ for all $\vec{x}, \vec{y} \in W$ and $c \in \mathbb{R}$

Given $\vec{x} \in W$ we need to show $\vec{x}' \in W$.

Consider $\vec{x}' = (-1)\vec{x} + \vec{0}$ taking $c = -1$ and $\vec{y} = \vec{0}$.

We calculate: $\vec{x} + \vec{x}' = \vec{x} + (-1)\vec{x} + \vec{0}$

$= \vec{x} + (-1)\vec{x}$ # zero vect. A3

$= 1\vec{x} + (-1)\vec{x}$ # Identity A8

$= (1-1)\vec{x}$ # Distributive A6

$= 0\vec{x} = \vec{0}$ # By Lemma.

Thus W contains the additive inverse of $\vec{x}$.

A.3 Teaching Development Grants

Teaching Development Grant Application:
Active MAT A29 Calculus I for Life Sciences

- Title of Proposal: Active Applications for MAT A29: Calculus I for the Life Sciences
- Courses in which the grant will be applied: MAT A29
- Number of Students Directly Affected: 590 (average enrollment of last four years)
- Summary of Proposal:

  We propose to develop teaching materials for MAT A29 that are: (1) directly relevant to the life sciences, and (2) support active learning in the classroom. We will hire three students with experience in the life sciences to assist us in developing these materials. These teaching materials will be released as an Open Educational Resource (OER) by September 2025.

- Provide a brief description of the program enhancement activity/project:

  Before describing our project, we outline the issues with MAT A29 as it is currently taught. Presently, very little emphasis is placed on applications of the material to the life sciences. This leads to students feeling that the course is irrelevant to their studies (Aikens et al, 2021). The course is currently taught with a traditional lecture-based approach (or a pedagogy rooted in transmissionism). Many studies such as (Kramer et al, 2023) or the landmark (Freeman et al, 2014) suggest that students in STEM are better served by an active learning-based approach to course delivery.

  We propose to develop material for active learning in the classroom. Our approach to active learning will be guided by the Mathematical Association of America's Guide to Evidence-Based Instructional Practices in Undergraduate Mathematics (Ludwig et al, 2018). The literature on active learning shows that active instruction is beneficial to student achievement, but it is especially so for marginalized populations in STEM such as women and BIPOC students (Kramer et al, 2023). The MAA Instructional Practices Guide explicitly frames active learning as an inclusive and anti-racist pedagogy. We will adopt a worksheet-based format for active learning. Students will be provided with one worksheet per hour of lecture time. Each class session will begin with a "mini-lecture" of no more than fifteen minutes. Then students will work in groups on the worksheets. Each class session will end with a "review" of no more than twenty minutes. Solutions to the worksheets will be available as PDF slide presentations for students to review. The crucial point of our proposal is that each worksheet will have at least one application of calculus to the life sciences.

  We propose to hire a teaching assistant, with experience in the life sciences, to support the development of course resources for MAT A29. The teaching assistant will bring disciplinary knowledge to the collaboration and provide examples of models in the life sciences that can be treated with elementary mathematics. We will hire an additional two work study students to help us prepare and verify solutions to the in-class activities. The three hires will meet with us weekly throughout the Summer 2024 term, to share progress on the project. The hires will collaborate with us through a shared Overleaf account, which has already been paid for by Dr. Glynn-Adey.

Now, we describe the duties of the three hires in more detail. The teaching assistant, ideally an upper-year student double majoring in life sciences and mathematics, will help directly with course development and apply their disciplinary knowledge of the life sciences. Drs. Kielstra and Glynn-Adey will provide them with a list of calculus textbooks that align with the pedagogical goals of the course. The TA will select exercises and modeling tasks suitable for MAT A29 which foreshadow material that the students will actually encounter in their life science courses. Their direct experience with the life sciences curriculum will be essential to ensure that the materials we produce are relevant to students in MAT A29's future courses in the life sciences.

The two work study students, ideally lower-year students with an interest in the life sciences, will act as "play testers" for the course materials. Each week, they will be provided with the handouts produced by the course instructors and TA. The work-study students will attempt the tasks in the handouts, prepare their own solutions, and review the material produced by the course instructors and TA. They will not develop any course resources but will provide feedback on the resources prepared by the course instructors and TA.

We intend to make the resources we produce with this grant available to the public as an OER through eCampus Ontario by September 2025. The materials will be prepared throughout the Summer 2024 semester, and be ready for classroom delivery by the Fall 2024 semester. We anticipate that we will both teach MAT A29 in the Fall 2024 semester when it is a multi-section course. However, due to high drop and failure rates, MAT A29 is typically offered in both the Fall and Winter terms. The materials will be revised in the Winter 2025 term by Thomas Kielstra, who we anticipate will serve as the course instructor. In the Summer of 2025, we will revise the materials again, and submit them as an OER to eCampus. The source code and artifacts (such as images) of all course materials will be distributed under the Creative Commons BY-NA-SA 4.0 license on GitHub and Parker Glynn-Adey's website.

References

Aikens, Melissa L., Carrie Diaz Eaton, and Hannah Callender Highlander. "The case for biocalculus: Improving student understanding of the utility value of mathematics to biology and affect toward mathematics." *CBE—Life Sciences Education* 20.1 (2021): ar5.

Freeman, Scott, et al. "Active learning increases student performance in science, engineering, and mathematics." *Proceedings of the national academy of sciences* 111.23 (2014): 8410-8415.

Kramer, Laird, et al. "Establishing a new standard of care for calculus using trials with randomized student allocation." *Science* 381.6661 (2023): 995-998.

Ludwig, L., et al. "Guide to evidence-based instructional practices in undergraduate mathematics." *Washington, DC: Mathematical Association of America* (2018).

- What are the educational outcomes you hope to achieve?

First and foremost, we will improve the student experience of MAT A29. Presently, it is seen as an irrelevant burden, an unnecessary hurdle to entering the life sciences. We want to change this perception and make the class seem relevant by creating a collection of course materials that show students the full relevance of calculus to the life sciences. If we achieve this goal, it will result in a higher retention rate, and increased student achievement (Aikens et al, 2021). Our proposal directly addresses the following goal of the Campus Curriculum Review Working Circle.

(Curriculum Development) 1.3. Expand students' exposure to diverse knowledges and perspectives in the Sciences and Management by enhancing discipline-specific pedagogical, curricular, and co-curricular resources that can assist faculty and staff in undertaking this work and building opportunities for cross-disciplinary and cross-departmental mentorship and dialogue.

Second, we will create an active and inclusive classroom experience. The literature on active learning shows that active instruction is beneficial to student achievement, but it is especially so for marginalized populations in STEM such as women and BIPOC students (Kramer et al, 2023). The MAA Instructional Practices Guide (Ludwig et al, 2018) explicitly frames active learning as an inclusive and anti-racist pedagogy. The TA will help us design materials in alignment with the Guide. Given this, our proposal directly addresses the following goal of the Campus Curriculum Review Working Circle.

(Pedagogical Development and Related Supports) 2.5. Develop dedicated mentorship and educational opportunities for TAs at UTSC centered on equity, accessibility, anti-racism, and anti-colonialism.

A.4 Advice

Advice for Students

Parker Glynn-Adey

March 10th 2023

1 Introduction

Students often ask me for advice on how to improve in mathematics. I find that I keep recommending the same strategies across a broad range of courses. This document attempts to collect up these strategies. They may or may not work for you. Try them out in whatever order suits you best. Some of the strategies explicitly suggest that you try out other strategies. For example, Watch Your Self Talk suggests that you Set a Timer. If any of these things help, or you have a strategy that you think should be on here, please let me know.

Summary

1. Be Kind to Yourself
2. Watch Your Self Talk
3. Read Slowly
4. Read with a Pencil and Paper
5. Set a Timer
6. Identify The First Tricky Thing
7. Teach Other People (or Rubber Duckies)
8. Learn to Ask Precise Questions
9. Keep Seeking Help
10. General Life Advice
    1. Do Nothing: Meditate
    2. Do Something: Exercise

2 Be Kind to Yourself

Studying mathematics is hard. Being a student of mathematics is even harder. It is so easy to get bummed out and think that you will never make any progress. We all make errors in mathematics. However, errors lead to learning and progress. From time to time, you should acknowledge that you're doing an awesome thing by learning mathematics. **Be kind to yourself, forgive your own errors, and acknowledge that hard things are difficult.**

3 Read Slowly

When you're reading the textbook, or sources online, read slowly. There is a great quote from Bill Thurston, which talks about the rate at which we need to read mathematics.

I was really amazed by my first encounters with serious mathematics textbooks. I was very interested and impressed by the quality of the reasoning, but it was quite hard to stay alert and focused. After a few experiences of reading a few pages only to discover that I really had no idea what I'd just read, I learned to drink lots of coffee, slow way down, and accept that I needed to read these books at 1/10th or 1/50th standard reading speed, pay attention to every single word and backtrack to look up all the obscure numbers of equations and theorems in order to follow the arguments.

When you're reading material in mathematics, expect it to take a long time. This is common. You're not a slow reader. Mathematics is just very compact and requires careful attention to detail.

3.1 Read with a Pencil and Paper

To help stay active while reading, keep a pencil and paper handy. I think it is particularly important to work with physical paper and pencil. Paul Halmos has some good advice here.

Reading with pencil and paper on the side is very much better - it is a big step in the right direction. The very best way to read a book, however, with, to be sure, pencil and paper on the side, is to keep the pencil busy on the paper and throw the book away.

4 Set a Timer

It is interesting to experiment with spending a small limited amount of time studying. Set a timer on your phone[1], for a period of twenty or thirty minutes, and study without distraction until the timer goes off. When you are studying, it is easy to spend a lot of time and be disappointed with the results. One can get stuck in endless study sessions which switch for studying to hanging

[1] Alternatively, you can find a timer online. For example, https://www.online-timer.ner/.

out, to looking things up online, to eating, to reading the book, to working on some problems, to continued studying, to catching up on the news, and back to studying, *etc*.

If you pick a small amount of time, say twenty minutes, then you can guarantee that for those twenty minutes you are purely studying. Before starting the timer, turn off anything that could distract you and sink in to the material. During that timed interval, you should be fully present with the material that you're studying. After the timer goes off, you can decide if you want to start up another timer or switch tasks. One way to think about working with a timer is that you're purposefully interrupting yourself to see if you're still on task.

During the timed interval, it is helpful to set a concrete task to work on. You can write this task out before starting the timer, and check it if you feel yourself getting pulled off task. Some tasks that you might consider are:

- solve a particular problem,
- produce examples of a definition,
- read a single page of a book with pencil and paper.

While the timer is running, it is helpful to consider your self-talk (see Watch Your Self Talk) as it arises.

5 Watch Your Self Talk

We all talk to ourselves almost constantly. I call this kind of talk self-talk. It generally comes in two varities: neutral self-talk and negative self-talk. Most of time, self-talk is just mindless chatter about our surroundings or experiences. "Oh, that person has a nice hat. I am *really* early for class. I wonder what that noise is." When we are working on something new to us, this self-talk can become counter-productive or harmful. "I'm so dumb. I have no idea what is going on. I'll never make POSt." These examples of negative self-talk probably seem quite contrived. Negative self-talk is often nasty, clever, and attempts to undermine our efforts. I want to make it clear that we *all* experience negative self-talk. Here are some real examples of counter-productive self talk that came up for me, Parker Glynn-Adey, as I was writing this note about advice for students:

- "All of this stuff is obvious and stupid."
- "Writing this is a waste of time, I doubt anyone will actually do any of this."
- "I'm not a counsellor or a guru. Who am I to give this advice to anyone? I suck at this."

So, what can we do about negative self-talk? We can Be Kind to Ourselves! We can fight it with positive self-talk! If you notice that your self-talk is getting out of hand and starting to sound negative, you can fight it with some positive self-talk. You'll need to play with this technique to find your own town for positive self-talk. It helps to rehearse it our loud a couple times. "I'm so dumb. Oh yeah? I am learning. I'm studying manifolds. This is the opposite of dumb." Or something like, "I'll never pass this course. I'm doomed. Well, I'm able to devote the next half-hour to studying. If I study for the next half hour, then I will improve my odds of passing the course."

In this last example of positive self-talk, I included in another learning strategy. If we're working with a timer (see Set a Timer) then we can use the timer to limit the scope of our negative self-talk. We can say: "Yeah, this problem is hard. I don't understand it right now. But, in fifteen minutes, once my timer goes off, I can leave it behind."

6 Identify The First Tricky Thing

Getting completely lost is common when learning a new mathematical concept by reading a textbook, or reviewing your lecture notes. As you read along, a haziness or confusion starts to develop imperceptibly. Things make less and less sense as you read. Eventually, you get so confused that you have no idea what anything means. At this point, your negative self-talk can take over and force you away from the new material.

One strategy for getting out of this situation without letting negative self-talk crush us, is to to back-track to the beginning of the material and identify the first tricky thing. By reading carefully, with pencil and paper, we can usually find a specific sentence where the material got tricky.

Here is an analogy for this process. We can imagine the material that we want to learn as a road at night. The road is lit by streetlamps, which represent sentences in a reading. At the beginning of the road, everything is bright and comprehensible. At the end of the road, everything is dark and incomprehensible. When we noticed that we've come to a dark part of the road, we can go back to the beginning, and pay attention to the individual street lamps. There will be some first street lamp that is not turned on. That's the "first tricky thing" and we want to pay attention to it.

Once we know where the first tricky thing is, we can set about trying to understand it. With any luck, we can figure it out and make some progress. That street lamp turns on, and the road becomes a little bit brighter.

Knowing the location of the first tricky part can help us ask a precise question.

7 Learn to Ask Precise Questions

Mathematics is a language for asking and answering extremely precise questions. By getting better at asking questions, we can improve our ability to answer questions. Often, when a student comes to asking for direct one-on-one help with a course, I encourage them to come back the next week with three questions about the course material. Usually, their first batch of questions is quite vague. Asking questions is a skill that improves with practice.

As you're reading a piece of mathematics, it is helpful to identify the first tricky thing. Once you know its location, you can ask specific questions about it. By formulating a precise question about the tricky thing, you can hopefully figure it out. At the very least, you have a question to ask someone.

Suppose that you were reviewing your lecture notes about rotations. At some point, the notes assert:

$\det(R_\theta) \neq 0$ and so rotations are invertible.

If you wanted to understand this point better, you might formulate a question about it. Consider the following questions which get more and more specific.

1. I don't understand invertiblity.
2. Why are rotations invertible?
3. Why do the rotation matrices always have non-zero determinant?

The first "question" (which is really a statement) is so broad as to be unapproachable. If you asked a professor or TA this "question" then they would probably try to answer it. However, they would spend a lot of time talking about things other than your main concern: rotations.

The second question is more specific. It addresses your concern about rotations and invertibility; however, it could be addressed geometrically. The second question is still not specific enough to help address your point. And so, we formulate the third question which is very precise. It is precise enough to address exactly the part of the notes that is unclear to you.

8 Teach Other People (or Rubber Duckies)

The best way to learn a subject is to teach it.

Humans are social creatures. We do our best work *with* other people. For this reason, some people do their best thinking only when talking out loud to an audience. If you can convince a friend or room mate to help you study, then ask them to patiently listen while you explain stuff. Promise that you will return the favour and listen to them explain stuff after. It is especially helpful if they don't know the material which you want to learn.

A patient audience, who will listen to you endlessly, is a rare thing. Finding an audience to teach is the hard part of this approach. Luckily, there is a simple solution. Get a rubber ducky[2]. Explain the problem the subject that you're learning to the rubber ducky. Be sure to talk out loud and clearly explaining

[2]It doesn't have to be an actual rubber ducky, but it should be lively and friendly. Our brains process visual images containing eyes and mouths differently from other visual stimuli. My rubber ducky is a little three-eyed monster. I would advise against explaining stuff out loud to an empty room or a pet rock. It can be done, but it's not as effective as explaining to a happy little rubber duck.

everything to the ducky. Initially, it will feel a little bit crazy. After a while though, you'll get comfortable with the process.

9 Keep Seeking Help

There are so many resources available at UTSC to help you succeed in mathematics.

1. The Math Help Center IC 404
2. CTL's Math and Stats Support
3. Facilitated Study Groups
4. TAs' office hours
5. Professors' office hours
6. Upper year students

However, it is easy to avoid all these resources and feel totally isolated. We feel that seeking help is a waste of time or that it diminishes us. The truth is that you don't need to face your courses alone. When I was growing up, my instinct was to avoid seeking help and figure things out on my own. Eventually, I learned to combat this instinct and actively seek help.

Seeking help is a skill. You can get better at it through practice. Part of seeking help is learning to asking precise questions that will benefit you. Another part of seeking help is to learning to identify who[3] can help you and when.

Initially, asking for help makes us feel vulnerable or exposed. Society makes us feel that ignorance or not-knowing is a bad. It is worthwhile to pay attention to your self-talk surrounding help seeking. With practice, we can get very comfortable seeking help. Initially, it helps to be mindful of the fact that the university is an institution of learning. Its role in society is to produce and disseminate new knowledge. By seeking help, by seeking learning, you are actively furthering that mission. Professors love to help and discuss ideas[4].

10 General Life Skills

This sub-section contains some general advice which is not explicitly mathematical or related to study skills. I hesitated to include the following material in this page. However, my life has been dramatically improved by practicing a little bit of each of these things. So, I've included a little bit about exercise, meditation, and appreciation.

[3] Identifying who can help you is a major accomplishment once you reach the research level of learning. After you complete a PhD, there might only be a handful of people on Earth who know your field of study as well as you. It is helpful to be on good terms with these people. I had several research breakthroughs that consisted *entirely* of finding someone else that was better prepared to answer than me.

[4] Often students ask if they can ask a "quick" question.

10.1 Do Something: Exercise

Warning: I am not a medical doctor. Even though the amount of exercise that I am going to recommend is minimal, I don't know your physical condition or any issues that you might have. It is always worthwhile to talk with a doctor before beginning a new exercise program. Talk to your doctor, and listen to your body.

Try the following experiment, if your circumstances permit it[5]. Ask yourself how energetic and aware do you feel right now? Mentally make a note of how you are feeling; assign yourself a score from one (barely awake) to ten (hyper energetic and aware). Get up from your computer, or put down your phone, and try out the following. Jump up and down twenty times in a row. And once you're done, ask yourself again, how energetic and aware you feel. In my experience, the little bit of exercise caused a noticeable and beneficial boost in my energy level. After jumping around a bit, I have more energy and a greater sense of awareness and focus. As you've just seen, by doing the experiment, a little bit of exercise can change your energy level and ability to focus.

I recommend doing some amount of exercise, even an itty-bitty little bit, every day. Just walking a half-hour per day is known to have significant medical benefits. You can choose your own exercises to do. The exercise program that you choose does not need to be complicated and you don't need to go the gym[6].

I recommend the following simple exercises as a good place to start: walking, jumping jacks, and squats. None of these require any equipment, and they can be done anywhere. A minimal prescription for daily exercise, might be something like this:

- 10 minutes walking
- 3 sets of 25 jumping jacks
- 3 sets of 10 squats

If you try this prescription out immediately before studying, then I am certain that you will feel the increase of energy and awareness. Once you get in a daily groove of exercise, continue to experiment and explore. If you're interested in learning more about exercise, then I would be glad to suggest additional resources.

10.2 Do Nothing: Meditate

The act of sitting down, breathing deeply, and clearing your mind is much harder than you would expect. I recommend that everyone try meditation, at least a

[5] As I write this, I tried out the experiment myself. So, you can be sure that at least one person somewhere has done this. It's 08:40 in the morning, I'm sitting by the bank of the River Lea near Three Mills in London, England. Initially, I rated my awareness and energy level 5/10. After jumping twenty times, I feel closer to 7/10.

[6] As a UTSC student, you have free access to Toronto Pan Am Sports Center. If you want to get in to serious gym stuff, like weight-lifting or team sports, they have a tonne of great resources.

few times, to appreciate how difficult it is.

As an exercise, try the following simple meditation technique:

- Find somewhere that you can sit comfortably without interruption.
- Set a timer for two minutes.
- Pay attention to your breath:
  - Notice the sensation of breathing in.
  - Notice the sensation of breathing out.
  - When you get distracted, acknowledge that you are distracted, and return to focusing your attention on your breath.
  - When the two minute timer goes off, you are done meditating.

If you try this exercise, you will *immediately* get distracted. It happens instantaneously. As soon as you want to sit and breathe, you'll start thinking about all sorts of random things[7]. Once you are distracted, acknowledge it and think to yourself "Wow — I'm distracted." Turn your attention back to your breath. Keep gently returning your awareness back to your breath until the timer goes off.

The point of this sort of meditation is to build up the ability to direct your thoughts. Gently returning your attention to your breath is a means of learning of this ability. Once you've tried meditating a few times, you'll become more aware of what it feels like to be distracted. If you try to do short bursts of focused work, as I recommend in Set a Timer, then meditation will help you realize when you are distracted.

Meditation can help with test anxiety, emotional self-regulation, and a whole host of other issues. You don't need to do much of it for the effects to be present. As little as five minutes a day can have significant positive effects.

This document is available online at:
`https://pgadey.ca/notes/advice-for-students/`

[7] As I write this, I tried out the meditation exercise myself. Immediately, I got distracted because I wasn't sure if the timer that I set worked. (The timer that I used doesn't display any count down and I haven't used that particular timer software in a long time.) I got distracted thinking about about: whether the baby would cry, whether my runny nose would become a problem, a friend of mine has a young daughter who is having trouble sleeping at night, by the fridge turning on, *etc.* I got distracted thinking about all the things that I got distracted by. I got distracted by wondering if I should try to keep a count of the number of things that I got distracted by. The list of distractions just *keeps on going.*

Advice for New Teachers

Parker Glynn-Adey

April 11th 2024

1 Be kind

This is the highest rule of teaching (and life). Be really kind and forgiving. If you're too kind, nothing bad will come of it. If you're not kind enough, everyone suffers.

What does it mean to be kind to your students? Treat them as people with complex lives. They've probably got their own jobs, and their own love lives, and their own hang-ups. So, whenever possible, cut them some slack and lift them up. Compliment them when they do something cool. This can be really small: if a student wears really cool sneakers, let them know. If a student comes up with a genuinely interesting question, let them know. If you can choose an easy and instructive example, choose it over a hard and complicated one.

Remember that there is a very strong power dynamic at play in teaching. You've got the ability to bury these students, fail them, or make them miserable. Don't. Just be kind.

2 Be honest

This advice also sounds like it is common sense. However, in teaching, it is very tempting to bend the truth. If you're teaching a course in Advanced Nose Picking[1], it is tempting to pretend like nose picking is very important. Yet, in reality, nose picking is just a small part of life. Remember that they're taking your course to pursue their life and career goals, not to become nose picking experts.

It is hard to be honest when faced with the perennial question: "Will this be on the test?" It is tempting to (always) say: "Oh yes! Listen up or you will fail." And the reality is that there is always more material than you can possibly assess, so sometimes that answer should be: "No — This is not on the test, but it is worth knowing for reasons X, Y, and Z." Most people will tune you out the moment that you say something will not be assessed, but that's their loss.

[1] This is a pretty silly example course. It might get edited out later on. For now, I'm going to riff of this nose picking joke for the whole post.

And lastly, it is helpful to let your students know that you're new at this teaching thing. If you run in to a tricky point that you can't explain, or the learning management system blows up, or you get totally mixed up and tell people something nonsensical, just say: "Hey — I'm pretty new at this teaching thing. Could y'all be patient with me? I'll try to get this sorted out for our next class." If you are kind to your students, then they'll be kind to you.

3 Active is better than passive

I love this quote from Paul Halmos:

"The best way to learn is to do; the worst way to teach is to talk."

All the evidence seems to indicate that active teaching, where students are engaged in activities throughout class, produces much better results than passive teaching, where students are primarily listening and making notes. The landmark meta-study of this effect is: Freeman et al. (2014). One study that says that we must establish a new standard of care in calculus classes Kramer et al. (2023) and argues that if "traditional lecturing" were a drug on trial with "active learning" then it would be professionally negligent to prescribe "traditional lecturing".

On reflection, it isn't especially surprising that "doing" is more effective than "listening". If you reflect on how you've learned any complex skill, then you'll find that it involved a lot of trial and error, re-adjustment, and *doing*.

What does this practically mean for classroom teaching? Throughout your class, interleave small activities. In an hour, you might pose three to four active tasks, mixed in with explanation and guidance. To get started in this teaching style, start small. Give your students a task relevant to the material, set a timer (to create a sense of pressure), and ask them to try the thing. The classic task is think-pair-share: students are given a task, asked to think about it, discuss it in pairs, and then the pairs share their thoughts with the class. It takes some time to develop a knack for finding good activities[2].

4 Give time for people to form questions

This one is very practical. Give people time to form questions. If you say: "Any questions?" then count mentally (or on your fingers) to ten or twenty before moving on. It takes time for people to think about what they don't understand and put it in to words. (Think about how frustrating it is when doctors or bankers rush you through the "Any questions?"-question. Don't be that person!)

[2]One meta-observation is that selecting active learning tasks is something best learned by doing!

5 Walk around the room and chat with people

This is another easy to implement aspect of active learning. When you assign a task, and give it a time limit, then you're left facing the question: What should I do while the students work on the task? You could sit at the front of the room and twiddle your thumbs, or you could engage the students. Walk around the room a bit, and chat people up. Ask them how the task going. Compliment them on their fashion choices. Just be present with them. You'll probably get a lot of questions from shy people, and you'll get to meet your students.

6 Drink lots of water

A friend of mine, Tyler Holden, taught me this early on and I've found it extraordinarily helpful: bring a water bottle with you and sip from it often. There are two reasons to do this. First, it is good to stay hydrated. Second, it is important to pause your teaching. Walking over to your water bottle, opening it, taking a sip, and closing it, gives a nice cover for pausing for ten or twenty seconds while people catch up with the last slide or the example.

7 Teach the assessment

This one is controversial. The anti-"teach the test" people will tell you that you are not preparing your students for real life. They'll claim that it is important for students to be able to respond to novel tasks, and apply their knowledge to new situations. This is totally reasonable. I'm not about to say: "Teach your students the exact material on which you will assess them and nothing else."

The point that I want to make is about the perceived fairness of your assessments. It is all about the subjective impression that the students have of the material. You want the students to see that the instruction is relevant to the assessment. As a teacher, you can easily dupe and blindside your students. There are all sorts of reasons people do this: to control course averages, to assert dominance, or to teach people that life is hard. This is the point that I want to make clear: Students find it very disheartening when the assessment doesn't line up well (in their view) with the instruction.

And so, try to align your assessment with your instruction. This has the added benefit of giving you an actionable plan for creating a course: Create the assessments such as tests or assignments, and then plan backwards from them to create your instructional content.

8 The curse of knowledge

As an instructor, it is very easy to fool yourself in to thinking things are straightforward for your students. This effect is called the curse of knowledge.

To go back to the Advanced Nose Picking example from above: if all your instruction is centered on picking human noses, then you shouldn't assess the students with a "real life" example of picking equestrian noses. For an expert on nose picking, the situation is obvious: Horses lack fingers. Therefore, horses must use the nearest available limb and so they lick their noses. Mucus is pretty much the same across the whole mammalian family, and it dissolves in saliva. The horses get by just fine by licking their noses to pick them.

Anyone taking Advanced Nose Picking would be able to figure out this *obvious* generalization of nose picking. Unfortunately, it is not so. There is a lot of implicit expertise being deployed here; you need to have a bit of horse sense.

The flip side of the curse of knowledge is that the students don't know the material. This sounds so obvious that it is embarrassing to write it down, but it's helpful to remind yourself from time to time: The students don't know the material. If you find yourself talking with a student, red in the face, asking the same question over and over, remind yourself: "The student doesn't know."

9 Write detailed lesson plans

The very first lesson that I ever taught was a complete train wreck. I had asked a colleague, a leader in mathematics education, if they could write a letter of recommendation for me. The colleague noted that they'd need to see me teach, and offered me a chance to teach two back-to-back sections of their class. I had never taught before; I had no idea what to do. After a week of flailing around, I read up on the topic, sketched some notes on an index card, and decided to work out examples on the fly. The examples blew up in my face, I gave lousy explanations, and everything was a total disaster.

It sounds like I'm catastrophizing the situation, but I'm not. This is the feedback that I got from the expert colleague in-between the two sections. The colleague was furious, and they even offered to take over and give the second lecture. I persisted and gave the second lecture. It too was a train wreck, but a smaller better-managed train wreck. It was more like a toy train set crashing than a full passenger train.

The students probably learned nothing from my haphazard lecture. I learned[3] that it pays to take the time to prepare detailed lesson plans. My workflow for preparing lectures has evolved a lot over the years. Initially, I wrote everything out long hand in a notebook. Now, I typeset everything, use version control, and post material far in advance. Everything depends on what you're teaching and how you plan to teach it. The best way forward is to experiment.

[3]It is surprising that one needs to learn to prepare lesson plans. When I learned this lesson, I had attended thousands of hours of lectures and had naively assumed that they were improvised. There is a deep cognitive bias at play here: we don't perceive the preparatory work of a performance. This is a common illusion in writing: a good piece of writing just flows and seems effortless. It feels as though the author merely started typing and the finished work emerged. Similarly, in music, one sometimes feels that the musicians are there just improvising, when in reality you are seeing the end result of endless rehearsals. (Thanks, Alex, for pointing out this deep cognitive bias.)

If you're working towards an assessment: try to imagine what experiences your students will need to undergo in order to do well on that assessment. Usually, this takes the form of examples and tasks. Pick illustrative examples, write out all the examples in advance, see all the computations through to the end, and prepare some tasks for the students to do. Write it all up somewhere (on a blog would be great) and you're good to go. Be patient with yourself, and start small though. It takes a long time, over many iterations, to find a workable way to store lesson plans. Usually, you can go about preparing week-to-week: using your spare time in Week N to write the material for Week $N+1$.

10 Write a detailed syllabus

In addition to detailed lesson plans, it is helpful to have a detailed syllabus. I think of the syllabus as the rules of the boardgame that is my course. If any weird or unexpected corner cases turn up, I can point to the syllabus and say: "Look, this is the policy. I committed to it in advance. I cannot change it now."

In addition to any policies that your institution mandates must appear in your syllabus, it must contain: 1. A grading scheme for the course. 1. Your policy on late submission of course work, missed term tests, and other unfortunate events. 1. A week-by-week summary of the course material. 1. Your communication policy outlining how students should contact you.

After some experimentation, I started to write FAQ section into my syllabus. The point of the FAQ is to catch all the corner cases. The sections have grown as I've taught and re-taught various courses. One year, a student insisted that slipping papers under my door was an acceptable means of getting me to grade them. The next year, I added a clause to the syllabus clarifying that such work will not be accepted.

One last point about writing a good syllabus: You can use it to plan backwards. Set the week-by-week syllabus for the course. Then, decide on what your assessments will be. And then go about preparing detailed lesson plans.

11 Arrive early

It helps to be at the right place *before* the right time. If you arrive early and get your tech setup, you are free to chat with students as they arrive. This saves you a tonne of e-mail, and builds rapport. At my institution, instruction begins at ten minutes after the hour. I try to consistently arrive on the hour, and then I have ten minutes to get setup and hang out.

If you can't get in to the room early, because classes are scheduled back-to-back or you've got a long commute: don't try to juggle socializing and setting up. First setup your teaching materials, then chat with people. Trying to do both at the same time just slows things down.

12 Never rush

If you're deep in to a lecture on marsupial nose picking, and running short on time, it is tempting to plow ahead blast through the remainder of your slide decks: "Wombats do this, and opossums do that, and quokkas are a special case. All macropods use their non-prehensile tails." The words begin to rush out of you. You stop making time for questions. No one has any idea what quokka even *is* at this point.

When we rush, we're slipping in to transmission: the philosophy of education that says knowledge is transmitted *magically* from instructor to student. It is a mistake to think that if the instructor says all the content (once and only once), then the students will learn all the content. The truth is: no one is learning anything in a rush. Most people's pre-theoretical approach to teaching some form of transmissionism, and it's the default mode of instruction that we slip in to when feeling rushed.

Teaching and learning are iterative, reflective, and interactive processes. You can't rush them. If you find yourself short on time, grab a sip from your water bottle, make time for questions, and say that you'll catch up with the rest of the material during the next class. In my opinion, it is better to dismiss a class early than it is to rush.

13 Students primarily copy material from the board

One non-obvious finding from the education literature is that students primarily retain material that is written down on the board or in the slides. This creates a strong asymmetry between teacher beliefs and student action: teachers believe the verbal aspect of a class is primary, and students act as though the written aspect of a class is primary. The effect is so strong, that students and teachers still act this way even once you make them aware of it Fukaway-Connelly et al. (2017).

How did learning about this effect impact my teaching? I started to write a lot more down, even some silly stuff. For example, before learning about this effect, I would often talk about how mind-boggling big infinity really is. I would jump around making hand gestures of largeness. As that material wasn't written down, it didn't make it in to students mental models of the material. Now, I write something silly on the board like:

> Infinity is REALLY big: $10,000,000,000 < \infty$. Inifinity is bigger than EVERYTHING finite.

14 Be kind

This advice is so important that I've included it twice. Be kind. That's all. That's my advice about teaching: be kind.

A.5 Representative Scholarship

Investigating Mathematical Reading Comprehension

Parker Glynn-Adey[2], Micheal Pawliuk[1], Alexander Rennet[1], Jaimal Thind[1]
[1]University of Toronto - Mississauga, [2]University of Toronto - Scarborough

UNIVERSITY OF TORONTO

Introduction & Context

We studied students' **Mathematical Reading Comprehension (MRC)** and **overall success** in a flipped, recently redesigned large math course.

MRC Research Question: *"Did our redesign (and subsequent improvements) increase students' success and MRC?"*

`Course: MAT223, Linear Algebra I @ UTM`
`Period: F/W semesters, 2019-2022`

Methods

Instruments:

1. **Perceptions of Learning Mathematics Instrument** (Code et al, 2016), adapted with MRC-focused questions.
2. **MRC Test** (Self-designed, course-independent)
3. **Assessment of MRC problems** on course assessments (21-22 only).

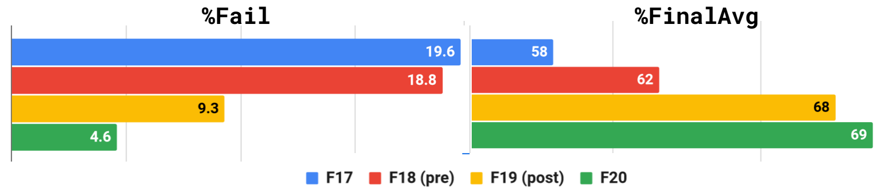
Delivery:

- **Instruments 1 & 2** were administered **Pre/Post-**semester online.
  (Course materials for all three years **targeted MRC** using 'scaffolded' pre-class readings and quizzes, and targeted instruction.)
- Students' performance on **MRC-focused problems (Instrument 3)** was compared early and late in the semester (in 21-22).

Results

Course redesign was successful (in line with literature, e.g. Freeman, et al 2014).

Figure 1: Grade Data for Pre/Post Redesign



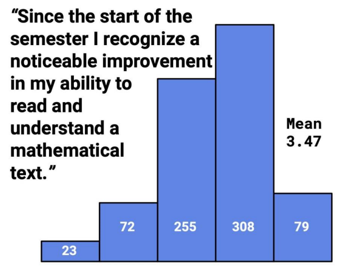
Instrument 1 (Perceptions Survey)**:**

- Student responses showed self-assessed improvement in MRC (Figure 2).
- Overall changes in measures of confidence were similar to expected results (decreases, but by less than non-active courses).
- **ELL students** showed a statistically significant improvement in confidence reading mathematics pre-COVID (Figure 3).

Instrument 2 (MRC Test): no statistically significant change.

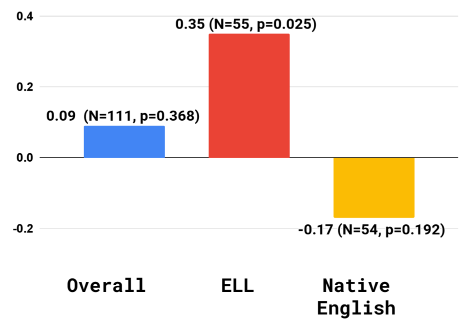
Instrument 3 (MRC Assessments)**:** students' performance showed no statistically significant improvement, and 1&2 were not improved during 21-22.

Figure 2: Self-Assessed MRC Improvement



`2019-22 Data, Likert Scale:`
`1(strongly disagree) - 5(strongly agree)`

Figure 3: Average Pre/Post Change in Confidence



Discussion

- **Pre/post-COVID data** is hard to analyze**.**
- The timing and lack of incentives may have resulted in **unreliable MRC Exit Test data.**
- **MRC-focused assessments are taxing** (Directly assessing MRC increases the difficulty level of course assessments).
  Students performed worse in various measures in 21-22, including confidence.
- Very positive result (at least pre-COVID) suggests that **ELL students benefit from a focus on disciplinary reading** and that this can support equity/accessibility.
- **Improving and assessing MRC is hard!** Many of our instruments and methods could use significant refining.
- Unclear how students make use of "scaffolded" readings; a **qualitative tool** (e.g. focus groups), could help.

Code, W., et al (2016). The **Mathematics Attitudes and Perceptions Survey**: an instrument to assess expert-like views and dispositions among undergraduate mathematics students. International Journal of Mathematical Education in Science and Technology, 47(6), 917–937.

Freeman, S., et al (2016). **Active learning increases student performance in science, engineering, and mathematics.** Proceedings of the National Academy of Sciences, , 111(23), 8410–8415.

`alex.rennet@utoronto.ca`
`jaimie.thind@utoronto.ca`
Department of Mathematical and Computational Sciences, UTM

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ORIGINAL PAPER

Tangible connections within the mathematical horizon: Exploring the Dihedral Calculator

Ami Mamolo[1] · Parker Glynn-Adey[2]

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Abstract
We report on a group theory activity in which learners explored dihedral symmetries through a tangible geometric model. This approach has historical roots in the work of Felix Klein's Erlangen Program and his *Elementary Mathematics from an Advanced Standpoint*. We situate our study with respect to this history as well as current educational research in spatial visual reasoning, abstract algebra, and teacher knowledge. Our findings highlight opportunities that tangible geometric models can provide for fostering structural and interconnected understanding characteristic of teachers' *knowledge at the mathematical horizon*.

1 Introduction

This research contributes to conversations around the role university mathematics can play in supporting teacher practice (Wasserman, et al., 2019) and fostering awareness of mathematical connections (Bass, 2022), structures (Taylor, 2018), and ways of being (Mason & Davies, 2013). In particular, we are inspired by the power of geometric models (Whiteley, 2019) and their potential to address the "double discontinuity" (Klein, 1945) between university and secondary school mathematics, which continues to be problematic (Liang, et al., 2022). To address the discontinuity within teacher education, there is a need to foster a general awareness of mathematics as a connected landscape (Bass, 2022), in addition to fostering specific connections within and outside of mathematics, termed intra- and extra-mathematical connections, respectively (De Gamboa, et al., 2022). In our research, we conceptualize this awareness as part of teachers' knowledge at the mathematical horizon (KMH), which is characterized by an understanding of mathematical structure, practices, and values that allow for an interconnected view of the mathematical world and how to be within it (Zazkis & Mamolo, 2011; Mamolo & Taylor, 2018).

In our review of the literature, we found extensive research which highlights the power of, and competencies for, teaching mathematical modelling for fostering extra-mathematical connections (Kaiser & Schukajlow, 2022; Kaiser et al., 2022), yet we found less attention paid toward the intra-mathematical connections made possible via modelling approaches. Nevertheless, intra-mathematical connections are recognized as similarly motivational for learners (Schukajlow, et al., 2022). In this paper, we seek to extend understanding of how intra-mathematical connections between concepts and practices can be fostered through the creation and use of a tangible geometric model.

In line with Sullivan et al.'s (2013) notion of a purposeful representational task, we consider how geometric models can provide opportunities to represent and broaden connections between core mathematical concepts, such as permutations and reflections. We also consider how geometric models can provide opportunities to connect school mathematics practices to universal ways of being enacted by mathematicians, such as seeking alternative representations, visualization, and play (Taylor, 2018; Whitelely, 2019).

2 Review of the literature

2.1 Mathematical models for mathematics education

The history of model making and use in mathematics dates back to the early 18th century. As Schubring (2010)

✉ Ami Mamolo
ami.mamolo@ontariotechu.ca

1 Ontario Tech University, Oshawa , Canada

2 University of Toronto Scarborough, Scarborough, Canada

recounts, the *Modellkammer*, a collection of mathematical models introduced by J.A. Segner that emphasizes applications of mathematics to modern technologies, was an early feature of the progressive teaching approach at Göttingen University. By the 20th century, however, the Modellkammer was dissolved by then-director H.A. Schwarz. Instead, Schwarz began to establish a collection of geometric models, which Klein lobbied to expand, noting the "necessity of "Raumanschauung" – spatial intuition – for successful mathematics teaching" (ibid.). For Klein, "The tendency to crowd intuition completely off the field and to attain to really *pure* logical investigations seems to me not completely feasible" (1945, p.14). As Cumino et al. (2021) note, the latter half of the 19th century saw progress in the study of geometry, which led to the creation of models for algebraic surfaces and curves. Yet, the shift to more abstract and analytic approaches of the early 20th century saw a decline in model use and production. Nevertheless, they note that scholars pushed back against verbal and abstract modes of teaching to advocate for "a multimodal learning of mathematics, developed (especially for geometry) through the use of concrete materials and movement" (Cumino, et al., 2021, p.153). Klein also emphasized the importance of teachers' accruing experiences building and using models (Mattheis, 2019).

Model building has at least two senses in the mathematics education literature: building physical models in the sense of making objects, and working with mathematical models of real-world scenarios. We note that Klein was interested in models in both of these senses; the physical models were held in the Modellkammer and the importance of modelling competencies was stressed in Klein's Meraner reform (Mattheis, 2019). Research regarding modelling in this second sense is flourishing (Cevikbas, et al., 2022), and attention has focused on conceptualizing and fostering modelling competencies (Garcia, et al., 2006; Kaiser & Schukajlow, 2022; Maaß, 2006; Niss and Højgaard, 2019). The role of teachers, and the importance of supporting and enhancing their modelling competencies for teaching, is a key theme. This theme resonates with Schubring's (2010) outlook on the Modellkammer and Klein's reform initiatives: "one has to return to teacher formation as key prerequisite for any sustainable improvement of mathematics instruction" (p.8). Mathematical modelling plays a role in connecting school mathematics to the broader world, where the possible connections are influenced by the sense in which modelling is interpreted (Garcia, et al., 2006).

In our research, we seek to extend conversations around mathematical modelling in teacher education by exploring learning opportunities that can be fostered through building a physical geometric model, which we see as related to the competency of solving mathematical problems within a mathematical model (Cevikbas, et al., 2022). In our case, the problems to be solved relate to the algebraic structure captured within the spatial visual representation of the geometric model.

2.2 Spatial visual models and teacher education

Spatial visualization is central to mathematical reasoning (Arcavi, 2003; Cumino, et al., 2021) and "has always been a vital capacity for human action and thought, but has not always been identified or supported in schooling" (Whiteley, et al., 2015, p.3). Recognized challenges in geometry instruction have been linked to procedural and rote approaches that emphasize deductive aspects of the subject while neglecting underlying spatial sense (Del Grande, 1990). Subsequently, research on teaching and learning of geometry has focused on how to foster spatial-visual approaches through uses of technologies, as well as on the professional development experiences needed to support teachers (Jones & Tzekaki, 2016).

Nagar et al. (2022) note that teachers' past experiences with mathematics influence their interactions with, and expectations for, geometric models. Sinclair et al. (2011) found that preservice teachers who had previously experienced mathematics via teaching methods that tackled topics separately and disjointedly encountered difficulties reasoning with, and inferring from, geometric models. They recommended that teachers encounter more robust and extended experiences exploring geometric models of typically algebraic tasks so as to "develop exploration strategies, build connections, and learn to notice mathematical details [of the models]" (p.156). They call for learning opportunities that can help "teachers revise their approach to mathematics" (ibid. p.157).

Recent research investigating the double discontinuity between university and secondary school mathematics highlights the importance of coherent experiences and values amongst learning environments, in particular because such experiences inform how preservice teachers envision their future roles (Liang, et al., 2022). The use of tactile models, or manipulatives, has been recognized as a useful pedagogical approach to help scaffold difficult lessons or concepts (Anghileri, 2006; Whiteley, 2019) though their use for exploring mathematics at the post-secondary level is not as well accepted. The predominance of lecture-based approaches in university mathematics courses leads students to believe that the use of tangible models is no longer appropriate as mathematics becomes more advanced and abstract. Incorporating instructional methods that support computational fluency with geometric intuition at the undergraduate level via the use of purposeful representational tools (Sullivan, et al., 2013) can help legitimize the relevance of such approaches to mathematical activity in general. As Watson and Ohtani (2015) observe, “Tasks shape the learners’ experience of the subject and their understanding of the nature of mathematical activity” (p.3). Mamolo et al. (2015) observe that geometric and spatial visual models provide learners of varying mathematical sophistication with a concrete tool through which to both represent and advance their conceptual understanding of major disciplinary ideas that cut across curricula. They emphasize that conceptual thinking involves connections across mathematical strands and a flexibility and fluency in navigating amongst multiple representations of the same idea. Geometric models in which mathematical structure appears tangible to the learner can provide a mechanism for fostering such connected and conceptual understandings of mathematics.

In this research, we investigate an instructional approach informed by the Erlangen Program (Klein, 1893; 1945), which uses tactile models as purposeful representational tools of symmetric groups in abstract algebra. Abstract algebra is commonly required in teacher preparation programs as it engages “students in the mathematical activity of defining a structure as a means of learning about it” (Zbiek & Heid, 2018, p.190). In particular, we are interested in how a tactile model can be used to cultivate geometric and spatial-visual intuition of algebraic structure, to help legitimize the epistemological value of model-making in mathematics, and to foster a sense of cohesion of ideas within the greater mathematical world.

2.3 Structural understanding and abstract algebra

Researchers have pointed to the relevance of structures in abstract algebra to school teaching, noting that an ultimate goal of school algebra is to foster understanding of mathematical structures (Usiskin, 1988). Groups were among the first algebraic structures to be axiomatized in the nineteenth century (Wussing, 2007). Historically, the discovery of group theory proceeded from particular isolated groups (e.g. of permutations) towards a general theory of abstract groups (Kleiner, 1986). In modern abstract algebra courses, students are introduced to group, field, and ring theories, often as mandatory part of mathematics degrees and teacher education. Lee and Heid (2018) advocate for instructional approaches that focus on developing a structural perspective of abstract algebra, which they articulate as “recognizing and having a tendency to look for mathematical objects, as well as knowing and using the relationships among mathematical objects” (p.295). Nevertheless, learners struggle with abstract algebra (Dubinsky, et al., 1994) and the challenges of reasoning with and acting upon mathematical objects at such a high level of abstraction (Hazzan, 1999). The struggles often relate to the process-object duality recognized in the APOS Theory (Dubinsky & McDonald, 2001), and articulated by Sfard (1991) as operational versus structural understandings, respectively.

In APOS terms, the process-object duality refers to the dual nature of mathematical ideas and representations as both procedures to be carried out (processes) and as encapsulated entities with their own properties (objects). For instance, when considering reflective symmetries, an individual with a process conception might think of a reflection as a procedure of flipping in order to determine a new image. In contrast, an individual with an object or structural conception might think of a reflection as a group element with order two on which other actions or processes may be applied. Symmetries admit other connections to object-process duality as well. Breive (2022) discusses a tension between how individuals perceive symmetry and how we describe it. Individuals can visually appreciate symmetry as a whole, but “the linearity of speech does not allow us to refer simultaneously to two different points, rather a symmetrical interaction between two entities is transformed into one entity that interacts with the other entity” (Breive, 2022, p.318).

We suggest that tangible geometric models which can portray symmetries without relying on the linearity of speech may help facilitate shifts in attention needed to begin to conceptualize symmetries structurally as objects in addition to processes. With respect to Sfard’s (1991) work, a structural understanding relates to a *trans-object* level of mathematical analysis, in which “relations across the objects are established in the learner’s mind and a collection of the object and others cohere and form a developed schema” (Lee & Heid, 2018, p.293). Such an understanding connects with the disciplinary knowledge for teaching that fosters a sense of the interconnectedness of mathematical ideas, practices, relationships, and domains – their *knowledge at the mathematical horizon*.

3 Theoretical framework

Klein’s Erlangen Program promoted a conceptualization of mathematics that emphasized its connections. Klein was interested in showing “the mutual connection between problems in the various fields” (1945, p.1–2) and supporting teachers in developing “the ability to draw (in ample measure) from the great body of knowledge there put before you a living stimulus for your teaching” (ibid., p.2). This connects to Klein’s idea of fostering an advanced perspective of elementary mathematics applicable to teaching situations. As Schubring (2010, 2019) notes, the English language translation skews some of the original intent of Klein’s work, misrepresenting notions of “elementary” and "advanced" as "simple" and "academic", respectively. In contrast, Schubring (2019) asserts that Klein's perspective of an *advanced standpoint* is one that provides a *higher view* of mathematical interconnectedness, including connections across disparate mathematical strands and amongst school and university mathematics; his notion of *elementary mathematics* is one that speaks to the *fundamental ideas* of the discipline.

Following Klein, Zazkis and Mamolo (2011) argued that the mathematical subject matter knowledge acquired in university studies can help provide a higher view of fundamental ideas in (school) mathematics. They conceptualize *knowledge at the mathematical horizon* (KMH) as an advanced understanding of mathematical structure, practices, and values that situates the mathematics of the moment within a greater mathematical world, and which can be leveraged in teaching situations in order to support and promote students' mathematical activities. They extend the work of Ball and Bass (2009), who introduced KMH as knowledge that "engages those aspects of mathematics that… illuminate and confer a comprehensible sense of the larger significance of what may be only partially revealed in the mathematics of the moment" (p.5).

As per Ball and Bass (2009), KMH includes four elements: (i) a sense of the mathematical environment surrounding the mathematics of the moment, (ii) major disciplinary ideas and structures, (iii) key mathematical practices, and (iv) core mathematical values and sensibilities. In the perspective developed by Zazkis and Mamolo (2011; also Mamolo and Taylor, 2018), KMH is closely connected to a teacher's focus of attention and "her ability to flexibly shift attention such that relevant properties, generalities, or connections, which embed particular mathematical content in a greater structure [can be] accessed in teaching situations" (Mamolo & Taylor, 2018, p.434). Shifts in attention are key mechanisms through which individuals develop mathematical awareness for teaching that includes both subject matter mastery and an ability to articulate how to foster such mastery in others (Mason, 1998).

To understand some of the shifts of attention necessary for mathematical understanding, Zazkis and Mamolo (2011) connected the four components of KMH to Husserl's philosophical notions of inner and outer horizon, extending it to encompass the abstractness of a mathematical object. An object's inner horizon is conceptualized as specific features of the object itself and the attributes evoked by its particular representation. As such, an object's inner horizon depends on our focus of attention, and may shift as attention also shifts. The outer horizon represents the "greater mathematical world", including the general structures, values, and sensibilities, which are relevant to, and exemplified by, the particular object. Mamolo and Taylor (2018) articulated connections across conceptualizations of KMH, as depicted in Table 1.

With these links, Mamolo and Taylor (2018) analyze how group theoretic ideas are a valuable component of mathematical knowledge for teaching. They articulate pathways from specific "locations" of the mathematical landscape of the moment, through the greater mathematical world, and back again, analysing examples that connect group structures to inverse functions (Zazkis & Marmur, 2018), isomorphisms to functions (Wasserman & Galarza, 2018), and Pythagorean triples to ring structures (Cuoco, 2018). Zazkis and Marmur (2018) suggest that group theory "can serve as a mathematical guide for teachers in situations of contingency" (p.379), while Mamolo and Taylor (2018) elaborate on the central importance of paying explicit attention to the mathematical structures of abstract algebra.

In abstract algebra, "we compare and contrast structures, exemplify and extend them, investigate implications, push boundaries, and play with relationships, all with a sort of directness and cohesion" (Mamolo & Taylor, 2018, p. 438). These ways of being mathematical are connected to schoolteachers' practices and decision-making. Planning decisions such as which definitions or properties to address, what emphases to place (Wasserman & Galarza, 2018), task design decisions that illuminate rather than cloud underlying solving methods (Cuoco, 2018), and in-the-moment

Table 1 Interpreting components of KMH (Mamolo & Taylor, 2018, p.435)

KMH components (Ball & Bass, 2009)	Our interpretation of components	Connections to inner and outer horizons
Mathematical environment surrounding current "location"	Knowledge of how the current subject matter relates to previously learnt and future concepts, within and across specific grades	Is influenced by focus of attention (inner horizon) and understanding of the lay of the land (outer horizon)
Major disciplinary ideas and structures	Knowledge of the underlying structural components of mathematics, such as connections between seemingly disparate content	Structure embeds specific content within the greater mathematical world (outer horizon)
Key mathematical practices	Including conjecturing, generalizing, and proving	Ways of working with specific practices (inner horizon) within a greater mathematical world (outer horizon)
Core mathematical values and sensibilities	Including precision, axiomatic thinking, and questioning conventions	Ways of being within the greater mathematical world (outer horizon)

responses to unanticipated student questions or confusions (Zazkis & Marumur, 2018) are shaped and influenced by the underlying structure, and disciplinary practices and values embodied in abstract algebra.

While much of Klein's considerations for developing connected understandings focused on specific mathematical ideas such as logarithms, he also took more general approaches when considering the higher standpoints afforded by geometry (Allmendinger, 2019). In this paper, we also take a general approach and consider how a geometric model can foster connections between abstract algebra and geometry, as well as between permutations and group elements, and between mathematical ways of being that are experienced in early schooling and at university levels. In line with Zazkis and Mamolo (2011), we conceptualize KMH "as an advanced perspective on elementary knowledge, that is, as advanced mathematical knowledge in terms of concepts (inner horizon), connections between concepts (outer horizon), and major disciplinary ideas and structures (outer horizon)" (p.12). We suggest that an advanced standpoint, or higher view, of fundamental ideas can foster KMH, both in terms of teachers' knowledge of disciplinary ideas, practices and sensibilities, as well as how ideas are connected and situated within a greater mathematical world. In this paper, we consider how tangible, geometric explorations of group theory may provide opportunities to foster KMH, with particular attention to importance of mathematical structure and interconnectedness. As such, we explore the research question:

> *In what ways can tactile experiences with geometric models in undergraduate mathematics provide for opportunities to foster knowledge at the mathematical horizon?*

4 Methodology

4.1 Design of the dihedral calculator

Klein introduced an approach to geometry, which was independently developed by Poincaré and Lie (Hawkins, 1984), that focused on classifying geometries by their symmetry groups. This changed the emphasis of geometry from the study of particular spaces and their associated metrics, to the study of spaces and their symmetry groups. Norton (2019) notes that such an approach "focuses our attention on the dynamic operations that define mathematical objects rather than the static figures that we use to represent them" (p.27). Piaget (1970) noted this as a "radical change of the traditional representational geometry into one integrated system of transformations" (p.22). This shift situates geometry within the framework of algebra and brings it in line with contemporary undergraduate education. For Klein, the "strong development of space perception, above all, will always be a prime consideration" (ibid., p.4). Indeed, strengthening learners' "spatial intuition" was a key objective of Klein's (Mattheis, 2019, p.93), as well as of his contemporary Peter Treutlein (Weiss, 2019), who leveraged paper folding to foster spatial intuition and teach modern approaches to geometry. Folded-paper models are notable as the transformation involved in producing a 3D shape from a 2D sheet of paper has precise mathematical meaning (Cumino, et al., 2021). The Dihedral Calculator is an example of paper craft with a deliberate purposeful design, which we developed to facilitate exploration of fundamental ideas in abstract algebra.

The dihedral groups D_n are one of the first examples of groups studied in undergraduate abstract algebra, and represent the group of symmetries of regular n-gons in the plane. The group structure of D_n can be depicted geometrically as polygons with dotted lines marking the lines of reflection (see Fig. 1 by Keith Conrad).

The purpose of the Dihedral Calculator is to make the geometric transformations of the dihedral group palpable by allowing learners to experiment with rotations and reflections. As a model, the dihedral calculator consists of two regular n-gons: the base and the disk, made out of paper. The front and back of the disk are labelled, as in Fig. 2, and each vertex of the disk is assigned a unique label. Notice that the vertex labels on both sides of the disk agree at the vertices. The vertices of the base are labelled like the vertices of the disk, and there is a pair of reflections *s* and *t*. To assemble the calculator, place the disk on top of the base. The disk should be able to move freely, rotate, and be turned over. The model is designed so that positions of the disk, which correspond to symmetries of the *n*-gon, can be viewed and described as permutations of the vertex set and as products of the reflections *s* and *t*. Writing the reflections *s* and *t* as

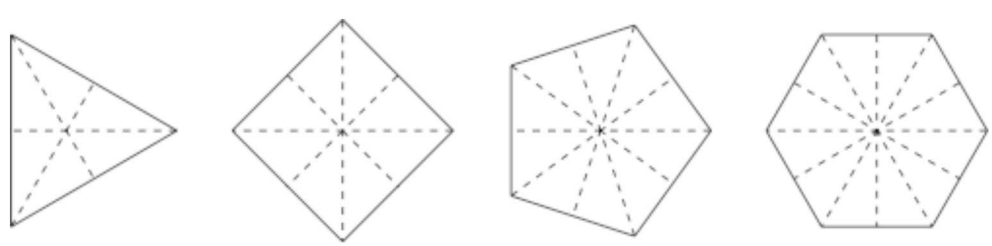

Fig. 1 Geometric representations of D_3, D_4, D_5, and D_6

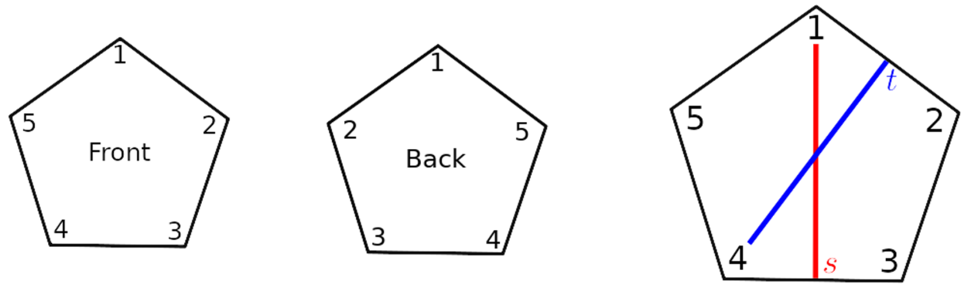

Fig. 2 The disk (left and center) and the base (right) with their labels

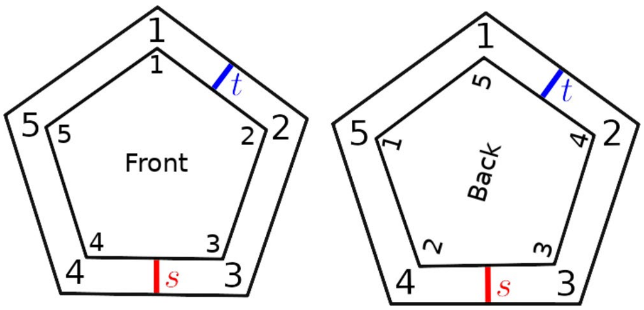

Fig. 3 The dihedral calculator assembled (left) and after the transformation *sts* (right)

permutations in cycle notation, we have $s = (25)(34)$ and $t = (12)(35)$.

Consider the position of the disk illustrated in Fig. 3 (right). This position can be written as a permutation of the vertices of the disk as follows: $p = (15)(24)$. Notice that we map the vertices on the base to the values on the disk; the importance of this convention is discussed in Subsection 5.1. One can verify that $sts = p$ by direct calculation with permutations. The physical model allows individuals to instantiate concepts from abstract algebra, such as permutations and cycles, in a concrete physical form. This physical representation builds a familiarity with core concepts of abstract algebra while encapsulating dihedral structure and structure-preserving mappings that can act on the regular *n*-gon. It also helps to develop a sense that groups naturally occur as the symmetries of real world objects.

The Dihedral Calculator provides an instance of a group admitting distinct presentations – via permutations and via reflections, fostering awareness of the mathematical value of multiple perspectives on a single problem. As elements of the symmetry group, the reflections *s* and *t* have order two, however, this is not the only relation they satisfy. Experimenting with the physical model shows that *st* produces a counter-clockwise rotation by $2\pi/5$ about the center of the disk. If the transformation *st* is iterated five times, then the disk rotates through 2π radians and returns to its initial position. This highlights an additional relation among the generators *s* and *t*. Working at the level of permutations, we see that: $st = [(25)(34)][(12)(35)] = (54321)$ which is a cycle of order five. Thus, we can present the group by its generators and relations as:

$$D_5 = \left\langle s, t | s^2 = t^2 = (st)^5 = e \right\rangle$$

4.2 Data collection and analytic approach

This study took place in a fourth year geometry course at a large Canadian university. The course is a degree requirement for all mathematics majors and its prerequisites include introduction to proof, linear algebra, and group theory. The textbook used was Sosinskiĭ (2012), which defines a "geometry in the sense of Klein" to be a pair (X : G) where X is a set and G is a group of automorphisms of that set. To make the high level of abstraction inherent in this perspective more concrete, a component of the course involved model building to foster tactile experiences of the groups involved. We focus this paper on one model from the course. A written description of how to assemble the model and a short silent video showing two hands performing the main steps of the construction were provided to students. Students were asked to construct a Dihedral Calculator from paper and use their construction to answer questions about group elements and structure.

This course ran September to December 2020; during the height of the COVID-19 Pandemic and all instruction and research activity was shifted to remote online environments. Given the nature of the online constraints during this time, data collection was limited to participant artefacts, which were submitted through the course's learning management system. Our data consists of participants' responses (N = 39) to the written component of the tasks along with photographs of their models; their written work was in response to both mathematical activities as well as feedback on their engagement with the model. For the calculational part of written component, questions addressed the behaviour of combining reflection and rotations. For instance, participants were asked to use their models to compare and contrast the group elements sr^k and $r^k s$, where $r = st$ is a rotation, and their respective behaviours. In addition, participants were asked to explore equivalency classes of D_5, as well as to pose their own original question based on using the model to solve a related mathematical problem. The tasks were designed so that students would provide multiple representations of their responses – briefly with fifty-words-or-less written descriptions, computationally with algebraic notation, and visually with diagrammatic representations and photographs of their physical model. Photographs of the physical models were taken at various stages of the exploration, including when the model was in the identity position and when it was positioned to depict non-trivial group elements. Given limitations imposed on our data collection procedures that prevented us from observing students interact with the model in real time, we relied on comparisons of photographs taken at multiple stages of the exploration for our analyses. For instance, to infer how the model could have been used in order to depict a nontrivial group element, we compared photographs of the final position of the model with photographs of the identity position. We then recreated possible steps that would yield the position of the nontrivial group element from the identity position. The axiomatic structure of group theory, and the uniqueness of a group's identity element and inverse elements, allowed us to conduct this analysis with a high degree of confidence and reliability.

We thematically examined the data, first coding independently and then meeting several times to compare. We analysed the photographic data of how participants constructed, labelled, and depicted their models, focusing on spatial visual features of the models, such as the position of the disk and orientation of the labels. We crossed analysed these features with the respective drawn diagrams, written descriptions, and algebraic calculations. We first coded with respect to components (i) and (ii) of Table 1, examining structural similarities and discrepancies amongst the photographed models, the diagrams, and the algebra, seeking to interpret what possible connections within the mathematical horizon could be afforded by the model. We attended to participants' notational choices, both in how they depicted their models and images, as well as in their algebraic calculations. Subtleties in how the models were built (with what materials), positioned (disk relative to base), and labelled (disk and base), provided insight into participants' reasoning and different aspects of the mathematical horizon which could be fostered through an Erlangen-inspired approach to abstract algebra. We then re-examined the data and coded with respect to components (iii) and (iv) of Table 1, analyzing the data for evidence of connections beyond their particular model to general ideas, such as the relevance of modelling practices and spatial visual reasoning in mathematics.

5 Findings

We examine ways in which tactile experiences with a physical geometric model of the Dihedral Calculator can provide for opportunities to develop KMH. We analyze our data with respect to the four components of KMH: (i) the mathematical environment surrounding the current 'location' of learning; (ii) major disciplinary ideas and structures; (iii) key mathematical practices; and (iv) core mathematical values and sensibilities. As we are concerned with teachers' horizon, we include in our analyses consideration of inner and outer horizons with an eye toward how our purposeful representational tool can facilitate shifts in how students attended to the mathematics at hand (inner horizon) and what aspects of the greater mathematical world were brought into view as a result (outer horizon).

When considering KMH through the lens of inner and outer horizons, we see the interconnected nature of the four components of Ball and Bass's (2009) conceptualization. For instance, a structural understanding of mathematics contributes to an individual's knowledge of the mathematical environment, and awareness of mathematical sensibilities guides ways of working with specific practices. As such, we organize our findings in two sections. Section 5.1*Highlighting key concepts with purposeful representational tools* analyses instances of structural understanding that can emerge from students' engagement with the Dihedral Calculator and what aspects of the mathematical landscape were brought into view by the purposeful design of the model. Section 5.2*Exemplifying the duality of mathematical ideas and their representations* analyses the mathematical sensibilities that can be fostered through exploration with the model and the key mathematical practices that can be brought into view.

5.1 Highlighting key concepts with purposeful representational tools

Our findings revealed that the experience exploring group theoretic ideas via a tactile model provided new opportunities for students to make connections amongst geometry and algebra, as well as amongst specific ideas within algebra, corresponding to "major disciplinary ideas and structures" (Table 1, component ii). In general, students ($n=27$) reported in their feedback that they "learned a lot" from making the model, and they appreciated the surprising connections amongst abstract algebra and geometry. For instance, students remarked on how the course drew their attention to "how different branches of math can relate" and how "things tend to fit together nicely in almost a magical way" suggesting a broadening of their outer horizons and

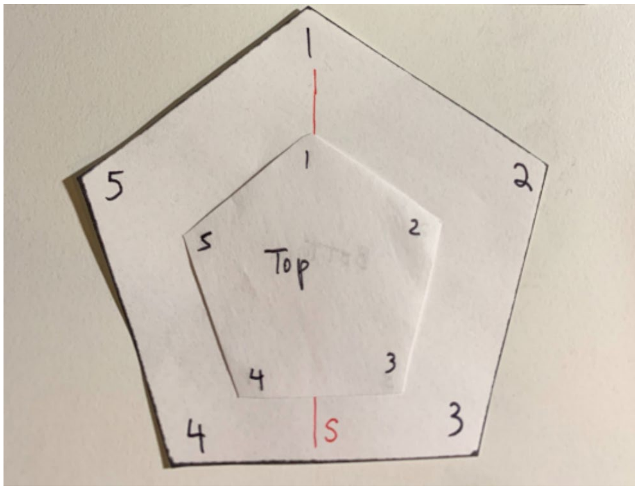

Fig. 4 A student's construction of the dihedral calculator, marking an axis of reflection, s

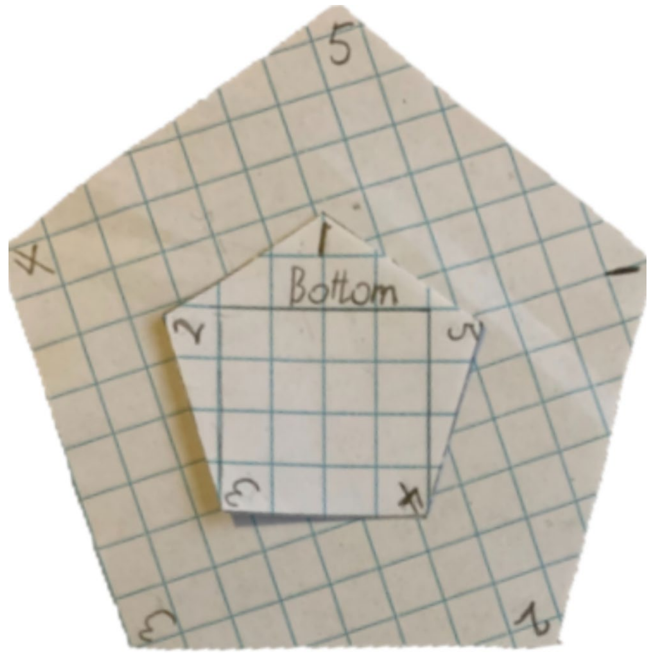

Fig. 5 Connecting permutations, reflections, and rotations via the position of r^2sr

expectations for the interconnectedness of the mathematical landscape.

An important aspect of the mathematical landscape (Table 1, component i) captured by the Dihedral Calculator is the connection amongst permutations, reflections, and rotations. In particular, the model provides students a way to develop a feel for how the group operations of D_5 behave, emphasizing structural aspects that exist in the outer horizon of every group. To answer the questions, students needed to play with the disk, spinning it or flipping it, to intuit (in the sense of Descartes, see Garber (1998) the position of the disk and structure of the accompanying group element. Depicted in Fig. 4 is a typical paper construction of the Dihedral Calculator, depicting the identity position. In general, the position of the disk corresponds to a permutation of the vertices of the disk, and in this way, the model captures the group structure and provides a tactile mechanism to determine the behaviour of group elements, which correspond to major disciplinary ideas (Table 1, component ii).

The model required students to read the permutations as maps from the base to the disk so that each generator could be replaced by its representation in cycle notation in order to obtain a permutation that accurately described the position of the disk. The calculator was used to explore equivalence classes, another major disciplinary idea, by rotating or flipping the disk and noting its final position, which offered an embodied experience connecting permutations, reflections, and rotations. For instance, when asked to calculate r^2sr , students were required to physically rotate the disk, flip the disk along the axis of reflection, and then rotate it twice to find the final position. One such example is provided in Fig. 5, where the student used their physical model to determine the final position of r^2sr, which they wrote as (15)(24). In group-theoretic terms, the convention of reading from the base to the disk is consistent with having both the group generated by r and s and the permutation group act on the right as group actions, which was a strategic design choice for this purposeful representational tool. The design decision to read from base to disk also allowed for students to self-check their work – students who read the permutation from the disk to the base (as opposed to from the base to the disk) obtained cycles that were inverses of the intended permutations, instantiating a common challenge learners face when composing group elements. The majority of students (approximately two thirds) either labelled their disks correctly right away or noticed a discrepancy when self-checking and adjusted their labelling, which was evidenced by erasing marks on the models or diagrams and crossed out written work in the algebraic computations. For the calculator to produce consistent results as permutations, the two sides of the disk must also have their vertices labeled consistently. That is, each vertex must have the same label on both sides of the disk. A handful of students labeled the disk so that the numbers appeared in clockwise order on both sides of the disk. This produced a model where the vertices were given different labels on either side of the disk. We suggest that students who made this labeling error struggled to connect permutation and generator/relations approaches to generating a group. Thus, while the Dihedral Calculator can help connect these ideas within the horizon for some students, for other students a more explicit articulation of these connections and their relationship seemed needed.

The physical representation of the Dihedral Calculator allowed students to view operations in a spatial visual way and as such apply actions on these operations, a key consequence that marks and results from encapsulating processes into mathematical objects (Dubinsky & McDonald, 2001). Shifting between process and object conceptions of group operations connects to both inner and outer horizons of the dihedral group. On one hand, the ontological shift from process to object can be described as a shift in attention between intended properties of the operation (inner horizon), such as reflection as something to carry out (process) or as something upon which actions may be carried out (object). On the other hand it also instantiates a phenomenon that exists across the greater mathematical world (outer horizon): the dual nature of mathematical ideas and their representations. In what follows, we highlight instances where students internalized the physical actions carried out with the models and in doing so engaged with key mathematical practices, and we link an understanding of mathematical structure with an awareness of core mathematical sensibilities.

5.2 Exemplifying the duality of mathematical ideas and their representations

Our findings showed evidence of students (approx. one third) internalizing the actions of rotating and flipping the disk into an imagined process, which could then be executed to understand the group element in a structural way as a combination of operations. We see this as engaging in "key mathematical practices" within KMH (see Table 1, component iii). We view the internalization of model elements and

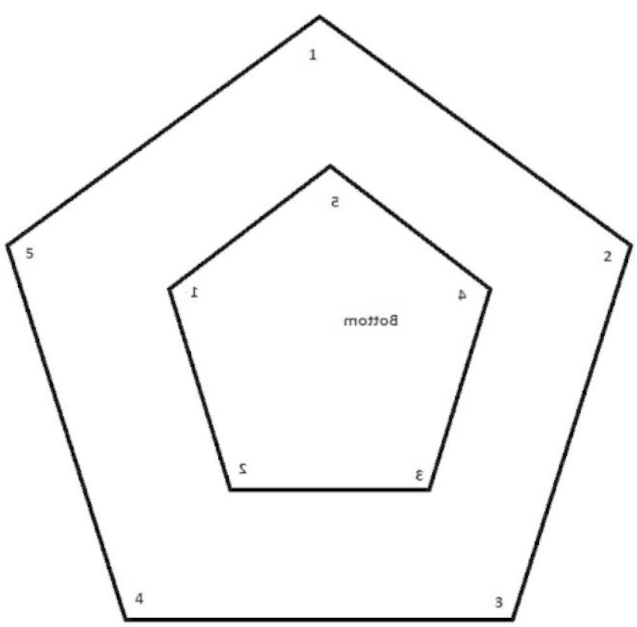

Fig. 6 A virtual model depicting the position of r^2sr

actions as indication of student learning, as it allowed students to express their KMH without the need of an external physical guide. For instance, the example depicted in Fig. 6 is interesting, as this student chose to depict their model virtually, without a physical model with rotating disk. We suggest that this student could separate their physical model from their conceptual representation of it, and make this separation in a way that acknowledged the relevant group of symmetries. In this virtual model, the student demonstrated the flipped orientation of the disk with backwards (reflected) text for the bottom and its numbered vertices. In the paper model (Fig. 5) depicting r^2sr, the student positioned the calculator such that the disk was oriented with the vertex 1 pointing up, while the base was rotated. In contrast, the virtual model (Fig. 6) keeps the base constant and imagines the computations acting on the disk to depict its new orientation for the element r^2sr. Internalizing the action of rotating the disk involved coordinating multiple steps and keeping track of how each step influenced the position of the disk. We note that the labels of the disk are reflected yet not rotated, suggesting that the student enacted the group operations internally and then labelled the diagram, rather than creating a virtual model in which the image was rotated using computer software.

Another example of internalizing algebraic structure was observed when students were asked to draw the ten dihedral symmetries of a regular pentagon, and a student decided to take a short cut (see Fig. 7). An accurate drawing of the physical model would show the labels of the vertices and faces rotating. In drawing the ten positions of the disk, this particular student began by rotating the "top" label, as depicted in their second drawing. They then chose to label the subsequent faces of the disk but not orient them consistently with the physical model. This student could abstract from the physical model to convey the necessary information to depict the required position of the disk, while ignoring non-mathematical details such as the labels "top" and "bottom", which prior research with pre-service teachers suggests is challenging to do (e.g., Sinclair, et al., 2011). In a comment below the figures, the student acknowledged that they were making an aesthetic choice that was inconsistent with the physical model, but still conveyed the essential structural information of the model.

The experience reasoning with a physical model in an undergraduate course also broadened students' perspectives about what mathematical practices and sensibilities are inherent in advanced mathematics, including leveraging visual reasoning to inform proving approaches and to represent group structures. Many students (n = 14) went on to pose interesting problems related to building models. As one student put it:

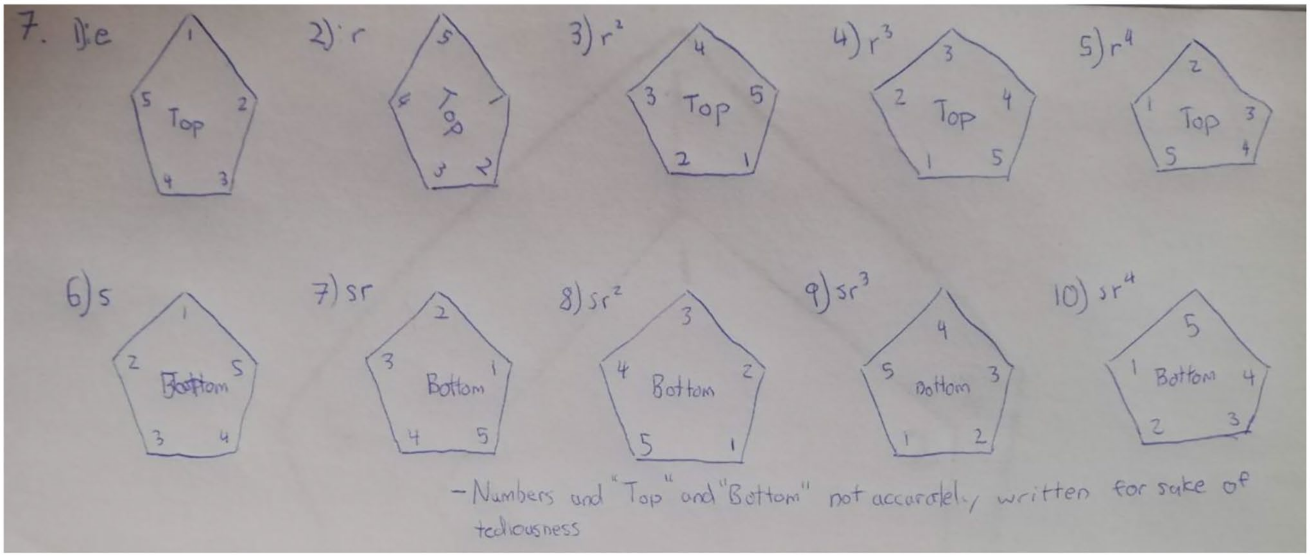

Fig. 7 Student visualizes rotating the disk while leaving the labels invariant. Note the student's comment: Number and "Top" and "Bottom" are not accurately written for the sake of tediousness

Fig. 8 Student intuits general structure of group elements and equivalences

> It is evident that flipping and rotating clockwise k times (sr^k) is the same as rotating counter-clockwise k times and flipping ($r^{-k}s$)
>
> Similarily, rotating clockwise k-times and flipping ($r^k s$) is the same as flipping and rotating counter clockwise k-times (sr^{-k})

> I enjoyed making the models in this course because I get to take a break from just doing math problems, and I think it's very helpful to visualize what we are learning… because it gets confusing just imagining it in my head, for example counting rotational symmetries.

In this comment, we observe the common notion that mathematics is about "doing problems… in my head". The model making experience offered "a break" from math in the student's point of view, yet in Klein's (1893) perspective, the model *is* the math. This student's comments highlighted the epistemological value of the model, as "it's very helpful to visualize what we are learning", thus giving credence to the use of models for exploring and explaining higher mathematics and broadening understanding of key mathematical practices and sensibilities for solving problems (see Table 1, components iii and iv). Further, the model provided opportunities for participants to generalize their reasoning. For instance, we noted participants (n = 19) speculating on the applicability of such a calculator for other groups, both of higher and lower order. One participant thought to "Design a Calculator for a 6 faces cube" and "find the corresponding rotational axis for all the symmetries". Consideration of how to extend the calculator to higher dimension suggests an expectation of an analogous group structure, despite the added complexities of modeling symmetries of 3D shapes, as well as of the appropriateness of using a model for such advanced mathematics. While tactile models, or manipulatives, tend to be recognized for their scaffolding properties and support for early mathematical experiences, the use of models at more advanced levels is less commonly seen or appreciated. Part of the horizon includes understanding of key mathematical practices, such as model making and visual reasoning, and when they can be applied (see Table 1, component iii). In addition, the physical representation can support students' generalizations about the structure of group elements such as sr^k and $r^k s$ by imagining how the calculator would apply. The majority of students articulated the general structure of both elements in terms of the model, and described how they would manipulate the disk to determine the final positions of each element. We saw evidence that this revealed for them equivalences amongst group elements, as exemplified in Fig. 8. The connections made across physical, virtual / visual, and symbolic representations of the group structure are interpreted as a broadening of participants' KMH and enacting "core mathematical values and sensibilities" (Table 1, component iv).

6 Discussion

In this paper, we examined ways in which tactile geometric models in undergraduate mathematics can help bring into view aspects of the mathematical horizon that are relevant for school teaching. Tactile geometric models can draw attention to mathematical structure, foster conceptual understanding, reveal and reinforce intra-mathematical connections, and make complex abstract ideas tangibly accessible (Mamolo, et al., 2015). In the context of teacher education, they broaden and strengthen reasoning skills and pedagogical approaches, giving credence to geometric sensibilities and ways of being in mathematics (Sinclair, et al., 2011). The Dihedral Calculator is a model that leverages connections between algebra and geometry that were advocated for in Klein's Erlangen Program and was designed to provide a tactile means through which to construct, represent, and act upon the dihedral group. Groups are recognized as major disciplinary structures within the mathematical horizon, and knowledge of group theory has contributed positively to teachers' abilities to respond to and support pupils' mathematical activity (Zazkis & Mamolo, 2011; Zazkis & Marumur, 2018). Our findings suggest that tactile models, when designed as purposeful representational tools, can provide for opportunities to develop KMH by bringing into view different features, connections, and practices associated with the object being modelled. In terms of inner and outer horizons, we note that the inner horizon encompasses the features of an object that are evoked, or brought into view, by the specific representation of the object, and these features help situate that object within a greater mathematical world (the outer horizon). We suggest that the Dihedral Calculator can serve as a tool to broaden both inner and outer horizons for undergraduate students and future teachers.

6.1 Broadening the view of the horizon

Through its design, the Dihedral Calculator brought to the foreground key components of the mathematical horizon, including the duality of mathematical representations and the connectedness of mathematical ideas. The construction of the Dihedral Calculator required students to coordinate permutations, reflections, and rotations simultaneously, and shift attention amongst these ideas. To use the model, the students needed to physically rotate and reflect the disk, and then interpret the result of these actions in terms of permutations in cycle notation. See Fig. 5 which shows the position of a student's model after performing three rotations and a flip. We saw evidence that at least some of these actions were internalized, such as in Fig. 6, where the position and structure of the object r^2sr was depicted without first having (physically) executed processes of rotating and reflecting. We suggest that the connection between the position of the disk and the structure of the accompanying group element can support a shift in attention between process and object conceptions of group operations, which is a well-established challenge in abstract algebra (Dubinsky & McDonald, 2001). This shift can broaden the inner horizon through awareness of reflection as a process to carry out and as an object upon which other processes may act, and it can broaden the outer horizon through awareness of this dual nature of mathematical ideas, symbols, and representations.

Our findings suggested a further connection made visible by the Dihedral Calculator: the notion that model-building and visualization are mathematical activities applicable to algebra. This was novel for many participants, who viewed it as a helpful break from their typical computation-focused problems. Students were no longer "just imagining" the group structure, the model helped them visualize what they were computing and brought into view "how different branches of math can relate". In our experience, both building the Dihedral Calculator and manipulating it required a coordination of algebraic and geometric ideas. On the one hand, building the model required interpreting the geometric structure algebraically. On the other hand, playing with it required interpreting algebraic structure geometrically.

Our research suggests that a tangible geometric model can offer more than pedagogical scaffolding and can help strengthen the expectation that the mathematical landscape is a connected one. Further, in prior research about KMH, attention to practices and sensibilities has focused primarily on elements such as conjecturing, generalizing, precision, and questioning conventions (Ball & Bass, 2009). We add to this literature by attending to model-building and visualization, respectively, as practices and sensibilities relevant for providing a higher view of the connections within the mathematical landscape. We suggest that models designed as purposeful representational tools can help reveal *that* ideas connect, as well as *what* and *how* ideas connect within the mathematical landscape.

6.2 Fostering KMH from outer to inner

Fostering a connected view of mathematical ideas is seen as an essential and valuable role that undergraduate mathematics education can play in the formation of future teachers (Bass, 2022). Through its design and use, the Dihedral Calculator can bring into view relationships that situate groups in a landscape that includes links amongst permutations, reflections, rotations, group actions, group elements, algebra, and geometry. Strengthening such links can contribute to knowledge at the mathematical horizon in two ways. First, it can provide for a more robust outer horizon by forging new connections across mental schemas of previously disparate ideas. A schema of processes and objects (Sfard, 1991; Dubinsky & McDonald,

2001) developed in a learner's mind for permutations can establish a new pathway to a schema for symmetry and a new pathway to a schema for group elements and operations. Such pathways can contribute to a more connected world in which the mathematics of the moment is situated. This greater world is understood as the outer horizon and informs what teachers view as relevant or helpful for their students' learning trajectories (Zazkis & Mamolo, 2011). Second, being able to access a richer collection of connections can influence what ideas or strategies (mathematical or pedagogical) may come into view when considering a specific question or problem, thus contributing to a richer periphery, or inner horizon, of related ideas, properties, practices, and so on. A robust inner horizon allows teachers to anchor their broad knowledge of mathematical ways of being in the moment and shift attention amongst these different relevant ways (Mamolo & Taylor, 2018); we suggest it can support Klein's (1945) goal to foster teachers' abilities to draw amply from a great body of mathematical knowledge and experience.

Our data was drawn from students' written, pictorial, and physical artefacts and our analyses attended to ways in which the Dihedral Calculator could provide for opportunities to develop KMH. We focused our analyses on components of KMH that were inherent in the model itself and that were suggested by students' decisions constructing, labeling, and positioning their models in connection with their algebraic and written descriptions. While our study was limited by restrictions that prevented us from physically observing participants create and interact with the models, we nevertheless suggest there is insight that can be gained into ways in which a tactile model can foster knowledge at the mathematical horizon.

We propose that the epistemological value of the Dihedral Calculator is that it captures and can convey aspects of the greater mathematical world (outer horizon) that can influence what ideas, connections, or practices are evoked and attended to (inner horizon). For instance, the sensibility of visualization exists in the outer horizon and is associated with mathematical practices such as visual reasoning, visual discernment, or visual proof. We hypothesize that the spatial visual representation of the dihedral group can direct attention to the applicability of spatial visual practices in algebraic contexts and thus broaden what connections become available (or in view) in the inner horizon. Although spatial visualization has been described as central to mathematical reasoning, the dearth of spatial visual approaches in secondary and university mathematics leaves learners ill equipped to reason geometrically in algebraic contexts (Sinclair, et al., 2011). We suggest that engaging with tangible geometric models at an advanced level can offer important experiences for future teachers whose stereotype of mathematics is that of a computational or procedural subject. We also suggest that the Dihedral Calculator can support generalizations from acting on D_5 physically to intuiting the group structure for arbitrary elements such as sr^k (see Fig. 8), as well as in imagining extensions from identifying symmetries of 2D figures to ones of 3D shapes, such as designing an analogous calculator for the symmetries of a cube. Thus, affording opportunities to develop a higher view of fundamental group theoretic ideas, while also providing opportunities to advance understanding of the interconnected nature of mathematics. There is a need for more research which can capture and unpack students' physical interactions with the model, as well as instances where they may have internalized structural aspects of the model. Research that includes the use of video recording and follow-up interviews, for example, could provide further insight into students' reasoning with tactile purposeful representational models in undergraduate mathematics and its relationship to developing KMH.

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Using Departmental Publications to Foster Student Creativity in Mathematics

Zohreh Shahbazi
University of Toronto Scarborough

Parker Glynn-Adey
University of Toronto, Mississauga

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Using Departmental Publications to Foster Student Creativity in Mathematics

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Using Departmental Publications to Foster Student Creativity in Mathematics

Zohreh Shahbazi

Computer & Mathematical Sciences, University of Toronto Scarborough, CANADA
shahbazi@utsc.utoronto.ca

Parker Glynn-Adey

Mathematical & Computational Sciences, University of Toronto, Mississauga, CANADA
parker.glynn.adey@utoronto.ca

Synopsis

This paper discusses the design and implementation of mathematical departmental publications. We argue that these publications foster students' creativity and written communication skills.

Introduction

Professional mathematicians communicate their ideas by writing and publishing their work. At the undergraduate level, the idea of creating and publishing in mathematics can seem intimidating to students. In order to develop students' writing and communication skills in mathematics, the Math and Statistics Learning Centre (MSLC) at the University of Toronto Scarborough (UTSC) launched two publications: *MSLC Magazine* and *Math in Action Journal*. In this paper, we will discuss the design and implementation of these publications. We will describe our strategy for fostering a culture of creativity through mentoring students who are writing for publication, and our conscious efforts to develop students' higher order thinking skills such as analysis, evaluation, and creativity. We will explore the value of departmental publication both for teaching undergraduate students to engage creatively with mathematics and for improving academic communication.

It is our thesis that, as Firmmender states, "Engaging students in written communication of their mathematically creative ideas positions them as practicing professionals." [6]

Outline of article

This article contains four sections. In the first section, we discuss models of creativity that we use to help our students with their writing. In the second section, we outline the purpose and history of our departmental publications. In the third section, we provide suggestions for starting a departmental publication and include some case studies of writing with high school and university students. In the final section, we offer additional advice for working with student authors who are preparing their first pieces for publication.

1. Creativity and Mathematics

One goal of education is to guide students to discover their creative faculties, their ability to create new ideas and methods, and their ability to think outside of the box. To work towards this goal, we need first understand what we mean by ingenuity and creativity. We also need tangible models for the creative process so that we can describe it to our students.

Creativity in mathematics can mean connecting previously understood notions and facts or coming up with new abstract theories. Once one has done creative work in mathematics, it is important to communicate the results as precisely and clearly as possible. Thus, written communication has a vital role in facilitating the creation of mathematical knowledge.

Ervynck recognizes three stages for mathematical creativity [5]. Stage 0 is the initial technical stage, which consists of some kind of technical or practical application of mathematical rules and procedures, without the user having a deep awareness of the theoretical foundation. Stage 1 is the stage of algorithmic activity, which consists mainly of applying mathematical techniques, such as explicitly applying an algorithm repeatedly. Stage 2 refers to conceptual and constructive activity in which mathematical creativity occurs and consists of non-algorithmic decision making [15]. An example of Stage 2 mathematical creativity could be observed in the work of professional mathematicians such as Maryam Mirzakhani who brought together several mathematical disciplines, including hyperbolic geometry, complex analysis, topology, and dynamical systems. In addition to synthesizing previous knowledge, Maryam communicated her works to the community and left us a record of her work. Our goal for mathematics undergraduate students is to work at Ervynck's Stage 0 of mathematical creativity. Our departmental publications offer undergraduates opportunities to independently learn, connect some of the facts and notions in their own way, and discuss practical applications and then communicate their work in form of presentations and written materials. We attempt to support gradual growth in students' creative processes [14].

We will explain our understanding of creativity through Bloom's Taxonomy of Learning [2], Edward Bono's Thinking Hats model [7], and Tony Buzan's Mind Maps [4]. We share all these different models with our students, so that students can choose the model which works best for them. Not all students will respond equally to all models. Each one provides a lens for examining the creative process.

Bloom's Taxonomy of Learning. This model is well known to educators. In this taxonomy, the affective, psychomotor, and cognitive domains of learning are broken down to their various stages. The psychomotor domain deals with learning physical skills, the affective domain is about the growth of feelings and emotion, and finally the cognitive domain is about mental and thinking skills. Figure 1 describes the hierarchical relationship among each domain levels progressing from lower level to the top level.

Figure 1: Cognitive, Affective, and Psychomotor Hierarchies in Bloom's Taxonomy.

The typical activities in cognitive domain consists of remembering basic facts, explaining concepts, using them in new situations, drawing connections among ideas, and justifying and finally evaluating a standpoint. This domain of learning culminates in producing an original work. In our work, in addition to the cognitive domain, we also pay attention to the affective domain by trying to help students develop positive feelings about learning mathematical topics [2].

Edward Bono's Model of Thinking. In this model of creativity and problem solving, we classify the process of thinking creatively into six stages and represent each stage with a coloured hat. We can imagine ourselves constantly changing these hats until we find an appropriate solution to a given problem. The blue hat is responsible for defining a problem clearly, managing our use of time and flow of ideas, and assisting in communication between the different hats. Figure 2 describes the dynamic relationship among various moods of thinking according to Edward Bono' Model.

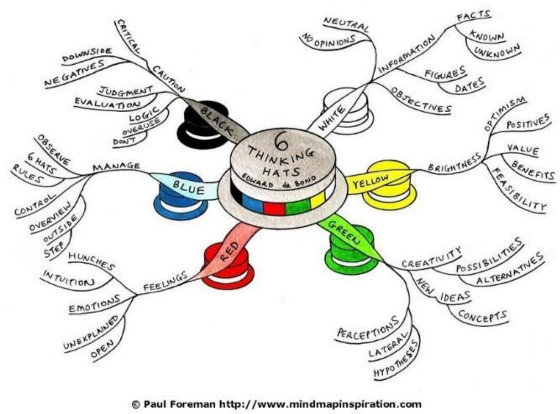

Figure 2: Six Thinking Hats [7], from https://abroun21.files.wordpress.com/2012/04/six-thinking-hats-mindmap2.jpg.

Some of the types of questions that we might ask while wearing this hat are: "What is the problem that we are facing? What are our goals and desired outcomes?" The white hat is the stage where we record whatever data our inquiry uncovers. The types of questions that we ask while wearing this hat might be: "What do I know and what don't I know about this problem?" The yellow hat represents the initial optimistic stage of thinking about a problem. It provides us with a roadmmap to our ultimate goal, and gives us the motivation to work towards it. It deals with questions like: "What are the benefits and potential positive outcomes of our proposed solutions?" The black hat represents the pessimistic stage of thinking, which is every bit as necessary as the optimistic stage, because it can help us to avoid making mistakes as we carry out our solutions. The types of questions asked at this stage might include: "What are some possible flaws of this way of thinking? What are the drawbacks of our method?" The red hat is all about emotions and intuition. The sort of questions that we might ask at this stage include:

"Is this the right approach? Intuitively, what seems to be the best way to solve this problem?" Finally, the green hat is concerned with creativity itself. While wearing this hat, we strive to bring new methods and ideas to the foreground. While wearing this hat, we might ask questions like: "Could we solve this problem more easily using another method? What other approaches might we consider?"

Mind Map Thinking. According to Tony Buzan [4], "creativity, by its very nature, implies getting away from the norm. Normal is that to which your brain has become used to; that which gives you no surprises..."

We use mind map tools to foster the process of creative thinking. To make a mind map, we begin with a concept that we want to explore. The central concept is then connected to related ideas by branches. By connecting the central idea in a mind map to more topics, we can hope to come up with novel insights. For example, Figure 2 is showing a mind map to describe a process of creative thinking.

Theory into Practice. The creativity models are not just theories for us. We share our understanding of the creativity with our students.

As a specific example of applying Bloom's creative model in students work, we will discuss the process of developing one of the materials in the *Math In Action Journal* (MIA). We will look at the article "Exploring Menelaus' Theorem in Hilbert Geometry" which was written by Yumna Habib, a UTSC student, in 2017. Zohreh was Yumna's supervisor for all stages of this work.

In her work, Yumna first provides the statement of Menelaus's Theorem in Euclidean geometry. She then defines the hyperbolic ratio of three distinct points on a given hyperbolic line. Then, she states and proves an analogue of Menelaus Theorem in Hyperbolic geometry. At the end, she mentions that Menelaus Theorem could be used as an evaluation tool for a given Hilbert Geometry to possess a hyperbolic structure. Yumna used multiple resources for her work including an article from the *Journal of Geometry* [10]. The understanding of the Euclidean version of Menelaus's Theorem in Yumna's work stands in the lowest two levels of Bloom's Taxonomy. When she tries to discuss and prove the Hyperbolic Menelaus's Theorem, she extended her previous knowledge in a new situation in order to make new connections. She continues the process of creating more links by stating the interesting fact that Menelaus's Theorem serves as a test tool for Hilbert geometries with hyperbolic structures. This latest connection is an example of creative thinking according to Bloom's taxonomy of learning in cognitive domain.

As an example of applying Edward Bono's Thinking Hats model, Zohreh discusses the various stages of thinking with her students using colored hats to offer a tangible way of thinking about creativity. The hats help to make people more conscious of their creative process. For example, if a student is struggling to get started in solving a problem or understanding a concept then we might encourage them to "use their white hat" and gather more information about the topic of study. If a student is unable to see the flaws in their own work, then they might look at their work while "wearing the black hat".

Zohreh encourages students to create mind maps as a place to begin their writing assignments. Mind maps can help to make conscious the associative process involved in creativity. For example, the video presentation and the extended abstract are the final product of Yumna's associative process of creative thinking.

2. History and purpose of these publications

The Math and Statistics Learning Centre (MSLC) at the University of Toronto Scarborough (UTSC) was established as a collaboration between the Department of Computer and Mathematical Sciences and the Centre for Teaching and Learning both to support students taking undergraduate mathematics courses and to enrich their learning experience in mathematics in general. Various small group sessions, review modules, and seminars have been developed and offered. Special attention has been given to understanding the sources of students' challenges in learning mathematical concepts and exploring new methods to engage students. MSLC teaching team consists of a coordinator, two assistant coordinators and graduate and undergraduate teaching assistants and peers. Each year over 2,500 students' visits are recorded for tutoring by at least 1,500 hours of support in various shapes.

In order to develop students' writing and communication skills in mathematics and nurture their creative thinking, the MSLC launched its annual magazine and *Math in Action Journal* in 2014 and 2017 respectively. This paper discusses the design and implementation of these publications, which were initiated by MSLC Coordinator Zohreh Shahbazi.

Shahbazi is also an Associate Professor, Teaching Stream at the Department of Computer and Mathematical Sciences at UTSC. Parker Glynn-Adey joined MSLC as an Assistant Coordinator and has helped with preparation of the materials for the *MSLC Magazine* and *Math in Action Journal* since 2018.

We encourage the reader to look at these publications before continuing with this article. Familiarity with these publications is helpful, but not necessary.

3. *MSLC Magazine*

The *Magazine* issues may be accessed at: https://www.utsc.utoronto.ca/mslc/mslcmagazine.

MSLC Magazine is a guide to the activities of the MSLC, and a showcase for student work. A typical issue includes material written by faculty, information about the programs offered by MSLC, and contributions from students. The magazine includes some lighter material such as cartoons and horoscopes to broaden its appeal. *MSLC Magazine* has a very narrow circulation. It is essentially an internal document. However, it is highly visible in the Center and copies are distributed continuously. As far as an academic publication goes, it has almost no external profile. The readers of *MSLC Magazine* are UTSC students and faculty. The *Magazine* is readily available in the MSLC, and we see students flipping through it in the Center while studying or waiting for tutors. Whenever a new issue comes up, a hundred hard copies are made available in the MSLC and UTSC Student Centre and a copy is posted online.

We view the mission of MSLC as primarily being a venue for students to publish. Students who submit their work are MSLC undergraduate teaching assistants, and MSLC users. Some students submitted their work as the result of projects and reading courses, which are supervised by the MSLC Coordinator or Assistant Coordinators.

We also published the work of a high school student in the 2019 issue. This student worked with Parker to prepare her article. She participated in the University of Toronto Math Mentorship Program, an enrichment program for high school students who would like to learn about mathematics at a higher level and do some original research. We will discuss this work in the Case Studies section in the paper (Section 7).

As Table 1 shows, there is always a faculty submission to the magazine. These pieces give faculty a space to do recreational writing and help to engage our colleagues in the creation of the issue. In addition, the faculty articles set the required standard for students' work.

Year	2015	2016	2017	2018	2019
Students	4	4	6	4	6
Faculty	2	2	3	3	3

Table 1: Submission numbers by year.

3.1. Where did MSLC Magazine come from?

To stimulate student creativity, and provide an opportunity to practice writing, Zohreh included a research assignment in her fourth-year classical geometry course MAT D02. This course typically consists of 30-40 computer science and mathematics students. Students are instructed to select a topic from eight available areas of geometry and write a ten-page report about it in the format of a typical mathematics journal article. The areas from 2018/2019 were: Finite Geometry, Affine Geometry, Combinatorial Geometry Polytopes, Contact Geometry, Algebraic Geometry, Complex Geometry, Projective Geometry. The task is very challenging for mathematics students, because they have had few opportunities to practice writing about mathematics.

To ease students into writing, the assignment is scaffolded, and they are provided with an adequate level of support. Part of the scaffolding consists of arranging mini presentations. These are four one-hour lectures spread throughout the term. In each session, students are divided into groups of three. Each student is given ten minutes to explain their topic to the group, and the other two students listen, record their observations and evaluate the presentation. This is an opportunity for students to practice expressing their thoughts in a low-stakes environment.

However, one research assignment in one course can only go so far in improving students' writing skills in mathematics. This inspired Zohreh to design and develop new tools to provide opportunities for her students to develop their skills further. *MSLC Magazine*, launched in 2014, is one of such tools.

Figure 3: Mini-Presentations.

The magazine includes articles related to mathematics or statistics, challenging problems, and more general math-related content. We also arrange interviews with students and teaching assistants to inquire about their experiences teaching and learning mathematics.

4. Creation process and formative outcomes

Each year, Zohreh forms and leads a committee including current and former students from mathematics, statistics, journalism and art together with faculty members to prepare the issue. Each member is assigned a specific role. For example, the communication officer is in charge of organizing meetings and collecting content from authors. The first task is to decide the theme for the year's issue. For example, the theme of the 2019 issue was creativity. Then, each member reached out to students or faculty members who might be able to contribute. During regular meetings, various issues were discussed, such as possible ways to improve and promote the magazine. Generally, the committee is eager to improve the magazine further.

What makes us excited about this initiative is looking back and seeing how far the Magazine has come. The *MSLC Magazine* has had a profound impact on some of our students by raising their confidence in producing new materials. This is evident from the fact that more students show interest in courses that require them to produce articles for the magazine. For example, 22 students took their project courses with Zohreh since 2015.

Figure 4: The Cover Page of 2019 *MSLC Magazine*.

Another formative outcome is that one of the students who wrote an article for the magazine won the UTSC library research award in 2017.

For some students, the experience of working with *MSLC Magazine* has significantly shaped their careers. Manaal Hussain was a MATD02 student and is now a program advisor in the Department of Arts, Culture, and Media at UTSC. Manaal is the *MSLC Magazine* designer. She collects the final versions of articles and images and designs the format of each issue. Below, Manaal shares her thoughts about how their experience with the Magazine has shaped her education and career.

> "My involvement with the Math and Statistics Learning Centre started back when I was an undergraduate student looking to find a space where I fit in. Experiencing the space as a student inspired me to seek opportunities within the Centre wherein I could further impact students in their learning. Not only did it help me grow professionally, it also helped me to explore my creativity when it came to teaching as well. I am very aware of the negative misconceptions surrounding mathematics, and statistics as fields of study. These misconceptions I often find discourage students from pursuing these fields, and those who do have a hard time looking for support. Working on development and design of the *Magazine* for the Centre gave me the opportunity to engage such students and show the fun and creative parts of the field and talk about how math could be more than numbers and functions."

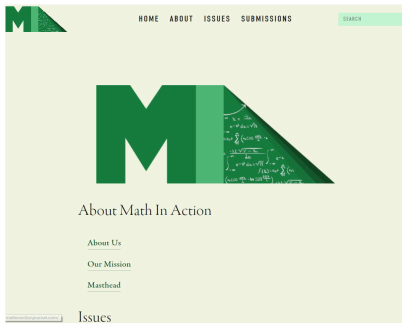

Figure 5: The Home Page of *Math in Action Journal*.

5. *Math in Action Journal*

The online journal may be accessed at: https://www.mathinactionjournal.com/.

Math in Action (*MIA*) is a video journal that provides a platform for mathematics undergraduates to share their work with peers and academics alike. Typically, it features research projects from senior undergraduate classes. Submissions to *Math in Action* are accepted in the form of video presentations with two-page extended abstracts, and students are encouraged to use creative approaches for presenting their work.

Upper-year students who have submitted to the *Math in Action Journal* can continue to expand their communication skills through providing feedback to their junior peers by reviewing and editing submissions.

Each year, Zohreh and Parker invite a couple of students who did exceptional work on their research reports in their upper level mathematics courses to submit their work to *MIA*. Then, they provide about 6-10 hours of support per student to supervise them further while they create their videos and extended abstracts.

Another mechanism to encourage submission to *MIA* is department reading or project courses. Each year several students take reading or project courses with Zohreh and Parker. They hold regular meetings with each student to discuss the topic from various resources such as journal papers. Zohreh and Parker encourage students to check the *MIA* items to broaden their familiarity with mathematics. In addition, the journal link is available on the main page of the Department of Computer and Mathematical Sciences.

5.1. Where did the Math in Action Journal come from?

To develop students' writing and communication skills further, Zohreh initiated *Math in Action Journal* in 2017. *Math in Action* is an undergraduate research journal developed using a Teaching Enhancement Grant. The journal aims to provide a platform for mathematics undergraduates to share their work with peers and academics alike. *Math in Action* has also been used for research assignments in senior undergraduate and graduate mathematics courses. We hope that this journal will help to foster student engagement in mathematics programs. It will also give researchers the opportunity to interact with students and inspire greater interest in their fields, with the hope of creating stronger generations of future researchers. Submissions to *MIA* are accepted in the form of video presentations, along with a two-page extended abstract. Students are encouraged to use creative approaches in presenting their work.

The idea of a video journal came after designing online calculus and precalculus modules using funding that Zohreh received from the Council of Ontario Universities (COU). The project team developed 12 online modules for undergraduate students looking to improve their skills in mathematics fundamentals. Each module consisted of video lessons and an animation. (The online modules may be found at: https://www.utsc.utoronto.ca/math-instruction/.) The experience in designing and launching mathematical videos and animations inspired Zohreh to develop a video journal in which students publish their research work as videos.

6. How to start a departmental publication

In this section, we will outline how to create a departmental publication in your department. Of course, every department is different. There are different resources available at every institution and everyone's time is limited. To start a departmental publication, you may adapt our advice to your situation.

The first, and most important part of starting a publication, is finding some enthusiastic students who are willing to contribute material. Parker has had some success working with students from mentorship programs and math clubs. As outlined above, student work can also be generated through projects in upper-year classes and reading courses. Olivia Rennie and Manaal Hussain, key members of the editorial team, were Zohreh's students and teaching assistants.

It is important to define the roles and responsibilities of each team member clearly in the beginning. For our publication, MSLC coordinators and assistants are reviewers of the submitted articles and make comments on accuracy and structure. The editor helps with the language improvement of written materials. The magazine designer collects the final versions of all articles and images and designs the format of each issue. The communications officer is responsible for scheduling meetings, collecting submissions, sending them to reviewers and editors for possible improvements, and making sure the tasks are done on time. The MSLC coordinator is the editor in chief of the magazine and oversees the whole process. The illustrator produces creative images, which resonates with the theme of that issue and articles.

We recommend seeking funding for your departmental publication. An approximate budget would be $3,200 per year for one issue. In addition to printing costs, there is considerable work for editing and laying out a publication. If possible, compensate your layout team. If you start an on-line journal, then the costs can be greatly reduced. We provide some approximate values for our costs:

- Printing cost $1,500
- Editorial work $500
- Communications officer $500
- Designer $500
- Illustrator $200

Contact other people in your department who might be willing to contribute material. Graduate students, post-docs, and colleagues can provide material which might help flesh-out a publication. When approaching others, be sure to make them aware of the low stakes of publishing in your departmental publication. Be sure that they are aware of the tone or style of your proposed publication.

Once you have a team ready, with some material ready to publish, begin considering layout and design. These are the final touches of starting a departmental publication. At UTSC, we worked with several graphics designers before contacting the university print shop. They had in-house designers who were able to assist us in illustrating the *MSLC Magazine*.

Outline of starting a departmental publication:

1) Apply for grants from your institution.
2) Find some enthusiastic students and colleagues who will contribute to the content.
3) Find a team of people to help edit the journal.
4) Find people with journalism or publishing experience to design the final product.

When starting a departmental publication, there is a rush to get the first issue out. Expect it to take time. Much of the initial work is about forming a team of people, and finding authors willing to write for your first issue. It might take two years to get the first issue published.

7. Case studies

Parker Glynn-Adey is an Assistant Professor at the University of Toronto. During the 2018-2019 academic year, he helped edit *MSLC Magazine* and review articles for *Math in Action Journal*. In addition to teaching at the university level, he mentors high school students in mathematics. In this section, Parker will reflect on writing an article for *MSLC Magazine* with Hillarey, a high school student, and preparing an article for *Math in Action Journal* with Henry, a third-year computer science student at UTSC. We will look at one article which eventually was published, and another which was never published.

7.1. Case study 1

Hillarey met Parker through the University of Toronto Math Mentorship Program, an enrichment program for high school students who would like to learn about mathematics at a higher level and do some original research. Together, they prepared a short article "String Figures and Tom F. Storer's Calculus" about knot theory for *MSLC Magazine*. This publication was Hillarey's first piece of creative writing about mathematics. She said that she learned a great deal about the publication process. In particular, she was surprised by how much creative effort is required to write an article for publication. The process of preparing an article for publication required many free choices and allowed for much more self-expression than writing a piece for a school assignment.

Working with high school students on a mathematical publication presents unique challenges. Among other issues: they are unfamiliar with the conventions of academic writing, they don't check their email, and they do not know how to use standard tools like LaTeX.

The situation with undergraduate students is very similar. Undergraduate students show more maturity in communication and willingness to learn new skills. At the University of Toronto, we find that many undergraduates, especially those in computer science, are familiar with LaTeX. Students who also work as teaching assistants are generally much more dependable in their responses to e-mails.

However, compared to undergraduates, strong high school students are also uniquely positioned to do good work. They have a lot of time on their hands and can engage in extensive reading and writing. A major theme of Parker's work with Hillarey has been "editing down" her writing. As her project neared completion, she made a point of writing much more than needed to convey a point and would edit it down to a manageable length with Parker's help.

To address this issue of unfamiliarity with writing conventions, Parker showed Hillarey a lot of material. Often he would send her away with three or four articles and advise her to look at them "with soft eyes." The goal was for her to notice patterns of writing that she could later emulate in her own article. Small tasks like identifying all the definitions in an article were helpful to develop a sense for the form of mathematical writing.

Communication with high school students is difficult because they do not yet have experience with conventional channels of academic communication. As academics, we are used to checking our e-mail frequently. When working with students, it is important to establish clear guidelines for communication. For example, by setting particular days of the week to check e-mail. It is also important to establish regular face-to-face meeting times with any student who you are writing with. Communicating revisions and modifications by e-mail is difficult and time consuming.

7.2. Case study 2

Henry was a third-year computer science major at UTSC when he started to work on a publication for MIA with Parker. He is familiar with LaTeXand electronic communication. Henry and Parker met during regular office hours held at the MSLC. One struggle that Parker had while working with Henry was keeping progress consistent. Some weeks, Henry would show considerable progress in his work, and other weeks there would be almost no progress. For students, and professors, it is easy to be stuck in the "busyness trap" where flexible tasks like writing can be delayed indefinitely [3].

When Henry first approached Parker for support while writing his article, he had an unformatted document that covered the main points he wanted to address. The article was going to be about Sturm chains and roots of real polynomials. He had the bare bones definitions needed, and a statement of Sturm's theorem without any motivation or explanation. There was significant work to be done in order to bring the article up to a publishable standard.

Henry and Parker met at the MSLC during Monday afternoons office hours. They met almost every week to tweak and improve the article. It was often difficult for Henry to engage in writing outside of office hours. The result was slow, but steady, progress on the article. As the article was nearing completion, and was about to be submitted to *Math in Action Journal*, Henry went on his co-op term. This is a full-time work placement for Computer Science students, which immerses them in the world of programming and corporate life. As a result, Henry's progress ground to a halt. He became hard to contact, and impossible to meet. Eventually, the Sturm chains project slowly collapsed.

The issue that bogged down the project was that there was nothing at stake for Henry. He had approached Parker with a desire to write-up an interesting result. It was motivated by curiosity and a desire to make something of a project. However, it was not associated with any particular course and did not need to be completed. The incentive of publishing an article as an undergraduate was not strong enough to keep Henry working on the article. After several e-mail exchanges, it became clear that Henry was no longer able to commit time to the project, and we had to lay it down.

This can happen to any writing project. There must be a real and significant reason to complete a project. For this reason, we find that students who submit their final course work to *MSLC Magazine* or *MIA Journal* tend to publish. They have already invested a significant amount of time and energy chasing after a final grade, and they would like to see their work in public.

For students who are not motivated by publication, it is difficult to generate enough intrinsic motivation to complete writing projects, which might last several months, and require consistent commitment.

8. Techniques for working with students

In this section, we offer some advice to help boost students' confidence in their mathematical writing. Supervision of students' work is most effective when the students feel that they are in a flexible and supportive environment. First and foremost, we must acknowledge the student's stake in their project. As Hennessey says "People are most creative when they are motivated primarily by the interest, enjoyment, satisfaction, and challenge of the work itself—i.e., by intrinsic motivation. [9]"

8.1. Teach them standard tools

To produce mathematical articles of publishable quality, students must learn the LaTeXtypesetting language. For Hillarey's article on knot theory, there were lots of knot diagrams to create. It would have saved weeks of editorial delays if she knew how to produce these diagrams herself. When mentoring a student through the publication process, it is helpful if they learn the standard tools, such as LaTeXor Mathematica [1], [8].

8.2. Hold them beside each other

When a student is preparing an article for publication, they often have difficulty in identifying what needs to be improved to bring it up to a professional standard. They do not know the conventions of mathematical prose and cannot sense what needs to be improved. Generally, people are much stronger at comparing things than evaluating them in a vacuum. An exercise that Parker has found helpful is to compare students' work with a model. By looking at an expert specimen together with a work in progress, authors-in-training can identify places where their work can be improved to bring it to a higher standard.

8.3. You must take the initiative

When helping a student to prepare work for publication, you must always take the initiative. Students are hesitant to contact their advisors or mentors. Set aside time to review the work in progress of your students.

Students submitting work to a departmental journal will need some support and feedback on their work. There should be enough lead time for your student to respond to any comments you make before meeting in person.

8.4. Monstrous first drafts

Perfectionism holds many people back from writing. Often, we feel that a perfect vacuum is better than an imperfect first draft. As mentors to writers, who are trying to express their thoughts creatively, we can only work with the materials that are presented to us. It is important to encourage first drafts and emphasize the revision process. If a student seems hesitant in their writing, you can even set them the task of writing a "monstrous" first draft. Encourage them to produce large amounts of writing, in order to have some material to work with. Do not be harsh or judgmental about these drafts. They're a sign of definite progress from the blocked writer [11, 12].

8.5. Read it out loud

Reading a piece of writing out loud is a powerful exercise. It reveals subtle flaws in pacing and sentence structure that would otherwise go unnoticed. When preparing an article for publication, we will read it out loud with a student two or three times. This exercise provides an opportunity to discuss the work line by line, sentence by sentence [13].

9. Conclusion

The process of mathematics publication is challenging for undergraduates. By starting departmental publications, we can create resources which are valuable for both developing students' written and communication skill and improving their higher order thinking skills such as analysis, evaluation, and creativity. Furthermore, such publications can be incorporated in course designs and research assignments to engage students in mathematics research.

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THE Teaching PROFESSOR

The Life Goals Exercise: Context for Students' Big Questions

Parker Glynn-Adey ▪ September 5, 2023

Students often ask teaching faculty to help them make important decisions that will affect their lives in significant ways. "Should I drop this course?" "Should I pursue teaching or industry?" "Should I do graduate studies?" These questions can only be answered in the context of a student's life goals. Of course, most people are vaguely aware of what they want to do with their lives. But to obtain concrete answers to these big questions, it helps to have explicit life goals. In this article, I describe a short writing exercise that helps students clarify their life goals.

The exercise

The exercise comes from Alan Lakein's *How to Get Control of Your Time and Your Life* (1973, pp. 31–33), a classic of time management and goal setting. Throughout, I will describe how I do this exercise with individual students. I find that it is best to do the exercise one-on-one during office hours. To do it, you and a student will each need

- writing materials, such as paper and pencil;
- a watch or timer; and
- 15 to 20 minutes of undistracted time.

I recommend doing the exercise on paper instead of electronically because I find that I engage more fully and write slower on paper. The exercise is described below in a format suitable for reading aloud with a student. It consists of three questions, which we freewrite about for two minutes each with additional time for revision.

1. At the top of a blank piece of paper, write, *What are my lifetime goals?* Write freely for two minutes without censoring yourself at all. You are not committing to these goals; you're just writing them down. Dream big. Perhaps you want to visit the moon or run a major company. Dream small. Maybe you'd like to talk with your family or bake bread. Try to list as much as possible.
2. Once you've completed your two minutes of freewriting, take another two minutes to revise and add to your life goals list. Perhaps there is a theme to your life goals that you only noticed once you were done with the first draft. Add that theme.
3. At the top of another blank piece of paper, write, *How would I like to spend the next three years?* Again, freewrite for two minutes. This question will help frame and refine some of the topics that came up in the first question.
4. Once you've completed your two minutes of freewriting, take another two minutes to revise and add to your short-term goals list.
5. At the top of another blank piece of paper, write, *If I knew now that I was going to suddenly die in six months, how would I live until then?* An implicit premise of this question is that your family will not be majorly affected by your death, and you don't need to make any preparations for it. You can imagine that you suddenly dematerialize while your family and friends continue on their merry way. The question seeks to determine how you would live if you had a dramatically reduced life span. Are there things you'd prioritize differently? Again, freewrite for two minutes.
6. Once you've completed your two minutes of freewriting, take another two minutes to revise and add to your six-month goals list.

Take a final two minutes to look at and revise your answers to all three questions.

At this point, I encourage you to do the exercise yourself before reading on. The three questions have a surprising amount of interplay and can generate rather startling answers. By doing the exercise by yourself, you will be much better prepared to deploy it in a teaching context.

How I use the exercise

In my experience, students usually ask big, life-altering questions during informal times, such as at the end of lecture or while walking back from class. Instead of immediately launching in to a half-baked answer unsuited to their particular goals, I ask them an even bigger question: "What do you want to do with your life?" This bigger question is usually met with some vague, off-the-cuff response. I then talk about the importance of explicit life goals for answering their question. We then arrange a meeting to do the life goals exercise and answer their original question.

Prior to our meeting, I send them a write-up of the life goals exercise. The intent of sending them the exercise in advance is to familiarize them with the process. At the meeting, I ask them their original question, we discuss the importance of context, then we do the exercise together. This is important; completing the exercise while the student completes it builds a sense of collaboration. (And let's be honest: What would be weirder than writing down your life goals silently while your professor fiddles with their email?)

Once the exercise is complete, I ask the student their initial question again. With the added context of the exercise, most students come up with a reasonable answer. I never explicitly ask the student to share their life goals with me. If the student doesn't immediately see the connection from their question to their life goals, then I talk them through possible connections between their studies and their goals. At this point, it is important not to be too heavy handed or preachy. The point of the exercise is to provide context, not a preamble for a lecture.

The life goals exercise is simple and takes almost no time. Yet, it has provoked some of the most meaningful conversations that I've ever had with students. It is a great help in contextualizing student's questions in a way that they themselves find meaningful and helpful. I encourage you to try it out next time a student asks you a big question.

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Parker Glynn-Adey, PhD, is an assistant professor, teaching stream, at the University of Toronto Scarborough Campus, where he teaches large, first-year math classes.

A.6 Student Collaboration

2022 MOVES

MATHEMATICS OF VARIOUS ENTERTAINING SUBJECTS

MOVES
MATHEMATICS OF VARIOUS ENTERTAINING SUBJECTS

MOVES Conference events will be held August 7 – 9, 2022 at the following locations:

The National Museum of Mathematics, 11 East 26th Street

The Graduate Center, CUNY
5th Avenue between 34th and 35th Streets

MOVES Conference Schedule of Programs and Activities

Welcome researchers, educators, and families to the 2022 MOVES Conference, hosted by the National Museum of Mathematics!

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TWO SIGMA

Sunday Afternoon

Million Millimeter March
Peter Winkler
Meet in ***Additions***, the shop at MoMath
2:00-3:00

Sunday Evening

Open House and Dessert Reception at MoMath

6:30 Museum open for conference attendees only; early registration begins.

Enjoy more than three dozen unique, hands-on MoMath exhibits without the crowds!

7:45 Dessert reception, Wafels & Dinges.

9:00 Reception ends.

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MoMath is pleased to host 29 research talks intended for all audiences over the course of the conference.

For the research track (room C201), follow the blue listings in the program.

Mathematical Activities

MoMath is happy to provide a change of pace with 23 guided mathematical presentations intended for all audiences (C201) and guided mathematical activities (C202).

For the activity track (room C202), follow the yellow listings in the program.

Note that masks must be worn at all times and that **all visitors ages five and up must show acceptable proof of full vaccination against COVID-19** at MoMath and the CUNY Graduate Center (vaxproof.momath.org). (Note: MoMath does not accept digital images or photocopies of vaccination cards unless they have been uploaded to the NYC COVID Safe app.)

Monday Morning

Conference Kick-Off
Proshansky Auditorium, The Graduate Center, CUNY

8:30–9:15 Registration

9:15–9:25 Welcome
Cindy Lawrence, Executive Director and CEO, National Museum of Mathematics

9:35–10:35 Opening keynote
How to Invent Puzzles — Scott Kim

The Graduate Center, CUNY, C204

10:45–11:10 *Combinatorial Magic with the SET Deck*
Zhengyu Li
University of Toronto, Mississagua
Parker Glynn-Adey
University of Toronto, Scarborough

11:15–11:40 *Geometry and the SET Daily Puzzle*
Liz McMahon
Lafayette College

11:45–12:10 *QUADS: A SET-like game with a twist*
Lauren Rose
Bard College

The Graduate Center, CUNY, C205

10:45–11:10 *Metamorphosis of a Tiling Font*
Erik Demaine
MIT

11:15–11:40 *A Pythagorean Triple Puzzle*
Joe Fields
Southern Connecticut State University

11:45–12:10 *Infinite Tiles of the Regular Rep-tiles*
Tony Hanmer

All MOVES attendees are welcome to enjoy FREE admission to MoMath throughout the conference; please just show your badge in *Additions*, the shop at MoMath, when you arrive.

The Graduate Center, CUNY, C201

10:45–11:10 *Riddle Me This: Human vs. Computer*
Zach Wissner-Gross
Amplify

11:15–12:40 *Pieces of a Puzzle Pieces Puzzle*
Skona Brittain
SB Family School

The Graduate Center, CUNY, C202
Family Activity Fair
10:45–12:10

Hyperbolic Rainbow
Chaim Goodman-Strauss
University of Arkansas

Mathematical Card Effects
Colm Mulcahy
Spelman College

12:10–1:45 Lunch, pre-ordered to eat with others onsite at CUNY **or** on your own.

Monday Afternoon

Proshansky Auditorium, The Graduate Center, CUNY

1:45–2:45 Keynote speaker
The Fascination of Puzzle Hunt Puzzles —
Tanya Khovanova

2:45–3:15 Coffee break

The Graduate Center, CUNY, C203

3:20–3:45 *From Geography to Santorini: Computational Complexity of Games on Graphs*
Nathan Fox
Canisius College

3:50–4:15 *A Combinatoric Approach to Solving Chess Puzzles*
Andrew Lee
Jericho High School

4:20–4:45 *Sum Amusements with Fibonacci and Other Linear Recurrence Sequence*
Edmund A. Lamagna
University of Rhode Island

The Graduate Center, CUNY, C204

3:50–4:15 *Parity Graphs for KenKen Puzzles*
David Nacin
William Paterson University

4:20–4:45 *Surprising Sudoku Connections*
Shelly Smith
Grand Valley State University

The Graduate Center, CUNY, C205

3:20–3:45 *Living on a Random Torus*
Saad Mneimneh
Hunter College

3:50–4:15 *Making Paper Spheres*
David Richeson
Dickinson College

4:20–4:45 *Knot Theory on the n*n*n Rubik's Cube*
David Plaxco
Clayton State University

The Graduate Center, CUNY, C201

3:20–3:45 *So You Want to be a Competitive Puzzler*
Timothy Miller

3:50–4:15 *A Game and A Proof and A Challenge*
Jeanine Meyer
Purchase College, SUNY

4:20–4:45 *A Periodic Table of Polyform Puzzles*
Kate Jones, presenting via pre-recorded video
Kadon Enterprises

The Graduate Center, CUNY, C202
Family Activity Fair
3:20–4:45

Piles of Fun: Takeaway Games you can Play on the Go
Corey Levin
Yorkville East Middle School

Activity Based on the Shape Family Multi-Media Kit in Ivan Moscovich's Book
Lauren Siegel
MathHappens Foundation

Placing Number Tokens on Catan: A "Tour" of Smaller Islands
Susanna Molitoris-Miller
Kennesaw State University
Brian Kronenthal
Kutztown University

Monday Evening

5:15 Conference Dinner – The Graduate Center, CUNY – *Pre-registration required*

Proshansky Auditorium, The Graduate Center, CUNY

Monday Night MOVES

7:00 PM

Miyamoto and the Machine

Screening of a documentary about KenKen, followed by a panel discussion with Robert Fuhrer and Aaron Kaswell

Performance by special guest Will Calhoun

Multiple Grammy-award winning drummer for the band Living Colour, progressive and electronic percussionist, producer, songwriter, teacher, and artist

Post-performance discussion hosted by Steven Strogatz

Join the conversation with Will Calhoun and Steven Strogatz, MoMath's 2021–2022 Distinguished Visiting Professor for the Dissemination of Mathematics

Tuesday Morning

Proshansky Auditorium, The Graduate Center, CUNY

8:30–9:30 Keynote speaker
***Experiencing math through mechanical puzzles and puzzling mechanisms* —** Oskar van Deventer

9:40–10:40 Keynote speaker
***Playing with Puzzles: a presentation to honor the memory of Maki Kaji* —** Yoshi Anpuku

The Graduate Center, CUNY, C203

10:55–11:20 *The License Plate Challenge*
Lidia Gonzalez
York College, CUNY

11:25–11:50 *Solving Soluna*
Lily Li
University of Toronto

11:55–12:20 *An Impartial Two-player Pebbling Game on Complete Graphs*
Wing Hong Tony Wong
Kutztown University of Pennsylvania

The Graduate Center, CUNY, C204

10:55–11:20 *The mathematical recreations of Gaspar Nycolas (16th century)*
Jorge Nuno Silva
University of Lisbon

11:25–11:50 *CIRM puzzles from 1935 and 1937*
Tiago Hirth
University of Lisbon

11:55–12:20 *Puzzling Mathematics in Ancient Greek and Latin*
Suzanne Sumner and Liane Houghtalin
University of Mary Washington

12:20–1:50 Lunch, on your own; see deals and discounts at back of program.

Optional field trip to Tannen's Magic Shop led by Robert Vallin: meet at 12:30 pm in front of C204.

The Graduate Center, CUNY, C205

10:55–11:20 *Counting Variations of Knights and Knaves Puzzles*
Taisha Charles
Albright College

11:25–11:50 *Counting Socially-Distanced Catan Configurations*
Brian Kronenthal
Kutztown University
Susanna Molitoris-Miller
Kennesaw State University

The Graduate Center, CUNY, C201

10:55–11:20 *Lewis Carroll's Amazing Card Trick*
Jonathan Matte

11:55–12:20 *Li'l Jacob, the Kid Brother Version of the Jacob's Ladder*
Paula Krieg
Artist

The Graduate Center, CUNY, C202
Family Activity Fair
10:55–12:55

Accidental Flexagon
Lauren Rose
Bard College

Flexagon Fun
Ann Schwartz
Fantastic Flexagons

KOLAMS: Play with Art and Geometry
Venkat Gopalan
Pennsylvania State University

Tuesday Afternoon

Proshansky Auditorium, The Graduate Center, CUNY

1:50–2:50 Keynote speaker
Unbelievable Math Puzzles — Peter Winkler

The Graduate Center, CUNY, C203

3:05–3:30 *Variants of the 15-puzzle and the effects of holonomy*
Henry Segerman
Oklahoma State University

3:35–4:00 *Gears that turn and Archimedean solids*
Gary Gordon
Lafayette College

4:05–4:30 *Exploring Bipartition Dominoes*
Brian Hopkins
St. Peter's University

The Graduate Center, CUNY, C205

3:05–3:30 *The Derivative of Dolly: Iconic Graphical Analyses of Iconic Songs*
Brooke Tortorelli
Monmouth University

3:35–4:00 *Creative Design with Polystix: Homogenous Cylinder Rod Packings as Art*
Anduriel Widmark

4:05–4:30 *Guess What, I'm a Flexagon!*
Ann Schwartz
Fantastic Flexagons

The Graduate Center, CUNY, C201

3:05–3:30 *Symmetry and Quadrilaterals*
Frank Corley
St. Louis University High School

3:35–4:00 *Rubik's Snake Loopology*
Mircea Draghicescu
ITSPHUN

The Graduate Center, CUNY, C202

Family Activity Fair
3:05–4:30

Log In! Exploring Logarithms through Experimentation and Play
Philip Dituri
Financial Life Cycle Education Corp.
Andrew Davidson

Origins and Implementation: Engagement with Ratio and Proportion: Building a Tool to Deepen Understanding
Lauren Siegel
MathHappens Foundation

Puzzling Pythagorean Proofs
Joe Fields
Southern Connecticut State University

MOVES Closing
Proshansky Auditorium, The Graduate Center, CUNY

4:30–4:45 **Concluding Remarks,** Cindy Lawrence, MoMath

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2024 MOVES

MATHEMATICS OF VARIOUS ENTERTAINING SUBJECTS

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August 11–13, 2024

Of Loops, Braids, and String Figures:
The Loopy Calculus of Cat's Cradle

Parker Adey[1] and Vanessa Schattman[2]

[1]Dept. of Computer and Mathematical Sciences, University of Toronto, Scarborough, Canada
[1]parker.glynn.adey@utoronto.ca [2]ness.schattman@mail.utoronto.ca

Abstract

This workshop explores string figures as algorithmic art. We share a novel application of braid groups to analyze string figure algorithms. Participants will engage in hands-on exploration working individually and in pairs. Participants will come away with some silly string art and serious string mathematics.

Workshop Outline

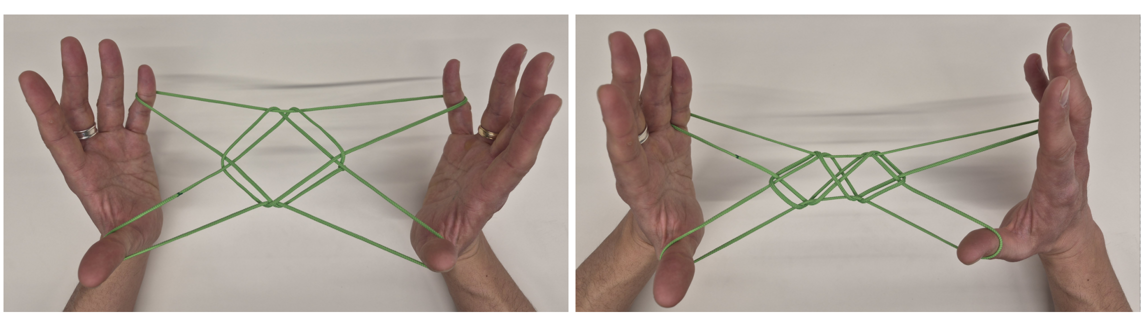
Materials

This workshop requires each participant to have a closed loop of string. Participants will be provided with pre-made loops. Generally, a good length of string for someone is the distance between their finger tips when their arms are fully extended side-to-side. We encourage people making loops at home to experiment with longer and shorter loops using a variety of materials. Polyester string can be made into loops by melting the ends together and is available in a variety of colours at a low cost.

Schedule

The workshop consists of three parts:

1. Teaching string figures (30 minutes): Participants playfully learn to create several warm-up string drawings culminating in the string figures Brokhos and Koura.
2. Heart-sequences and figures with partners (30 minutes): Participants pair up to replicate the figures by carrying out their loop-based constructions.
3. Applying loop braid relations to string figures (15 minutes): Working in pairs as before, participants explore the effect of loop braid relations on the figures.



(a) **(b)**

Figure 1: *Two string figures: (a) Brokhos and (b) Koura (Crayfish).*

Introduction: Heart-Sequences and Loop Braid Groups

String figures are a mathematical art form practiced around the world; for examples of string figures from Greece and New Zealand, see Figure 1. As we manipulate the string with our fingers, loops twist, cross, and pass through each other, creating elegant designs through sequences of loop manipulations. In 1988, Dr. Tom Storer, a mathematician at the University of Michigan, introduced the notion of a string figure's *heart-sequence* to describe a string figure's algorithm in terms of the motions of its loops. Storer described the loop manipulation sequence, or heart-sequence, of a figure as "a *gedanken*-experiment of fundamental importance in the deeper understanding of string-figures of the world" [11, p. 35]. This abstract loop-centric view has been used by anthropologists to clarify the relationship between different string figure corpora [12]. In this workshop, we present a novel connection between string figures, heart-sequences, and braid groups.

Topologists and artists will appreciate the similarity between the loopy motions of a string figure and the repeated intertwining of a braid. Describing string figures in terms of their heart-sequences allows us to use a powerful algebraic tool: braid groups. To see how a single loop of string can be decomposed into a family of braided loops, we will now describe a relationship between string figures and braids. In braid theory, this construction is known as a *plat closure* [3].

When we hold a string figure on our hands, there are a number of strings which pass from one hand to the other. Typically, the figure will be held aloft by several small segments of string which encircle individual fingers. These small segments are called *loops* in the string figure literature. Each loop has a near and a far string when the hands are held in normal position. If we imagine the near and far string of each loop as separate strings, running from hand to hand, then we obtain a braid with twice as many strings as loops.

In this workshop, we will work with the loop braid group L_n, a finitely presented subgroup of the usual braid group B_{2n} on $2n$ strands. The loop braid group appears in several fields of mathematics [6]. The classical braid group B_{2n} describes the motion of $2n$ strands. In the loop braid group L_n we imagine that adjacent strands pair up to form loops: the strands $2i-1$ and $2i$ form *loop* i for $i = 1, \ldots, n$. In Table 1, we introduce a *loop braid notation* for writing the generators of L_n. This notation was directly inspired by Storer's notation for heart-sequences [11, p. 25-27]. The generators are shown as braids in Figure 2. For the sake of comparison and reference, we give Hatcher and Brendle's notation for the generators of L_n and refer the reader to [4] for the full set of relations of L_n in terms of ρ_i, σ_i, and τ_i.

Table 1: *The Generators of the Loop Braid Group.*

Operation	**Loop Braid Notation**	**Hatcher and Brendle [4]**
Half-twist loop i clockwise.	$>i$	τ_i
Cross loop i over loop $(i+1)$.	$\overrightarrow{i}$	σ_i
Insert loop i over and down through loop $(i+1)$.	$\overrightarrow{i}\downarrow$	ρ_i

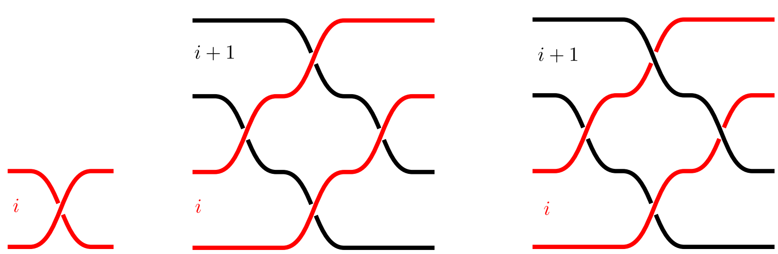


Figure 2: *The generators of the loop braid group: twisting* ($> i$)*, crossing* ($\overrightarrow{i}$)*, and inserting* ($\overrightarrow{i}\downarrow$).

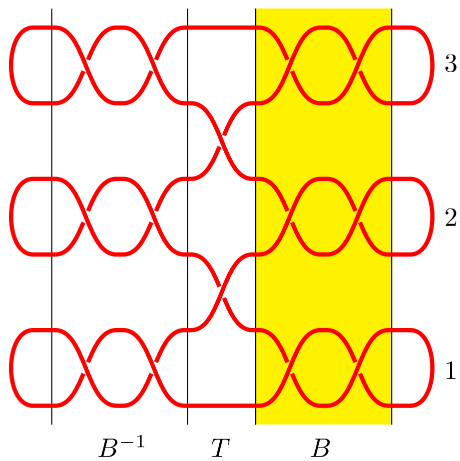


Figure 3: *The braid region B, its inverse, and the central tangle T.*

We illustrate this notation with an example. Most string figures start with *Opening A*, a three-loop figure with a loop on the thumbs, indices, and little fingers. See the section "String Figure Instructions" below for the construction of Opening A. Several figures from Oceania [10, 9] start with Opening A followed by the operation of twisting all the loops a full rotation. In the loop braid notation, we write: $(> 1)(> 1)(> 2)(> 2)(> 3)(> 3)$ for the operation of twisting all the loops a full turn away. Two half-twists of each loop are needed to produce a full turn. Notice that all these twist operations commute, a theme we will explore later.

In Figure 3, we have illustrated the complete string position labeling the loops $1, 2$ and 3 from bottom to top. Later on, when we draw the braid of a figure, we show only the braid formed by the right hand. We will also omit the labels of the loops. In a figure with a symmetric construction, where both hands perform the same operations, the right hand side of the braid is always the mirror image of the left hand side. We call the region on the right hand side of the figure, formed by loop manipulations, the *braid region* of the figure. The braid region is separated from its inverse by what we call the *central tangle*.

We require a slight extension of the loop braid notation to succinctly describe the loop moves occurring in string figures. As we introduce the extended moves, it is helpful to look at Figures 4 and 5 which contain examples of each type. To notate when loop i moves over loop $i-1$, we write $\overleftarrow{i}$. Similarly, we extend the notation to cover loops passing below other loops: $\underleftarrow{i}$ and $\underrightarrow{i}$. We will also need additional notation for when loops pass through other loops in various directions. To notate when loop i moves over and down through loop $i-1$ from above, we write $\overleftarrow{i}\downarrow$. As before, we want to extend the notation to cover loops passing below other loops and so we add: $\underleftarrow{i}\uparrow$ and $\underrightarrow{i}\uparrow$. This extended set of generators is essentially "syntactic sugar." All the extended moves we have just introduced can be expressed using the original generators of Table 1.

We can use the loop braid notation to create a finger-agnostic description of a string figure which describes its heart-sequence without any additional information; we describe only the motion of the loops relative to one another. This is the topological heart of the string figure algorithm.

In Figures 4 and 5 below, we give highly-condensed presentations of the string figure algorithms for Brokhos and Koura by showing only their braid regions. The complete string figures are shown in Figure 1. Both figures consist of three loops that are initially supported on the thumbs, index fingers, and little fingers. To improve the clarity of the diagrams, these loops are shown as red, orange, and black loops respectively. As before, the three loops are numbered 1, 2, and 3 from bottom to top.

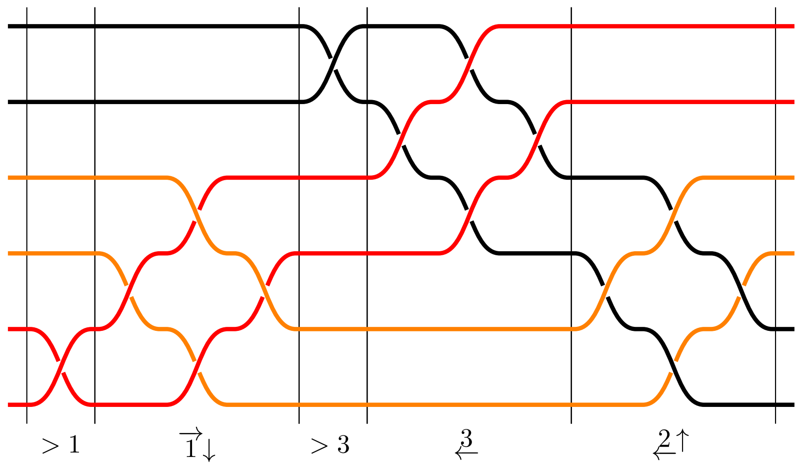


Figure 4: *The heart-sequence of Brokhos as a loop braid.*

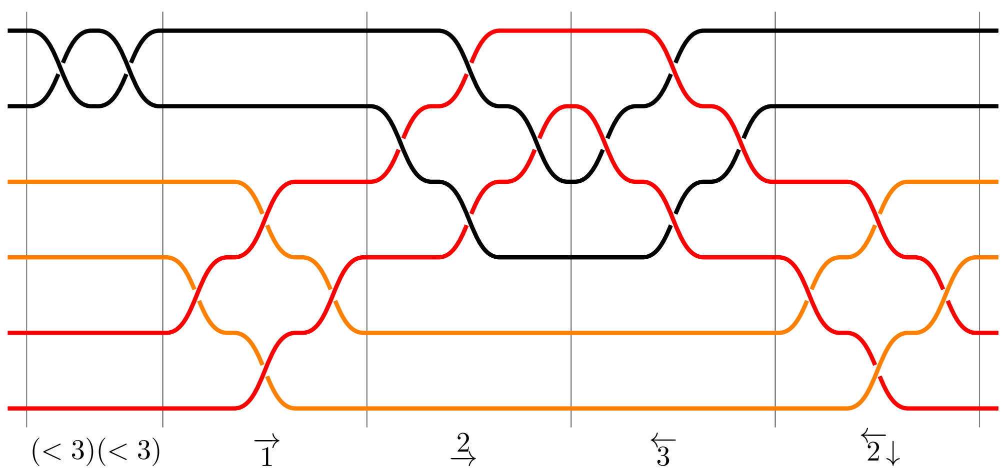


Figure 5: *The heart-sequence of Koura as a loop braid.*

Detailed Workshop Instructions

Playing with String

A closed loop of string can be made to look like many things; the string is a line that we can use to draw. Any workshop on string figures should begin with a few string drawings to get the audience's fingers warmed up and to create a light atmosphere. As we hand out the strings, we ask: "Can you make a string heart? A smile? Did you play with string when you were a kid too?" In any string workshop, especially in a school setting working with young children, we remind participants to never place the string around their neck. A well-made loop is much stronger than a human neck and can cause strangulation.

Once participants have had the chance to warm up their fingers and play with string drawings, we move on to teaching string figure algorithms: the Brokhos and Koura. The section "String Figure Instructions" contains the complete descriptions of these figures. Generally, learning string figures takes a long time. In a workshop setting there is always a wide spectrum of dexterity. It helps to have at least two people teaching the figures. Once some participants have learned a figure, they are encouraged to help other participants.

Heart-Sequences and Figures with Partners

While heart-sequences and loop braids are useful constructs for string figure analysis, they can be challenging to visualize and verify in practice as the loop-manipulations are awkward to carry out manually. In this workshop, we introduce a novel way for two people to play a string figure together, inspired by Australian artist-scholar Louisa Bufardeci's work on *tacking* string figures [5].

Participants will pair up and collaboratively create one string figure: each partner holds one side of the loops and uses their free hand to perform loop manipulations on the string figure. Using tacking, participants will explore the heart-sequences of the Brokhos and Koura figures they learned previously, verifying that the loop-braid instructions produce the same figures.

Applying Loop Braid Relations to String Figures

We invite participants to play with their tacking sequences and discover some of the braid relations themselves. While tacking Koura, the partners twist the little finger loop twice. Carefully keeping track of the motion of the little finger loop, participants will re-tack Koura, switching up the placement of the twist moves within the algorithm and verifying that the resulting figure is the same. We suggest the following explorations:

- Perform all the non-twist moves first, and twist the little finger loop before releasing the index loops.
- Place the twist moves anywhere in the algorithm of the participants' choosing.

Participants will discover that the Koura figure is invariant under the placement of the twist move. A twist can slip over top of a crossing as shown in Figure 6. Formally, this is fact is expressed by the twist commutativity relations in L_n: If $m \neq n, n+1$, then:

$$\left(\overrightarrow{n}\right)(>m) = (>m)\left(\overrightarrow{n}\right) \quad \text{and} \quad \left(\overrightarrow{n}\downarrow\right)(>m) = (>m)\left(\overrightarrow{n}\downarrow\right).$$

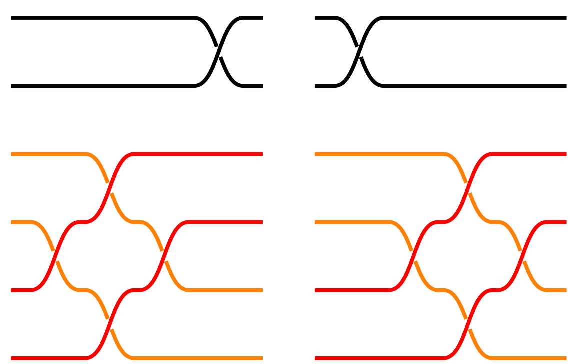


Figure 6: *The twist commutation relation* $\left(\overrightarrow{n}\right)(>m) = (>m)\left(\overrightarrow{n}\right)$ *for* $n = 1$ *and* $m = 3$.

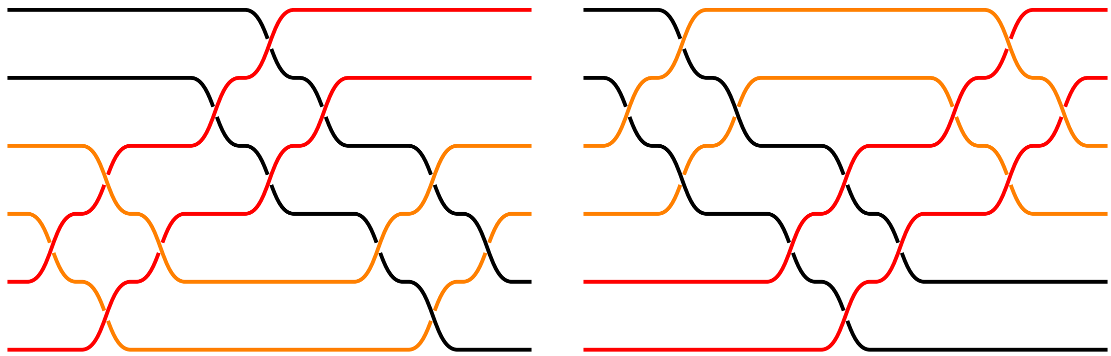


Figure 7: *A braid relation* $\left(\overrightarrow{1}\downarrow\right)\left(\overrightarrow{2}\right)\left(\underleftarrow{2}\uparrow\right) = \left(\underleftarrow{3}\uparrow\right)\left(\overrightarrow{1}\right)\left(\overrightarrow{2}\downarrow\right)$.

While tacking Brokhos, participants might notice that there are two large loop movements: the thumb loop moves down through the index loop towards the little fingers, and the little finger loop moves up through the index loop towards the thumbs. Participants are invited to switch the order of the two moves:

- What happens if the little finger loop moves through the index loop before the thumb loop does?
- What intermediary steps have to be tweaked to produce the final Brokhos?

After some playing, participants might notice that the crossing between the thumb and little finger loops must be preserved: while the loops can make their moves in any order, the little finger loop must always cross under the thumb loop in its path across the hand. This is the braid relation shown in Figure 7.

String Figure Instructions

We enthusiatically recommend that the reader join the International String Figure Association [1] in order to learn string figures. They have many publications, documenting thousands of string figures, which are freely available to members through their website. The vast literature on string figures uses a particular nomenclature for describing string figure constructions; we give a brief introduction to this nomenclature as well as the instructions for Brokhos and Koura.

When the hands are relaxed, with palms facing each other and fingers pointing upwards, we say that the hands are in *normal position*. A segment of string around a finger is called a *loop*. A segment of string running across the palm is called *palmar*. The segment of a loop nearest the chest is the *near string* of the finger holding the loop. The opposite segment, farthest from the chest, is the *far string*. If a finger has two loops, the loop closest to the palm is the *lower loop* and the loop nearest the finger tip is the *upper loop*.

Opening 1 and Opening A

Opening 1 and Opening A are two very common starting positions. To create Opening 1:

1. Put the loop around both little fingers. Separate your hands.
2. Put your thumbs into the loop from below, and draw the near little finger string back towards you.

To create Opening A:

1. Form Opening 1.
2. Put your right index finger underneath the string crossing your left palm. Pull your right hand back, drawing the left palmar string back with it.
3. Repeat this on the right side: your left index finger passes to the right, and enters the right index loop. The left index pulls back the right palmar string from underneath.

Brokhos

This string figure is the oldest recorded string figure construction. It was recorded by the Greek physician Heraklas circa 100 CE in a treatise on medical knots [8]. One can read a translation of the construction, with additional notes on its origin, in [7]. Figures 1a and 4 show the Brokhos.

1. Opening A.
2. Both thumbs cross over the near index finger string, cross under the far index string, and draw this far string back.
3. You now have two loops on your thumbs: an upper thumb loop that you just pulled back from the index finger, and a lower thumb loop that was there initially. Both index fingers cross under the near string of the lower thumb loop from below and pull it back towards themselves.
4. Release all thumb loops. You now have two loops on your index fingers and one loop on your little fingers.
5. Thumbs dive into the lower index loop from above. Rotate the palms outwards slightly. The thumbs take up the far little finger string from below and pull it back through the lower index loops to rest on the thumbs.
6. Release the little finger loops. You now have two loops on the index fingers and one on the thumbs.
7. Using the thumb and index of your right hand, take the upper index loop off your left hand, twist it half a turn towards the little fingers and put it on the left little finger.
8. Symmetrically, using the thumb and index of your left hand, take the upper index loop off your right hand, twist it half a turn towards the little finger and put it on the right little finger. You now have a loop on your thumbs, indexes and little fingers.
9. Release the index loops and gently pull your hands apart.

Koura (Crayfish)

Koura is sourced from Andersen's *Maori String Figures* [2, p. 58]. Figures 1b and 5 illustrate Koura.

1. Opening A.
2. Rotate the little finger loops a full twist towards the thumbs.
3. Thumbs pass over both index finger strings and under both little finger strings. Thumbs pull both little finger strings back.
4. Middle fingers cross over the index finger strings and pick up the far lower thumb string from below.
5. Release all thumb loops. You now have one loop on each of your little fingers, middle fingers and index fingers.

   The next move is tricky! It helps if your middle finger loops are a little higher up on the fingers than the index finger loops, and if you turn your palms slightly towards you as you execute the move.
6. Thumbs enter index loops from below, travel up into the middle finger loops, and take up the near middle finger string from below. Thumbs pull this string back down through the index finger loops and return to normal position.
7. Release middle finger loops and tighten slack. You now have one loop on each of your thumbs, index fingers, and little fingers.
8. Release index finger loops and pull gently until the figure emerges.

Conclusion

In this workshop, participants played with string figures on many levels: as artistic media, as physical implementations of algorithms, and as algebraic structures. String figures teach deep lessons about symmetry and algorithms. They are also well-suited for classroom use. They are fun, inexpensive, and mess-free. A class set of string figure loops costs almost nothing and can serve as a great way to start a semester. String drawings, or simply playing with string, can serve as light hearted ways to get people making art casually. Learning a small repertoire of string figures can build a sense of confidence and connection with cultures around the world.

At one level, we hope that this workshop gets more people playing with string and sharing the joy of string figures with their friends, families, and students. At another level, we hope that this workshop inspires participants to explore the many lingering questions pertaining to string figures and loop braids: What other loop relations remain to be found? How does one extract the heart-sequence from a finger-based algorithm? What finger-based algorithms can we derive backwards from a loop braid sequence?

Acknowledgements

We thank Oliver Cheng and Megan Shaw, as well as the anonymous reviewers, for feedback on early drafts of this paper. We appreciate the time it takes to learn string figures from written sources and offer feedback. We also thank Eric Vandendriessche and Alfredo Braunstein for the many conversations which helped to clarify our thinking about heart-sequences.

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[1] The International String Figure Association. n.d. https://isfa.org/.

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CERTIFICATE

Of Achievement

This certificate is proudly presented to

Samira Goder, Emily Forbrigger

for achieving **“Best Oral Presentation”** for their presentation: “Braid Theory & Groups” at the 2025 CMS Undergraduate Research Symposium.

Computer & Mathematical Sciences
UNIVERSITY OF TORONTO
SCARBOROUGH

A.7 Student Engagement

JOIN US FOR

CMS BOARD GAME NIGHT

The fun starts at 3:30 - 5:00 pm in the IC Atrium on **20JAN, 17FEB, and 17MAR**! Feel free to bring your friends and your own games*

Please register a week in advance.

*The department is not liable for any damage to personal games.

This event is organized by Parker Glynn-Adey at parker.glynn.adey @utoronto.ca.

CHECK OUT THE CMS UNDERGRADUATE SEMINAR SERIES

VIEW UPCOMING SESSIONS

Date	Speaker	Topic
September 21	Zack Wolske	An Introduction to Combinatorial Games
October 5	Scott Carter	Permutations with quipu
October 19	Blake Madill	An Algebraic Proof of the Fundamental Theorem of Algebra
October 26	Sarah Brewer	Star rosettes in GeoGebra: constructing traditional patterns with contemporary technologies
November 2	Alex Teeter	Spheres, Donuts and Crazy Bottles: An Introduction to The Classification Theorem of Surfaces
November 9	Ben Briggs	What's the deal with Homological Algebra?
November 16	Özgür Esentepe	What caused Coxeter many restless nights?
November 23	Brian Zhengyu Li	A SAT Solver + Computer Algebra Attack on the Minimal Kochen-Specker Problem
November 30	Jesse Maltese	An Introduction to Modal Logic and Its Applications

This event series is organized by Parker Glynn-Adey and Lisa Jeffrey.
For more information, please contact parker.glynn.adey@utoronto.ca or visit https://pgadey.ca/seminar/.

EVERY WEDNESDAY
1 - 2 PM
IC318 OR ONLINE

CHECK OUT THE CMS UNDERGRADUATE SEMINAR SERIES

VIEW UPCOMING SESSIONS

Date	Speaker	Topic
January 19* (THU - Online Only)	Anatoly Zavyalov	Automatic Sequences
January 25	Maitreyo Bhattacharjee	The Toeplitz Conjecture
February 1	Daniel Harrington	Exploring the QUAKE III Fast Inverse Square Root
February 8	Erik R. Tou	Making Juggling Mathematical
February 15	Justin Fus	Into the Infinite-Dimensional: An Intro to Functional Analysis
March 1	Andrew Feng	Proofs = Programs: The Curry-Howard Isomorphism
March 9* (THU - Online Only)	Logan Lim	Fun Applications of Geometric Algebra!
March 15	Aditya Chugh	Abstracting Reality: Symmetry Ideas in Physics
March 22	Faiza Robbani, Sharon Alex, Yunni Qu & Yushu Zou	Advantages of Working Remotely
March 29 & April 5	Reading Course Participants	Reserved for Reading Course Topics

This event series is organized by Parker Glynn-Adey and Lisa Jeffrey.
For more information, please contact parker.glynn.adey@utoronto.ca or visit https://pgadey.ca/seminar/.

A.7.1 Seminar: A Place to Sow Seeds

This subsection reproduces a reflection Seminar that initially appeared as:

> Adey, P. (2026) Undergraduate Seminar: A Place to Sow Seeds. FYMSiC Newsletter: Issue 18, First Year Math and Stats in Canada, February 2026, (link)

After a first year life science calculus lecture, a group of students comes up to you and asks: "Could you teach us about Hilbert space? Our chemistry professor uses it in all their papers, but we have no idea what is going on." What would do you tell such a group of eager students?

For the last five years, I've run a weekly meeting called Undergraduate Seminar. It is not a course, it is not a colloquium, but rather it is a place to chat about mathematics and anything else that appeals to the participants. It is a place to sow the seeds of learning and see what grows.

The setup is quite minimal. New people find out about seminar by word of mouth, invitation, and sometimes even by walking in on a seminar in progress to see what all the fun is about. Before each seminar, I make a pot of coffee and bring in a tray of cookies. The speaker for the week, if there is one, presents something to the group. Interruptions, question asking, and friendly heckling are encouraged. Afterwards, we chat about life, the semester, and arrange the speaker for the next talk.

At the beginning of a semester, we have an organizational meeting where we ask the regular participants if they have anything they would like to talk about or that they would like to hear talks on. At our last organizational meeting, we had about twenty talk suggestions and requests for a twelve week semester. Throughout the semester, we draw on the volunteers who suggested ideas for talks. As the organizer, I find it helpful to have a few talks ready "off the top of my head." In the event that no one is willing or ready to talk, as often happens around midterm season, I improvise on topics of interest; at the very least we have cookies and coffee to keep conversation going. And that's enough to hold a seminar!

In The Great Good Place (1989), Ray Oldenburg introduced the notion of "third places" — places which are separate from home and the workplace. A healthy and vibrant third place should be: opening and inviting, comfortable and informal, convenient, unpretentious, attended by regulars, focussed on conversation, and filled with frequent laughter. And that is exactly the way Undergraduate Seminar feels.

If this short communication has got you interested in starting a seminar, then I would love to chat with you about it and help you get it going. Please get in touch! We need more casual and friendly spaces for the mathematically inclined.

Acknowledgements This short communication wouldn't be possible without all the amazing people who participate in Seminar. Many thanks to the regulars. Lisa Jeffrey and Albert Lai co-organize Seminar with me. Thanks for all the help. This short communication was adapted from a talk given at the Winter 2025 CMS Supporting A Departmental Culture of Undergraduate Research in the education session Practical approaches to mentoring undergraduate research projects. Thanks to the co-organizers of that session: Elisa Bellah (University of Toronto) and Yuveshen Mooroogen (University of British Columbia).

A.8 Professional Development Activities

UTSC faculty writing group

Why a writing group?

Are you trying to find time to work on an article, a book, a grant, a syllabus, or any another writing project that needs your attention? We hope to facilitate your writing endeavors at this monthly writing group for UTSC faculty. During these meet-ups, participants will be provided with a quiet, comfortable space, as well as focused time in which to write. We hope this will offer an opportunity to build a supportive writing group.

What to bring to a writing meet-up?

For a successful writing meet-up, we suggest you bring with you: a laptop with charger, books and reference materials you may need, articles (printed, in PDF on your computer or USB key), pens and notebooks, music and headphones (if you want) and a reusable water bottle. **Snacks and beverages will be provided on site.**

The Pomodoro technique

Pomodoro means "tomato" in Italian. It is a time management technique designed by Francesco Cirillo at the end of the 1980s. It refers to the kitchen timer (in the shape of a tomato) originally used by Cirillo to delimit their periods of work and break. The Pomodoro technique is based on the idea that the more a task is important and complex, the more risks there are to sink into the maze of procrastination. This is why it is helpful to break this task down into concrete and time-bound steps.

Before each block of Pomodoros, we suggest you **first** identify realistic objectives to achieve (see SMART objectives below). **Second,** the writing group host will set the timer for 50 minutes. During this time, you will work on your objective until the timer rings. We suggest you refrain to work on tasks unrelated to your objective (e.g., answering emails and text messages). **After the 50 minutes period**, you will record what you accomplished and take a 10 minutes break to stretch, move away from your screen, grab a snack or something to drink.

SMART objectives

SMART objectives (Doran, 1981) is an approach meant to help set positive and achievable goals. It refers to: Specific (objectives should be straightforward and emphasize what you want to happen); Measurable (objectives usually have several short-term and on-going measurements so that you can see how you are doing in your aim to achieve them); Attainable (think about what is achievable); Realistic (Doing what's 'do-able.'); Timely (What is your time frame to achieve this specific objective?).

When and where?

Faculty who want to participate will need to register every month.

When: Oct 6, Nov 3, Dec 1 2022; Jan 12, Feb 2, Mar 2, Apr 6 and May 4 2023 between 2 - 5 PM. We suggest you arrive a few minutes before 2 PM as the first pomodoro will start on the hour.

Where: Ralph Campbell Lounge (BV380).

SIGMAA IBL

FALL WORKSHOP SERIES 2021

NETWORK

Stick around after the workshop for a 30-minute networking opportunity

Each workshop is 45 minutes long

https://cuboulder.zoom.us/j/93515879022

SEPTEMBER 8 @ 4 PM (EDT)

VIDEO LESSON STUDY OF INQUIRY IN ACTION

Elizabeth Thoren
Pepperdine University

Brian Katz
California State University, Long Beach

OCTOBER 7 @ 12 PM (EDT)

TILTING THE CLASSROOM: EXPERIENCES OF ENGAGING STUDENTS IN LARGE CLASSES IN THE UK

Lara Alcock
Loughbough University

NOVEMBER 1 @ 3 PM (EDT)

INQUIRY-BASED LEARNING MATHEMATICS AND BEYOND

Michael Starbird
University of Texas at Austin

DECEMBER 2 @ 6 PM (EST)

LISTENING TO OUR OWN ADVICE

Susan Crook
Loras College

Parker Glynn-Adey (University of Toronto at Scurborough) | **Lee Roberson** (University of Colorado Boulder)
Mami Wentworth (Wentworth Institute of Technology) | **Mel Henriksen** (Wentworth Institute of Technology
Nat Miller (University of Nortnern Colorado) | **Questions & comments:** Mami - wentworthm1@wit.edu

IBL SIGMAA presents:

2022
Spring Workshop Series

tinyurl.com/IBL-SIGMAA-Spring-2022

For questions, contact Lee Roberson
Lee.Roberson@colorado.edu

Zoom Link

Our Speakers:

SU DORÉE
Augsburg University

THURSDAY, JANUARY 27
3:30 PM EST
The Active Learning Pedagogy Sequence (the ALPS): A Model for Expanding the Use of Active Learning Structures in the College Mathematics Classroom

GEORGY KUSTER
Christopher Newport University

THURSDAY, FEBRUARY 24
3 PM EST
Connecting Teaching and Learning: Using Student Thinking as the Bridge

SANDRA LARSEN
University of Colorado Boulder

THURSDAY, MARCH 31
4 PM EDT
Back to the future: Teaching with inquiry during and beyond the pandemic

VICTOR PIERCEY
Ferris State University

TUESDAY, APRIL 26
3 PM EDT
Writing Materials for Learning with Inquiry

JANUARY 27: THE ACTIVE LEARNING PEDAGOGY SEQUENCE (THE ALPS): A MODEL FOR EXPANDING THE USE OF ACTIVE LEARNING STRUCTURES IN THE COLLEGE MATHEMATICS CLASSROOM

How can we create a classroom where all of our students are engaged, working hard, asking and answering each other's questions, learning mathematics deeply, and wanting to learn more? For many of us, active learning strategies are an essential part of the answer. In this workshop we present a framework, called the Active Learning Pedagogy Sequence (the ALPS), for understanding the ease/difficulty of implementing many different active learning structures. We will identify and try our hand at structures from several of the levels of the ALPS – both to play with mathematics ourselves and to think more about inquiry-based mathematics education. Whether you are just getting started in active learning or an "old pro" looking for new ideas, come ready to try your hand at an array of different active learning structures.

SU DORÉE

Workshop leader Dr. Suzanne Dorée is Professor of Mathematics and Chair of the Department of Mathematics, Statistics, and Computer Science at Augsburg University in Minneapolis, Minnesota where she has taught since 1989. She enjoys teaching students at all stages of their mathematical development using pedagogies that support active, inclusive, and inquiry-based learning. Dr. Dorée was recognized for her teaching in 2019 when she received the MAA Deborah and Franklin Tepper Haimo Award for Distinguished College or University Teaching of Mathematics. Dr. Dorée has a Ph.D. in Mathematics from University of Wisconsin. She dabbles in mathematical research on the state graphs of games, puzzles, and other dynamic processes, but her primary scholarship is in teaching and learning. Long active in the MAA, Dr. Dorée currently serves as Co-chair of the Transforming Post-Secondary Education in Mathematics (TPSE-math) Teaching Strategies and Practices Subgroup. She frequently runs professional development workshops for new mathematicians (and old) on active learning, teaching conjecturing (and other topics in inquiry-based learning), and mathematical speaking.

FEBRUARY 24: CONNECTING TEACHING AND LEARNING: USING STUDENT THINKING AS THE BRIDGE

Traditionally learning has been treated as a natural consequence of teaching. The goal of education however, is not teaching, rather the goal of education is learning. That is, as educators our role is to impact the minds of our students. By placing the focus of education primarily on learning we centralize the students in the classroom, and we can begin to aim toward intentionally engaging in teaching practices that support thinking and learning. This shift in focus requires a careful analysis of what exactly we desire our students to understand, how that understanding develops in our students, and how we as teachers can support its development. In this workshop, we will discuss the principles and practices of Inquiry-oriented Instruction, a student-centered form of instruction that relies almost entirely on student thinking and shares many characteristics with IBL. The goal of this workshop is to help instructors develop and/or implement lessons that foster and utilize student thinking to support learning.

GEORGE KUSTER

George Kuster is an assistant professor in the Department of Mathematics at Christopher Newport University. His research focuses on the teaching and learning of mathematics. Some past projects include how students use variational reasoning in differential equations, characterizing Inquiry-oriented Instruction, and helping teachers implement student centered teaching methods.

MARCH 31: BACK TO THE FUTURE: TEACHING WITH INQUIRY DURING AND BEYOND THE PANDEMIC

As we pass the two-year mark since the pandemic arrived in the US and instigated the Great Online Pivot in higher education, we can also look back on a great variety of teaching experiments – successful or not – that these two years represent. In the spirit of not letting a good crisis go to waste, I will draw on data about mathematics instructors' teaching practices to propose some important lessons about inquiry teaching learned from our collective crash course in trying to do it online. Using as guidance the four pillars of inquiry-based mathematics education, we will reflect together on what we did, what we learned, and how we can use these lessons as we move forward into a teaching and learning future that, whatever it looks like, will not look like the past.

SANDRA LAURSEN

Sandra Laursen maintains interests in both research and practice in science and mathematics education. As senior research associate and director of Ethnography & Evaluation Research at the University of Colorado Boulder, she leads research and evaluation studies focusing on education and career paths in STEM fields. Her research interests center on professional preparation for teaching, advancement and visibility of women and people of color in STEM careers and academia, and organizational change in higher education. She is also interested in inquiry-based teaching and learning, and in all ways to improve STEM education in and out of the classroom. She has a Ph.D. in chemistry by way of U. C. Berkeley and is proud co-champion of the 2021 Grinnell College Alumni Spelling Bee.

APRIL 26: WRITING MATERIALS FOR LEARNING WITH INQUIRY

You want to use inquiry in your course, but you can't find an activity to fit one of your learning objectives, or you can't find any inquiry materials for your course at all. How do you get started writing your own materials?

In this presentation, we will walk through and practice developing course materials ranging from single activities to an entire course. Along the way, we will identify resources we have for support as well as discuss some course design principles.

VICTOR PIERCEY

Dr. Victor Piercey is a professor of mathematics and the Director of General Education at Ferris State University in Big Rapids, Michigan. He holds a J.D. from Columbia Law School and a Ph.D. in mathematics from the University of Arizona. His teaching focuses on quantitative reasoning and actuarial science. He is the lead author of two unpublished inquiry textbooks (available to share): Quantitative Reasoning for Professionals (for a 2-semester hybrid quantitative reasoning/algebra sequence) and Financial Mathematics for Actuaries.

CMS Online Education Meeting | La Réunion d'éducation en ligne 2023 de la SMC

Saturday | Samedi November 25 novembre	Sunday | Dimanche November 26 novembre
13:00 - 13:15 Opening Remarks | Propos introductifs	11:00 - 11:15 Summary & Opening Remarks | Résumé & propos introductifs
13:15 - 14:45 Francis Su Education Plenary Conférence sur l'éducation	11:15 - 12:45 Cynthia Nicol Education Plenary Conférence sur l'éducation
14:45 – 15:00 Break | Pause	12:45 – 13:00 Break | Pause
15:00 - 15:25 *Session Block 1* | Sessions 1	13:00 - 13:25 *Session Block 3* | Sessions 3
15:25 – 15:30 Break | Pause	13:25 – 13:30 Break | Pause
15:30 – 15:55 *Session Block 2 | Sessions 2*	13:30 – 14:00 *Session Block 4 | Sessions 4*
15:55 – 16:30 Break | Pause	14:00 – 14:30 Break | Pause
16:30 - 17:30 Lew Ludwig Interactive Presentation Présentation interactive	14:30 - 14:55 Session Block 5 | Sessions 5
17:30 – 18:00 Discussion	14:55 – 15:00 Break | Pause
	15:00 - 15:25 Session Block 6 | Sessions 6
	15:25 – 16:00 Discussion

CMS Online Ed Meeting Talk Schedule| La Réunion d'éducation en ligne 2023 de la SMC

	Room 1 | Salle 1	Room 2 | Salle 2	Room 3 | Salle 3	Room 4 | Salle 4
Saturday | Samedi Session Block 1 | Sessions 1	**Ami Mamolo, Ontario Tech), Parker Glynn-Adey, (UTSC)** *Learning affordances of the Dihedral Calculator: A spatial-visual approach to groups*	**Pamela Brittain (Fields)** *Lessons from Fibonacci's Liber Abaci - And What it Can Teach Us Today*	**Paulina Chin (Maplesoft)** *Developing General-Purpose Software for Math Education: Ongoing Question*	**Paul Tsopmene (UBCO)** *Very Detailed Workbooks in Calculus*
Saturday | Samedi Session Block 2 | Sessions 2	**Sarah Mayes-Tang (UofT)** *The Impact of Multiple Problem Set Resubmissions in Proofs Classes*	**David Guillemette (Québec à Montréal)** *Arguments for a more explicit introduction of the history of mathematics in mathematics education coming from high school teachers*	**Yuliya Nesterova (Carlton)** *Lights-Out Mathematics: Helping Students with Aphantasia on Visualization Concepts*	
Sunday | Dimanche Session Block 3 | Sessions 3	**Taras Gula (George Brown), Miroslav Lovric (McMaster)** *Why should math educators care about what is happening in numeracy education research?*	**Brian Winkel (SIMIODE)** *Teaching differential equations in a modeling first and throughout context*	**Gordon Hamilton (MathPickle)** *Three Puzzles for Your First Outreach to an Elementary School*	
Sunday | Dimanche Session Block 4 | Sessions 4	**Trefor Bazett (UVic)** *Lessons from Social Media: Crafting Engaging Math Stories*	**Chantal Buteau (Brock)** *What do students learn from conducting programming-based mathematical investigations? What kind of investigations work best?*	**Frédéric Morneau-Guérin (Québec)** *Martin Gardner et la question du réalisme*	
Sunday | Dimanche Session Block 5 | Sessions 5	**Ahad Moosa (York), Nadya Askaripour (UTM)** *Exploring the Link Between Math Anxiety and testing strategies*	**Rebecca C. Tyson, Sarah Wyse (UBCO)** *Introduction to ODEs with climate change models: Linking 2nd year math students to the climate crisis*	**Zack Wolske** *Leading a Math Circle is a Walk in the Park*	
Sunday | Dimanche Session Block 6 | Sessions 6	**Diana Skrzydlo (Waterloo)** *Yes, It Blends!*	Connor Gregor, Caroline Junkins (McMaster), Lindsey Daniels (UBC) *A Diagnostic Tool that Scales Student Voice through Semi-Automated Text Analysis and Qualitative Clustering*	**Jeremy Chiu (Langara)** *Strategies for Active Learning in Math Classrooms*	

FIELDS

Mathematics Education Forum

November 26, 2022, 10 AM – 2 PM
Fields Institute, 222 College Street, Toronto

Tweet us @FieldsMathEd

Mathematicians Leading Mathematics Education

Organizers: Parker Glynn-Adey (University of Toronto) & Dragana Martinovic (University of Windsor)

Description: The objective of this month's Forum is to discuss the role mathematicians play in mathematics education. There are many famous mathematicians who contributed to mathematics education; Felix Klein, Hans Freudenthal, Erich Christian Wittmann, Leonhard Euler, György (George) Pólya, and Nikolai Lobachevsky come to mind, as well as Hung-Hsi Wu. However, "mathematics is not mathematics education" (Bass, 2005), so, what might have sparked their interest in the study of teaching, and how were they able to affect it?

Wittmann (2020) argues that "Connecting mathematics and mathematics education requires looking at mathematics from the point of education and also looking at mathematics education with a broad understanding of elementary mathematics," and cites Heintel (1978) who wrote that "taking subject matter fundamentally into account in building didactical models means breaking up the narrow boundaries of special disciplines, reconstructing 'deep-frozen' learning processes, and elaborating the social use of knowledge and also its limitations." Our speakers, who are mathematicians, also have interest in mathematics education and will present their views and involvement in it.

9:55 AM - 10:00 AM Welcome to the Forum
Description: Participants arrive and are accepted through the waiting room, view house rules, are present for the land acknowledgement, and receive an introduction to the day's sessions.

10:00AM - 10:10 AM Reports from OAME, OMCA, OCMA, OCMC, CMESG, CMS, CME, AFEMO, and others.

10:10 AM – 10:50 AM

My vision for the secondary math curriculum

Peter Taylor (Queen's University)

I will propose a curriculum that has not so much a different content as a different structure. And along with that will come different objectives.

This alternative curriculum will be richer, more sophisticated, more beautiful, more hands-on, and more active. It will be more engaging for those who are ready to engage. It will be more challenging to learn and more challenging to teach and at the same time it will welcome a more diverse classroom.

I am aware that this all sounds like a pipe-dream, but don't forget that IT WILL HAVE DIFFERENT OBJECTIVES. In a word it will value performance rather than knowledge. Instead of What do you know? it will ask What can you do?

Finally, this is not a new idea.

Bio: *Peter Taylor is a professor in the Department of Mathematics and Statistics at Queen's University, cross-appointed to the Department of Biology and the Faculty of Education. His longtime area of research is in evolutionary ecology but for the past few years he has spent most of his time developing curriculum for 9-12 mathematics. His heroes are the many high-school teachers he has been lucky enough to work with.*

10:50-11:30AM

The mathematical neighborhoods of school mathematics

Hyman Bass (University of Michigan)

What is the "geography" of school mathematics — number; measurement; algebra; geometry; trigonometry; probability; pre-calculus; calculus? Are these separate islands? Or, are they connected lands, the approach to the large continent of disciplinary mathematics? I fear that many students experience only the former. While I favor the latter, I appreciate the difficulty of providing an accessible opportunity to experience the wide sweep of this latter unifying perspective. I will report on my effort to provide such an opportunity. But accessible to whom?

Bio: *Hyman Bass is the Samuel Eilenberg Distinguished University Professor of Mathematics and Mathematics Education at the University of Michigan. He came to Michigan from Columbia University in 1999. He has held many visiting appointments, extensively in India and in France. His mathematical work is in algebra, with connections to algebraic geometry, number theory, topology, and geometric group theory. His educational interests, much of it in collaboration* *with Deborah Ball, include mathematical knowledge for teaching, task design, mathematical practices, "connection-oriented mathematical thinking," and social justice.*

Dr. Bass is a member of the National Academy of Sciences, the American Academy of Arts and Sciences, the (Third) World Academy of Sciences, and the National Academy of Education. He is past president of the American Mathematical Society and of the International Commission on Mathematical Instruction. He received the U. S. National Medal of Science in 2007.

11:30 AM – 12:00PM

Problem solving in the mathematical education of teachers.

Frédéric Gourdeau (Université Laval)

Abstract: For many years, I have been teaching a course where mathematical problem solving is central. It affects the way the class is run and the type of work we engage in. I will share some examples and my reflections on this type of works, and look forward to our conversation.

Bio: Frédéric Gourdeau is a professor of mathematics at Université Laval. He has been working in the mathematical preparation of teachers since 1995 and is a regular participant of the annual meetings of the Canadian Mathematics Education Study Group (CMESG/GCEDM). Very active in promoting mathematics, his interest in problem solving led him to set up the Association Québécoise des jeux mathématiques in 1998 (www.lamagiedesmaths.ulaval.ca) which provides resources for teachers and the general public, and runs the Championnat international des jeux mathématiques, a friendly mathematical competition.

12:00 PM – 1:00 PM

Lunch Break

1:00 PM – 2:00 PM

1:00PM – 1:30PM

Should mathematicians play a role in the education of future teachers?

Wes Maciejewski (Red Deer Polytechnic)

Mathematics courses taken by future K-12 teachers fall into two categories: "content" and "methods", the former being typically taught by members of a mathematics department, and the later by mathematics education faculty. In this presentation, I'll trouble this course distinction and examine the role mathematicians (ought to) play in the education of future teachers.

Bio: Wes received his PhD in mathematics from Queen's University and since that time has been very fortunate to work with mathematicians and mathematics educators all over the world. He's currently faculty at Red Deer Polytechnic in Alberta.

1:30-2PM

Panel with Speakers/Discussion

2:00 PM ADJOURNMENT

A.9 Service

Volunteers Needed!

Canadian Math Kangaroo Contest

Spread the joy of mathematics through this grade 1-12 contest and insightful speaker session.

(Training is on March 14th, Pi Day.)

Scan this code to register!

We need your help with event logistics, check-in coordination, contest invigilation, and date entry.

1265 Military Trail,
Toronto ON M1C 1A4
March 19th, 2023
8 AM - 4 PM

For more information, contact parker.glynn.adey@utoronto.ca

UNIVERSITY OF TORONTO SCARBOROUGH

Office of the Vice-Principal Academic & Dean

April 28, 2023

Dear Professor Glynn-Adey,

I write to thank you for your service as a panelist for the New Faculty and Library Orientation Summer 2022 program, a project of the Vice-Dean Faculty Affairs, Equity & Success in the University of Toronto Scarborough Office of the Vice-Principal Academic & Dean.

The concurrent roundtable titled *'First-Year Tips & Challenges, Teaching Stream'* was very well received and participants appreciated the insights you shared to help them set up for success. These efforts are among many in our office to ensure that U of T Scarborough faculty have the support to thrive in their academic and professional lives. Such thriving is fundamental to U of T Scarborough's pursuit of Inclusive Excellence.

Thank you for your service to your colleagues and campus. It's a pleasure to work alongside you.

Sincerely,

Jessica Fields
Professor and Vice-Dean Faculty Affairs, Equity & Success

Arts & Administration Building, 1265 Military Trail, Toronto, ON, M1C 1A4 Canada
Tel: +1 416 287 7027 · www.utsc.utoronto.ca/vpdean

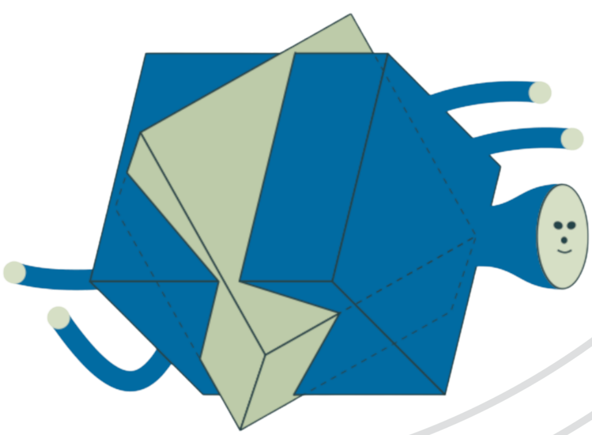
MATH IS FOR EVERYONE!

A SCIENCE LITERACY WEEK EVENT

Come celebrate all things math!

September 20 // 12-2 P.M.

Instructional Centre Atrium



- Light refreshments
- Math-inspired activities
- Connect with your campus math partners
- Find out all the ways UTSC supports math education.

Library
UNIVERSITY OF TORONTO
SCARBOROUGH

utsc.library.utoronto.ca

Thursday. Nov 4 2021. 2:00 - 6:00pm

Women and STEM: Making New Paths

Graduate Professional Day Conference at UTSC

University of Toronto Scarborough (UTSC) grad students are known for the depth and innovation of their research and training, and graduating from U of T is an important step towards a career. Yet not everyone progresses at the same rate: fewer women occupy higher-ranking STEM positions than men, and the percentage of women working in STEM in Canada has not substantially changed in the past 20 years.

Opening

2:00 p.m. – Land Acknowledgement

2:05 p.m. – Welcome

Speaker: *Mary Silcox,* Vice-Dean Graduate, UTSC and Professor, Anthropology

Workshop

2:15 p.m. – How to use LinkedIn for Job Networking

This session will cover challenges associated with being a visible member of an equity-deserving community on LinkedIn, and how to develop a network to seek out and grow new chances for success.

Speakers: *Monique Chambers*, Career Strategist, EDI, Academic Advising and Career Centre, UTSC

Jen Davies, Manager, Career Development Services, Academic Advising and Career Centre, UTSC

Break 1 (10 min)

Keynote speaker

3:15 p.m. – Women in STEM: Overcoming Barriers with Structural Change

Speaker: *Fiona Rawle*, Acting Chair, Toronto Initiative for Diversity & Excellence, Professor of Biology, UTM

Break 2 (15 min)

Panel discussion

4:45 p.m. – Navigating the 'Glass Obstacle Course'

Women in STEM face many barriers, from the unconscious biases of editorial committees to a lack of supervisory support, which some have likened to a 'glass obstacle course'. Join STEM professionals at different career stages to discuss how can we work together to dismantle the obstacles.

Speakers: *Lamia Akbar*, Environmental Health Advocate and President of the Graduate Students Association at Scarborough (GSAS)

Nadia Alam, Faculty, Department of Community and Family Medicine

Andreea Bosorogan, MSc student, UTSC; Social Events Chair, GSAS

Prof. Parker Glynne-Adey, Department of Mathematics, UTSC

Meet the speakers

Fiona Rawle
Acting Chair, Toronto Initiative for Diversity & Excellence, Professor of Biology, UTM

Nadia Alam
Faculty, Department of Community and Family Medicine

Lamia Akbar
Environmental Health Advocate
President GSAS

Andreea Bosorogan
MSc student, UTSC, Social Events Chair, GSAS

Monique Chambers
Career Strategist, Equity, Diversity & Inclusion, Academic Advising & Career Centre, UTSC

Jen Davies
Manager, Career Development Services, Academic Advising & Career Centre, UTSC

Parker Glynne-Adey
Faculty, Department of Mathematics, UTSC

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Academic Advising & Career Centre

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Learn about the speakers

Fiona Rawle
Acting Chair, Toronto Initiative for Diversity & Excellence, Professor of Biology, UTM

Fiona Rawle has a Ph.D in Pathology and Molecular Medicine and is the Associate Dean, Undergraduate, at the University of Toronto Mississauga, and a Professor, Teaching Stream, in the Dept. of Biology. Her research focuses on public communication of science, the science of learning, and equity, diversity, and inclusion in science. She has received numerous awards focused on teaching, including the University of Toronto's President's Teaching Award. Dr. Rawle is also the acting Chair of the University of Toronto's TIDE group (Toronto Initiative for Diversity & Excellence), through which she gives lectures and workshops on bias, equity, diversity, and inclusion.

Lamia Akbar
PhD student, Environmental Health Advocate and President, Graduate Students Association at Scarborough (GSAS)

Lamia Akbar graduated with Honours BSc. in Molecular Biology and Health Studies at the University of Toronto, and is currently a 3rd year PhD student at the Dept. of Physical and Environmental Sciences (Tsuji Environment & Health Lab) at UTSC. Her research focuses on the complex health impacts of broad environmental factors on Indigenous and racialized communities within Canada and globally. She is driven by justice-focused practice and has diverse experiences working in research-based and leadership roles in wet labs, clinical settings, and grassroots organizations. She is the current President of the GSAS where she works to build an equitable and proactive graduate student community.

Nadia Alam
Faculty, Department of Community and Family Medicine

Dr. Nadia Alam is a recognized physician leader, strategist and policy analyst, who has won awards in teaching, leadership and health system policy. She has published, presented and led tables in a number of fields, including primary care, integrated care, mental health, leadership, burnout and women in medicine

Andreea Bosorogan
MSc student, UTSC, Social Events Chair, GSAS

Andreea Bosorogan earned her BScH at Queen's University, and she's currently a Cell and Systems Biology MSc student, in the EGV lab, at the University of Toronto Scarborough. Her research focuses on the correlation between plant defense metabolites and bacterial diversities in plant-insect interactions. Andreea is also the social events chair for the Graduate Student Association at Scarborough, where she works to build a tighter-knit and inclusive community among graduate students. Recently, Andreea started posting about plant science and graduate student life on social media (Instagram & Tiktok: @ada_miza). Through her social media platforms, she intends to share her grad life journey as a woman and first-gen in science, normalize bench-work and raise awareness about the many topics in plant sciences.

Monique Chambers
Career Strategist, Equity, Diversity & Inclusion, Academic Advising & Career Centre, UTSC

Monique's portfolio includes supporting students from equity-deserving groups by equipping them with the tools they need to effectively strategize towards their careers of choice. She brings to her current role a wealth of experience and knowledge. Most recently, Monique was the Coordinator for Student Diversity and Inclusion Initiatives with Humber College, providing oversight to a Black-centered equity hub called "The BASE". She has over 16 years of experience working with marginalized communities within the education and community development sector.

Jen Davies
Manager, Career Development Services, Academic Advising & Career Centre, UTSC

Dr. Jen Davies has worked in post-secondary and community-focused career services since 2007, except for a stint teaching English in South Korea. In her spare time she completes Massive Open Online courses, and teaches online for one of her alma maters, Conestoga College. She has also studied at Queen's (Psychology) and McGill (Counselling Psychology), and completed OISE's Doctorate in Education in Counselling Psychology for Community and Educational Settings in 2019.

Parker Glynne-Adey
Faculty, Department of Mathematics UTSC

Parker Glynn-Adey (he/him) is an Assistant Professor, Teaching Stream, in the Computer and Mathematical Sciences Department at UTSC. When he is not hiking with his family, playing with his daughter, or writing, he can be found teaching mathematics, organizing mathematics education events, or chatting with students.

Women and STEM: Making New Paths
Thursday. Nov 4 2021. 2:00 - 6:00pm
Graduate Professional Day Conference at UTSC

To register, click here or scan QR code

A.10 Reference Letters

Samira Goder
1008201054

MATD93: Braid Theory & Applications

Course Reflection and Feedback

I never really considered doing a research course until my final year at UTSC, but I'm so glad I decided to do it. Taking MATD93: The Braid Group and Applications gave me my first real glimpse into the research process. I would struggle through the process of starting with an idea, feeling stuck, and eventually being less confused. Seeing a professor I admire go through the same experience made me realize that research is meant to be a slow process, which actually made it seem less intimidating and encouraged me to consider graduate studies after graduation.

This course also offered some great networking opportunities. I had the chance to present an introduction to our research on Braid Theory and Groups at the UTSC Research Symposium, where I recieved the "Best Oral Presentation" award. I was also encouraged to submit an abstract to the URSA Tri-Campus Conference in January 2026, with hopes of presenting my research there as well.

One thing I really appreciated about the course was its flexibility. Initially, we had planned to write a paper, which we still hope to do, but with the busy semester, we just didn't have the time to work on it. Professor Glynn-Adey was very understanding and offered to continue working on the paper with me until the end of Winter semester 2026. This flexibility took the pressure off, allowing us to focus on producing high-quality work rather than rushing to finish a paper during just the fall semester.

I also loved how hands-on the course was. Since our research focused on Braid Theory and its applications, each week we presented physical hand-braided samples that helped shed light on various aspects of our research. It was fun making connections between the braids and track plans, and I really enjoyed learning a new craft in the process.

Another aspect I appreciated was the collaborative environment. The course was shared with a mentor student of Dr. Parker Glynn-Adey, which created a relaxed, non-pressured environment. I enjoyed working alongside a younger student and was happy to see MATA29 students take an interest in Braid Theory by joining us for some meetings to learn how to braid. This collaborative effort made me feel supported and less isolated in my research.

If I could change one thing about the course, it would be the organization. While I appreciated the adaptability, I think having a clearer structure such as a set schedule for readings, research assignments, or an agenda for our weekly meetings. This would have been helpful in knowing what to plan for and look forward to in upcoming weeks.

Emily Forbrigger
De La Salle College "Oaklands"

MATD93: Braid Theory & Applications

Course Reflection and Feedback (from a high school student)

Last summer as my studying at UTSC began, the opportunity itself to engage in a university course was novel and exciting. Up to that point, all I knew about mathematical research came from my imagination and minor reading. Through this course, I learned so much about the research process and the mindset of studying pure math. These lessons would benefit any student interested in pursuing a future in mathematics, especially due to the accessibility, support, and enthusiasm provided by Dr. Glynn-Adey throughout the learning process.

Moreover, I agree with the undergraduate students who took part in this course; the hands-on, tangible style of both teaching and researching kept the work engaging while solidifying our understanding of course content. I enjoyed braiding samples for each meeting at the beginning of the semester, and I recommend keeping that consistent throughout the term. Also, braiding during meeting time was a great way to explore ideas together, though I also think that more structure and planned purpose prior to a braiding "lesson" would make a more efficient use of time. This is primarily because physically braiding can take a while, and sometimes I felt disappointed when we did not have an opportunity to discuss any ideas together because we went overtime with braiding.

Also, the field trip we took to the Canadian Textile Museum may have been my favourite part of the course. It illuminated to me how interdisciplinary the study of braids was, and I loved being able to combine my passions for math and the arts together. I suggest for the future that field trips become a regular addition to this course and other similar ones, especially in fields like pure math where tangible interaction with studied content is a rarity. Similarly, the opportunity to listen to Samira Goder's presentation during the UTSC CMS undergraduate research conference was exciting and enlightening. It was an incredible opportunity to talk with university students and faculty in a research context, and also to get to know them better as I saw them around the school.

Expanding on that note, throughout my time at UTSC, I have felt incredibly welcomed into the university environment, undoubtedly due in large part to Dr. Glynn-Adey. This is not directly related to the course, though it was through MATD93 that I had the chance to experience the university community I always envisioned. Each student and faculty member I interacted with treated me with hospitality and respect. I would recommend this experience to any high school student looking to learn more about post-secondary academic and social life.

Finally, I will talk in regards to course content and learning goals specifically. I found that the unique structure of the course kept every session exciting and fun, but it left some gaps in my knowledge about how concepts related and how we got from one topic to the next. I think that this type of "flow" was in part beneficial to the survey-like nature of this course in terms of the breadth of angles with which we looked at braid theory. However, a more clear development of ideas within the structure of the course would have helped me build my knowledge progressively and with more awareness of what broad categories and relations existed within what we learned of braid theory. For example, we focused a great deal of our time on the subgroup of the braid group called loop braids, but sometimes I would get confused which properties or ideas applied to just this group and which ones were generalized to all braids.

Overall, Dr. Glynn-Adey was extremely accessible and open to suggestions or feedback throughout the course. He stimulated my interest in braid theory from the beginning, and he challenged me to achieve my best in all the work I did. I am so grateful that I got to take part in this course, and my only critiques relate to developing its structure for the future.

MATD93: End-Of-Semester Reflections

Hi Parker,

How It Went

Overall, very well! This was an atypical reading course — a few weeks of research, a hectic scramble to submit a paper in week 7, then a second, smaller writing project. It had its pros and cons: I spent a lot of the semester stressed to my limit, but I enjoyed the accomplishment of submitting to a real-world conference and discovering new ideas independently. The semiweekly meetings worked well for me: regular accountability, a place to explore together, and a set of expectations before each meeting.

What I Learned

Perhaps this is best left as a list of noun phrases: String figure construction and analysis. Storer calculus and Jayne-Rivers-Haddon notation. Comfort and dexterity with string figure production. Basic group theory/knot/braid theory, from a background of basically zero. Techniques in LaTeX. How to structure the collaborative writing process. How to *teach* string figures: getting on people's level, figuratively and literally. How to, and that one ought to, scope out journals and tailor one's submissions accordingly. That research is some mixture of reading, playing with other people's ideas, following trails of possibilities as far as they take you, then sitting yourself down and WRITING IT. How to pronounce "Osage".

Improvements

One thing I would have benefitted from was a defined writing process from the start: what needed to be written when, the expectations for drafts at various stages, etc. It might have been useful to spend more time considering the outline of the paper, starting with a thorough journal/conference scope and dedicating time just to structuring the paper before we started writing. As it was, we went through several restructures after a considerable chunk of writing had already been done, though I acknowledge that the suddenness and urgency of Bridges didn't help.

General Reflections

Probably my most valuable takeaway from the course was a hands-on understanding of what math research looks like. The second-most valuable part, and what made this course so unique, was the opportunity to play by myself, come up with genuinely new stuff, and follow the cool ideas wherever they led. I was interested in research before, but now I know with certainty how happy it makes me.

Thanks for a great semester!

Ness

Computer & Mathematical Sciences
UNIVERSITY OF TORONTO
SCARBOROUGH

To Whom it May Concern:

I am writing this letter in support of Parker Glynn-Adey's application for tenure and promotion.

I was teaching MATB41 together with Parker. It pretty early became clear that the students preferred Parker's lecture as hardly anyone showed up for my lectures. Nevertheless, Parker did everything to help me fix the situation. He visited my lectures and gave useful advice afterwards. His support and encouragement meant a lot to me, and helped me not to lose motivation even when I needed to speak in a large but almost empty lecture hall.

I think the distribution of tasks among us was reasonable, I prepared the sample homework and exam solutions, while Parker took care of the tutorials and administrative tasks. He always made sure that I got proper credit for my work, for example by mentioning my name when sending out my solutions to the students. It made me feel appreciated. I was also impressed by how efficiently Parker handled the administrative tasks, and the communication with the students and the TAs. I also learned a lot from him about what can be reasonably expected from the students, what level of difficulty one should aim for such that the students are made to work hard but they are not discouraged or overwhelmed. In general, Parker helped me a lot to adapt to the local teaching practices.

Next semester I was teaching MATB24, where I developed my own version of the course. From time to time, I consulted with Parker. He was always very generous with his time, and his insights and suggestions helped me a lot to improve my course. Also, Parker has valuable knowledge about the skills of many of the TAs in the department. Getting some information on my TAs from Parker, helped me to assign certain more critical tasks to the right TA.

To summarize, Parker was a true mentor for me, who was always kind, encouraging and eager to help, and he is without doubt a valuable and important member of the department.

Mészáros András

András Mészáros, PhD

Postdoctoral Fellow,

University of Toronto Scarborough

November 8, 2023

University of Toronto Scarborough

1265 Military Trail

Scarborough, ON, M1C1A4

Dear Committee members,

I am writing to wholeheartedly recommend Professor Parker Glynn-Adey for promotion to the tenure track position in the teaching stream at the University of Toronto Scarborough. I have had the pleasure of working closely with Parker during our time teaching MATA22, an introductory course in Linear Algebra, and I am thoroughly impressed with his dedication, management skills, expertise, and passion for teaching.

In the capacity of our collaboration, Parker consistently demonstrated exceptional teaching as well as management abilities. He possesses a deep understanding of Mathematics and has the remarkable ability to convey complex mathematical concepts in a clear, concise, and engaging manner. His lectures, as well as the course content and structure, were not only intellectually stimulating but also fostered an inclusive and supportive learning environment, where students felt encouraged to participate and excel.

One of Parker's standout qualities is his commitment to student success. He goes above and beyond to ensure that all students comprehend the material thoroughly and have all the opportunities and tools to improve their understanding. His approachability and willingness to assist students inside and outside the classroom have significantly contributed to our students' positive learning experiences. Parker has also demonstrated a keen interest in pedagogical innovation, incorporating various teaching techniques and technologies to enhance the learning process effectively.

Furthermore, Parker is an excellent team player and collaborator. He works effectively with colleagues, contributing valuable insights and ideas to our teaching methods and curriculum development. His dedication to continuous improvement and willingness to engage in professional development activities have greatly benefited me as his collaborator, as well as our students.

In addition to his exceptional teaching abilities, Parker is also a person of outstanding character. He is reliable, dependable, and possesses excellent interpersonal skills. His positive attitude and willingness to take on leadership roles make him an asset to the department and the academic community.

I am confident that Parker will continue to make significant contributions in the years to come. He has my highest recommendation for promotion, and I am certain that his passion for teaching, combined with his expertise and dedication, will continue to inspire, and empower students.

Please do not hesitate to contact me if you require any additional information.

Sincerely,

Daniel Calderón

Daniel Calderon Wilches

PhD Candidate, University of Toronto

d.calderon@mail.utoronto.ca

(+1) 647-819-2214

In the winter of 2022, our collaborative efforts as a teaching team for MATA22 Linear Algebra were under the capable leadership of Parker, who held the role of Coordinator for the course. This position required him to oversee the academic needs of a substantial student body, comprising more than 600 individuals. Despite the challenges inherent in managing such a large cohort, Parker demonstrated exemplary organizational skills and a genuine commitment to the educational process.

One of Parker's noteworthy contributions to the course was the meticulous preparation of various instructional materials. His dedication was evident in the creation of comprehensive course notes that served as an invaluable resource for students navigating the complexities of linear algebra. Additionally, Parker curated challenging yet rewarding assignments and thoughtfully designed tutorial worksheets to reinforce key concepts covered in the lectures. These materials played a crucial role in cultivating a robust learning environment, providing students with the tools they needed to succeed.

The impact of Parker's efforts on our students was profound. The teaching materials he developed not only facilitated a deeper understanding of linear algebra but also inspired a sense of confidence among the student body. Through his commitment to excellence, Parker elevated the overall educational experience for everyone involved in the MATA22 course.

Beyond his role as a Coordinator and contributor to the course materials, Parker distinguished himself as a mentor and educator at UTSC. His guidance went beyond the classroom, demonstrating a keen interest in the academic and personal development of each student. This holistic approach to teaching left a lasting impression on those fortunate enough to be under his tutelage.

In conclusion, Parker's multifaceted contributions as a Coordinator, creator of educational materials, and mentor make him a truly outstanding teacher at UTSC. His impact on the MATA22 course and the academic community at large is a testament to his passion for education and his unwavering commitment to student success.

- Kaidi Ye

University of Toronto Scarborough - Department Computer and Mathematical Sciences
1265 Military Trail, Toronto, ON, M1C 1A4, Canada

November 8, 2023

To Whom it may concern,

I am writing on behalf of Parker Glynn-Adey to commend his course coordination and administration. As one of his repeat Teaching Assistants for both MATB41 and MATB42 at the University of Toronto Scarborough, I have had the honour of working with Parker over multiple terms and must praise his professionalism, kind demeanour, and teaching skills.

I started to work with Parker online during COVID and he was so very invested in making sure to help us TAs out and be involved in the work we were doing. In that particular term, he was creating new curriculum and so we were writing up solutions for tutorials together. However, as the term progressed, Parker noticed that I and a fellow TA were primarily contributing to this effort. Since he liked the work that we were doing though, he offered if we wanted to be assigned this task and gave us extra paid hours to compensate all our efforts. This is just an example of how thoughtful Parker is and how he makes a point to commend good work and make sure it doesn't go unnoticed.

Parker also really desires to teach his students well and truly cares about how they are learning in the course. During another term, he released a form to allow students to give feedback as to how they were feeling about the course thus far. As to be expected there were some unhappy responses, but nonetheless, Parker instantly made announcements in the class and changes in the course to do all that he could to make the course better and accommodate the concerns of his students. This was a testament to his authentic care to be the best teacher he can be even when dealing with more difficult students.

Lastly, I would like to note his true compassion and support. Even though I was one of his main TAs over the years and always tried to go above and beyond, in the final terms of my degree I found myself way overbooked and was having a very hard time keeping up with some of my responsibilities. I felt very badly, but Parker's compassion for my situation really helped me during that time. He did note to me how I should make a point to look ahead in the future and I learned from this, but he also did all he could to help me out while I was stuck in that situation and that meant a lot.

Parker is an exceptional individual who is extremely knowledgeable and professional in his work but is also such a good and kind person too. It was truly a pleasure working with him and I would love to again if ever the opportunity arose. I can assure you that he will excel beyond your expectations in any position you may have for him due to his professional competency and his kind and caring personality.

With kind regards,

Andrew D'Amario
Former Teaching Assistant
Department Computer and Mathematical Sciences
andrew.damario@mail.utoronto.ca

To Whom It May Concern,

This is Hooman (Seyedhooman Ahmadpanah) and I am currently a student and an employee at the UTSC. I was lucky enough to work closely with Professor Parker first as one of his students in MAT A29 and then later as one of his TAs for MAT A29. I also had the opportunity to be included in a few of his numerous extracurricular/student engagement activities, including weekly social events for CMS students, Math Kangaroo, and many more.

Professor Parker's pedagogical approach is remarkable and unparalleled in my academic journey thus far. His ability to tailor explanations to individual learning styles and needs is exceptional. He meticulously prepares his teaching resources, continually refining them each term to ensure they are as beneficial to students as possible. His dedication to students' academic and personal well-being is profound, extending his mentorship beyond the confines of the classroom. Mathematics or manners, there is always something to be learned from Professor Parker.

As his TA, I was met with an incredibly supportive and well-structured experience. Professor Parker possesses an innate ability to identify and nurture the strengths of his team members, aligning them with tasks that enable personal and professional growth. The chance he gave me to challenge myself has had a lasting and transformative impact on my academic trajectory at UTSC. To this day, I’m thankful to him, and I always will be. That year, he gathered some talented undergrads who would have never had a chance to showcase their potential without him. He trained a cohort of TAs; most still TAing and contributing back to CMS.

He has a unique ability to empathize and view situations from the perspective of his TAs, providing thoughtful support and comprehensive training that I found to be unmatched in subsequent TA roles. Professor Parker's influence on my approach to teaching and collaboration has been so significant that I have carried his methodologies into other TA positions, advocating for their adoption. The positive feedback from other faculty members who have implemented these strategies is a testament to their effectiveness and his visionary approach to education and team leadership. He is also highly organized, always being a few steps ahead of the TAs and students. You will never catch him rushing his work.

Professor Parker distinguishes himself through his dedication to knowledge and innovation in teaching. He regularly updates his teaching methods and integrates new technologies to improve the learning experience. A notable example is his use of an office camera to demonstrate mathematical concepts on a whiteboard visually. He complements this by using a custom program to capture and upload these explanations to his website, making them accessible to all students. This approach is just one among many of his innovative teaching methods that could greatly benefit the wider academic community. Professor Parker's innovative teaching transcends traditional methods, emphasizing accessibility and engagement for all students, thereby redefining educational standards.

In conclusion, individuals like Professor Parker are invaluable in academia and the wider world. His absence, particularly now during his sabbatical, is deeply felt within the CMS department and UTSC. I wholeheartedly recommend Professor Parker for his exceptional qualities as an educator, mentor, colleague, and friend. His contributions are not only essential but also irreplaceable. It would be a shame to imagine CMS without him.

Sincerely,
Hooman Ahmadpanah

To whom this may concern,

I trust this letter finds you well. I am writing to wholeheartedly endorse Professor Glynn-Adey for continuing status within the university. As a former Teaching Assistant for the MAT A22 course under Professor Glynn-Adey's guidance, I had the privilege of closely witnessing his exceptional dedication to both the students and his teaching team.

In our joint teaching experience, Professor Glynn-Adey demonstrated an incredible commitment to the success and growth of the students. He instructs his TAs to provide extensive support to students and works closely with his teaching team. His collaborative approach encourages active participation, ensuring that students receive the necessary guidance and resources to thrive academically. Additionally, he sets a commendable standard by treating his TAs with the utmost respect and professionalism, fostering a positive working environment and exemplifying an unwavering positive attitude that reflects in the teaching team's overall morale and dedication.

Furthermore, Professor Glynn-Adey demonstrated exemplary course administration, efficiently managing all facets of the MAT A22 course. Notably, he highly prioritizes student feedback, readily adapting his teaching methodology throughout the semester. His approachable and friendly demeanor fosters a supportive learning environment, as evidenced by his dedication to hosting frequent office hours, active engagement on Piazza, and additional personalized sessions in his office. Students find his lectures well-organized and his teaching style easily digestible. His commitment to thorough course coverage ensures that assessments are fair and comprehensive, reflecting his exceptional dedication to student success.

Beyond the confines of the lecture hall, Professor Glynn-Adey's unparalleled passion for mathematics becomes distinctly evident. His dedication to fostering student interest in math research is exemplified through his initiative in hosting the undergraduate seminar which serves as a platform for students to delve deeper into the world of mathematical research. Furthermore, his enthusiasm for mathematics transcends traditional academic boundaries. He frequently engages students by demonstrating the exciting and practical applications of mathematical concepts. Whether it involves performing and explaining captivating magic tricks, showcasing the art of juggling through mathematical patterns, or introducing stimulating math puzzles, Professor Glynn-Adey consistently inspires his students to discover the inherent fascination of mathematics beyond the classroom.

Professor Glynn-Adey is an integral part of our mathematics community, consistently breathing new life into the learning and exploring the fascinations of mathematics. I am confident that he would continue to make invaluable contributions to UTSC’s academic community and uphold the institution's commitment to inclusive excellence.

Sincerely,

Leon Lee
Former MAT A22 TA

A.11 Course Syllabi

University of Toronto Scarborough
Department of Computer and Mathematical Science
MAT A29: Calculus I for the Life Sciences

Lecture Schedule

LEC 01	Wednesday	15:00–17:00	HL B101
	Friday	14:00–15:00	HL B101
LEC 02	Monday	14:00–16:00	AA 112
	Wednesday	10:00–11:00	AA 112

Important Dates

- Reading Week: Saturday October 8th – Sunday October 16th

- Drop deadline: Monday November 21st

Professor's Contact Information

Parker Glynn-Adey (he/him)
Preferred Names: "Parker" or "Professor Parker"
E-Mail: parker.glynn.adey@utoronto.ca
Website: https://pgadey.ca/
Office: IC 344

Office Hours

Parker holds office hours 11:15-12:15 on Wednesdays and Fridays in IC 404. Office hours are a dedicated time that Parker is available to answer your questions, discuss course content, and generally be of support. If you would like help in the course but have a scheduling conflict that prevents you from attending office hours, please email Parker to schedule an appointment.

Textbook

OpenStax Calculus Volumes 1 and 2.
These books are open educational resources, and they're freely available online.

- OpenStax Calculus Volume 1 for Weeks 1-9.

- OpenStax Calculus Volume 2 for Week 10-12.

Prerequisite / Exclusions

Prerequisite: Grade 12 Calculus and Vectors. This course will be difficult without the pre-requisite. If you want to attempt the course without these courses, please contact Parker immediately.

Note that this course excludes most other calculus courses: MATA20H3, MATA27H3, MATA30H3, MATA31H3, MATA32H3, MAT123H, MAT124H, MAT125H, MAT126H, MAT133Y, MAT135Y, MAT137Y, MAT157Y, JMB170Y

Your Professor's Message

Hi! I'm Parker Glynn-Adey, the professor for MAT A29. This is one of my favourite courses at the University of Toronto. It was the first course that I ever taught, and I'm glad to be teaching it back teaching it again.

I like it so much because the students in this course are awesome. You want to be doctors, pharmacists, nurses, mental health workers, and all sorts of people in the life sciences. And that's awesome! I want to help you succeed in that. If I can get you started doing a bit of math, and you can use it on your mission in the life sciences, then I'll be tremendously happy.

I've tried to design this course so that you can succeed. I'm hoping that there are no surprises in the course, and that it does not stress you out too much. If you're feeling unsure about your ability to succeed, or you need someone to talk to about the course, then please come to me. I'd be glad to help. My goal is to help you succeed, to go on to finish your program in life science, and to support you on your journey.

Course Outline

- Limits and continuity.
- Differentiation: product, quotient, and chain rules.
- Applications of differentiation: plotting graphs, optimization, and related rates.
- Integration: substitution, integration by parts, partial fractions, and improper integrals.

Course Objectives

"A course in differential calculus for the life sciences. Algebraic and transcendental functions; semi-log and log-log plots; limits of sequences and functions, continuity; extreme value and intermediate value theorems; approximation of discontinuous functions by continuous ones; derivatives; differentials; approximation and local linearity; applications of derivatives; antiderivatives and indefinite integrals."

Student Learning Outcomes

By the end of the course, students will be able to:

- Calculate limits involving functions and sequences.
- Apply differentiation techniques to a variety of problems such as graphing and optimization.
- Differentiate functions involving logarithmic, exponential, and trigonometric functions.
- Integrate functions by finding anti-derivatives using substitution, integration by parts, and partial fractions.

Grading Scheme

Tests	$2 \times 20\% =$	40%
Exam	$1 \times 40\% =$	40%
Practice Tests and Exam	$3 \times 1\% =$	3%
Assignments	$(6-1) \times 3\% =$	15%
Class Surveys	$2 \times 1\% =$	2%

FAQ: What does $(6-1) \times 3\%$ mean?
It means that there are six assignments, but we drop one of them. Thus, we will only count your best five assignments towards your final grade. You can skip or miss an assignment without it impacting your final grade. The lowest assignment is dropped automatically when calculating final grades. Each assignment is worth 3% of your final grade.

Grading Policy

The grading scale for this course is based on a points system. Therefore, grades will NOT be rounded up or down. In general, grades are only changed due to a miscalculation. If you have concerns about your grade on an assignment, or term test, you have five days after the grade is posted on Crowdmark to contact the TA who graded the question. Therefore, do NOT wait until the end of the academic term if you have questions about your grade. For a list of who graded which questions, please see the Quercus page.

All grades will be distributed via Crowdmark. You will not see your grades on Quercus.

Assignments

Goal: these assignments give you the opportunity to deepen your understanding of topics covered in this course, and to practice. We use these assignments to determine if you can solve problems slowly, without time constraints.

Procedure: we will be using Crowdmark to grade assignment submissions. You will get a personalized submission link sent to your UToronto email address. Do NOT share this link with other students.

Due Date and the Zero Date: Every assignment has a due date and a zero date. Crowdmark and Quercus will list a due date on Friday. The following Monday will be the zero date. The due date is the recommended date for submitting the assignment. If you submit after the zero date, your assignment will receive a grade of zero. The due date and zero date are both in the early afternoon at 13:59 (EST).

Evaluation Criteria: The TAs will only grade two questions. This policy is called *subset grading*. Present your solutions in a logical and clear manner. Detailed solutions will be made available shortly after the zero date. Please pay attention to the following when writing assignments:

Format: solutions are neatly and correctly assembled and have a professional style. The graders should not struggle to read your work.

Completeness: all steps are clearly and accurately explained.

Content: the written solutions demonstrate mastery and fluency with the content of the course.

tl;dr: Do good work and submit it via Crowdmark as early as possible.

The Term Tests

The term tests will be written outside of regular lecture hours. The dates will be determined by the Registrar. We have requested that they will happen in Week 5, 6, or 7 and Weeks 8, 9, or 10. The term test may happen on a Friday or Saturday. If you cannot attend reasons of creed or religion, then you must contact Parker as early as possible to arrange for an alternative sitting.

If you miss a test then you must complete the Self-Declaration of Absence form on ACORN. If you miss the test for medical reasons, then you will need to send a UTSC Verification of Student Illness or Injury form to Parker:

http://www.utsc.utoronto.ca/~Eregistrar/resources/pdf_general/UTSCmedicalcertificate.pdf

Students who miss the midterm test will be asked to provide the Verification Form and a timetable for the next five days. You will be given only one opportunity to write the make-up test.

The Exam

The exam will be written during the fall exam period (December 8th to 20th) and will be conducted according to official UTSC Exam Regulations.

Electronic Aids

All calculators, laptops, phones, smart watches, and any device capable of sending and receiving messages or performing calculations will not be permitted during the term test or final exam. Possession of any electronic device during the term test or final exam is an academic offense. You may use these aids only for homework and study.

AccessAbility

If you have any reason to believe that you may require accommodations, contact Parker and/or the AccessAbility Services as soon as possible. We can discuss the particulars of your situation and, if needed, get you registered with AccessAbility Services. AccessAbility Services staff (located in AA 142) are available by appointment to: assess specific needs, interact with professors, provide referrals to medical professionals, and arrange appropriate accommodations. You can reach AccessAbility at: ability.utsc@utoronto.ca.

Academic Integrity

The Code of Behaviour on Academic Matters states:

"It shall be an offence for a student knowingly:

1. to forge or in any other way alter or falsify any document or evidence required by the University, or to utter, circulate or make use of any such forged, altered or falsified document, whether the record be in print or electronic form;
2. to use or possess an unauthorized aid or aids or obtain unauthorized assistance in any academic examination or term test or in connection with any other form of academic work;
3. to personate another person, or to have another person personate, at any academic examination or term test or in connection with any other form of academic work;
4. to represent as one's own any idea or expression of an idea or work of another in any academic examination or term test or in connection with any other form of academic work, i.e. to commit plagiarism (for a more detailed account of plagiarism, see Appendix A);
5. to submit, without the knowledge and approval of the instructor to whom it is submitted, any academic work for which credit has previously been obtained or is being sought in another course or program of study in the University or elsewhere;
6. to submit any academic work containing a purported statement of fact or reference to a source which has been concocted"

Summary

Do not manipulate document, use unauthorized aids, impersonate[1] someone else, copy solutions, or submit your own work from other courses. Simply put, do not cheat in this course.

Be careful!

1. Don't let people photograph your work. Make them write their own summary.
2. Don't hire a tutor to complete your assignments. Ask them to check your work.
3. Don't reproduce solutions found online. Discuss potential solutions with your TA or Parker.
4. Don't let a TA do your assignment. Try the questions before tutorial or office hours.
5. Don't bring unauthorized items to evaluations. Leave your phone and watch at home.

Helpful Resources

- The Math Help Room (IC404) is always helpful. The TAs and Parker hold office hours there.
- The Centre for Teaching and Learning has numeracy workshops.

[1]Believe it or not, people do this. Don't hire an imposter off Craig's List to write your test.

Communication Policy

All e-mail must be from an official University of Toronto account. You must include [MAT A29] in the subject line, or your e-mail might get lost. Please include your name and student number in every e-mail that you send. Be sure to include the precise question, and the problem or difficulty.

```
To:  parker.glynn.adey@utoronto.ca
From:  leonhard.euler@utoronto.ca
Subject:  [MAT A29] What is a derivative?

Hi!  I am Leonhard Euler (12932188) from MAT A29.
I need help with this question:  Find the derivative of f(x)=x^2.
My problem is this:  I don't know what the word ``derivative'' means.

Thanks!
```

Parker checks his e-mail between 09:00 and 16:00 during the work week. Parker will respond to all email inquiries within two business days. He will respond to emails sent after Friday at 16:00 by 16:00 on the following Monday morning. If you do not get a quick response, please follow up with another e-mail. Don't worry about contacting Parker. He is happy to help!

Feedback

All feedback is welcome in this course. You can submit anonymous feedback here:

https://pgadey.ca/feedback/

You may use the form to comment on lecture, ask questions about the course, or give me tips. You do not need to enter your name or email address unless you want a private response from Parker. Note that your anonymous feedback may be discussed (and answered) in lecture, or on Quercus.

Modifications to Course Delivery

Parker reserves the right to modify the course requirements, mode of delivery, and other related policies as circumstances may dictate with sufficient notification to all students. Given the COVID-19 Pandemic, he recognizes that unanticipated emergencies may arise that require modifications to our class schedule and/or requirements. Parker does not expect to invoke this clause, but if he needs to, you will be notified as soon as possible. Any change will be posted on our Quercus site and sent to your university email address.

Advice on Submitting Assignments

- Photograph all your pages early.
- Set aside at least a half an hour to upload your assignment.

- Check and re-check your submission.
- Submit your assignment as early as possible.

FAQ: The Due Date and the Zero Date

Why are the due date and zero date at 13:59 (EST)?

We do not want you to work late in the evening. If you submit your homework in the early afternoon, then you can use the evening to relax.

What happens if I miss the due date?

You will not be penalized for submitting your assignment any time before the zero date.

What happens if I submit my assignment after the due date but before zero date?

Nothing will happen, and your work will be accepted as usual.

What happens if I submit my assignment after the zero date?

You receive a mark of zero.

Why is there a gap between the due date and the zero date?

To give people time to figure out Crowdmark without being penalized. It is complicated software, and it is easy to make mistakes. We want you to have lots of time to upload your work and get a great mark.

FAQ: Errors While Submitting Assignments

What happens if my internet stops working on Friday?

You have extra time to submit your work. If your internet stops working on Friday, you will have all weekend to upload your assignment.

What happens if my internet stops working on Monday?

Due to cosmic rays, coming from deep in outer space, many people have problems with their internet at the very last minute. If your internet fails on Monday, and you are not able to upload your work, then you will receive a zero.

What happens if I upload all my work and forget to hit submit?

Your work will not be graded. You will receive a zero for the assignment.

Can I send you a screenshot to show that I completed the work on time?

Screenshots sent to us to prove that it was completed before the zero date will not be accepted.

What happens if I e-mail my assignment instead of submitting it to Crowdmark?

The instructional team of professors and TAs will not accept work sent by e-mail. All work must be submitted through Crowdmark.

What happens if I slip my paper under the professor's door?

Your work will not be accepted. All work must be submitted through Crowdmark.

What happens if I upload the files in the wrong order?

The instructional team will not correct your file order. You must check that you uploaded everything in the correct order.

What happens if I upload all the questions to one question's slot?
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What happens if the TAs cannot read my work?
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What happens if I don't submit some of my work by accident?
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FAQ: Term Tests

When will the tests occur?
At this time, we do not know when the tests will occur. It will be announced on Quercus as soon as we know the date. You can expect at least two weeks notice.

What happens if I miss a test?
You will need to use the Self-Declaration of Absence Form on ACORN within 24 hour of the test. For instructions, see this document.

What happens if I am unable to attend a make-up test due to a conflicting class or assessment?
You will need to choose which event to attend. If you have a conflicting class or assessment, then you might ask the other professor for an accomodation.

What happens if I am unable to write a term test or the make-up test?
You will receive a zero for that test grade.

Can I transfer the weight of a term test to my final exam?
No. We will not transfer the weight of a term test to the final exam.

If I score better on the final exam, can it replace my term test grades?
No. We will not replace a low term test grade with a higher final exam grade.

MAT A29 Reading Guide Please print this page for reference throughout the course.

Name: ________________________ Student Number: ___________________

Week	Suggested Exercises	Notes & Ask Prof / TA
1	§1.1 Review of Functions 14, 15, 17, 19, 46, 47, 53, 55	
	§1.2 Basic Classes of Function 59, 61, 69, 73, 83, 85, 87, 91, 93, 95, 97	
2	§2.2 The Limit of a Function 35, 36, 37, 46, 47, 48, 59, 60, 61, 62, 63, 64	Assignment #1 due Friday September 16th at 13:59 (EST)
	§2.3 The Limit Laws 83, 85, 93, 95, 111, 113	
3	§3.1 Defining the Derivative 11, 13, 19, 25, 27, 39	
	§3.2 The Derivative as a Function 57, 59, 61, 65, 71, 73, 93	
4	§3.3 Differentiation Rules 107, 111, 117, 123, 125, 127, 131	Assignment #2 due Friday September 30th at 13:59 (EST)
	§3.5 Derivatives of Trigonometric Functions 175, 181, 183, 191, 193, 209	
5	§3.6 The Chain Rule 215, 217, 221, 223, 235	
	§3.8 Implicit Differentiation 301, 303, 307	
	§4.1 Related Rates 1, 3, 5, 9, 17	
6	§4.3 Maxima and Minima 91, 93, 95, 105, 107, 109, 117, 119, 123, 125, 129, 145	Assignment #3 and Class Survey #1 due Friday October 21st at 13:59 (EST)
	§4.7 Applied Optimization Problems 311, 317, 319, 321, 353	

These questions are for your benefit. Attempting them will improve your learning.

MAT A29 Reading Guide Please print this page for reference throughout the course.

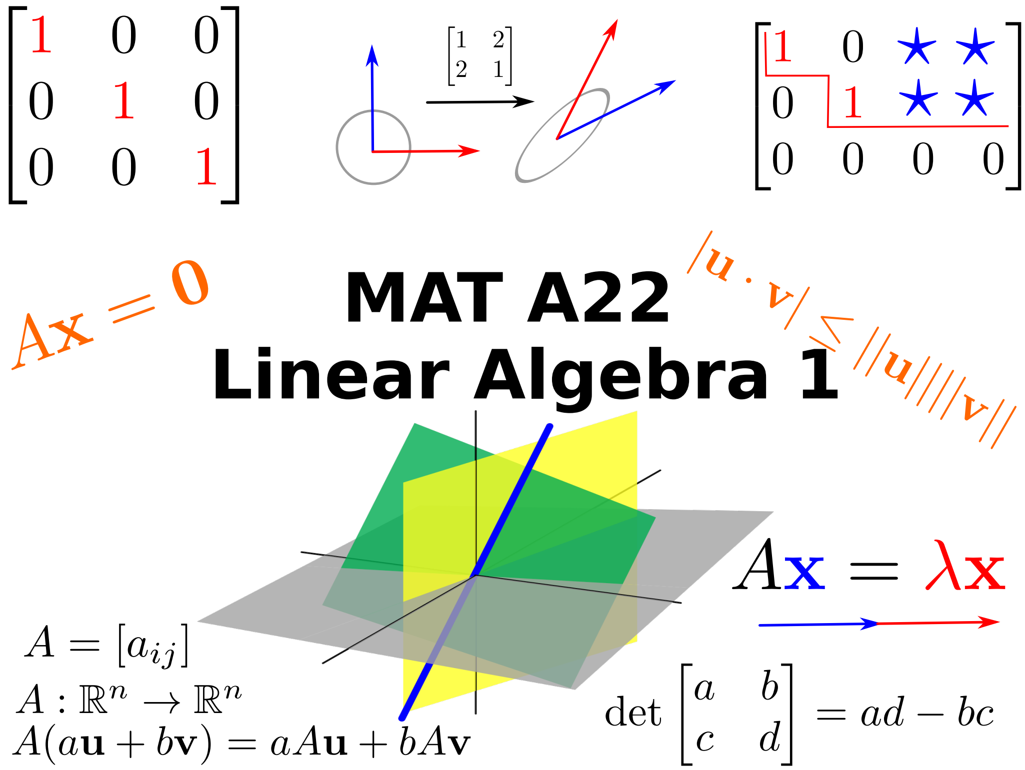
Name: ________________________ Student Number: ___________________

Week	Suggested Exercises	Ask Prof / TA
7	§4.5 Derivatives and the Shape of a Graph 201, 205, 207, 217, 219, 225, 227, 229, 241, 243	
8	§4.2 Linear Approximations and Differentials 51, 53, 33, 69, 73, 77, 83, 85	Assignment #4 due Friday November 4th at 13:59 (EST)
9	§4.10 Antiderivatives 465, 467, 471, 473, 475, 477, 483, 489, 491	
	§5.3 The Fundamental Theorem of Calculus 151, 153, 157, 161, 171, 173, 175, 177, 183	
10	§5.4 Integration Formulas and the Net Change Theorem 207, 209, 211, 213, 221, 231	Assignment #5 due Friday November 18th at 13:59 (EST)
	§5.5 Substitution 257, 259, 261, 263, 265, 269, 271, 273, 275, 279, 313	
	§3.1 (of OpenStax Calculus Volume 2) Integration by Parts 1, 3, 5, 7, 23, 28, 42, 43, 46	
11	§3.4 (of OpenStax Calculus Volume 2) Partial Fractions 182, 183, 184, 185, 196, 197, 198	
	§3.7 (of OpenStax Calculus Volume 2) Improper Integrals 350, 351, 352, 356, 362, 363, 366, 372	
12	§6.1 Areas between Curves 1, 3, 5, 7, 13, 15, 17, 21, 23, 27, 29	Assignment #6 and Class Survey #2 due Friday December 2nd at 13:59 (EST)
	§6.2 Determining Volume by Slicing 63, 67, 69, 71, 75, 77, 79, 83, 85, 89	

Record how many hours you spend on the course in the boxes below.

Week 1	Week 2	Week 3	Week 4	Week 5	Week 6
Week 7	Week 8	Week 9	Week 10	Week 11	Week 12

University of Toronto Scarborough
Department of Computer and Mathematical Science
MAT A22: Linear Algebra I for Mathematical Sciences



Lecture Schedule

LEC 01	Tuesday	08:00–10:00	IC 130
	Friday	08:00–10:00	IC 130
LEC 02	Wednesday	09:00–10:00	IC 130
	Friday	11:00–13:00	IC 130
LEC 03	Monday	09:00–10:00	HW 216
	Wedneday	15:00–17:00	HW 216

Important Dates

- Reading Week: Saturday February 18th – Sunday February 26th
- Drop deadline: Monday March 27th

Professors for the Course

Daniel Calderon Wilches (he/him)
Lecture: LEC 01
Preferred Names: "Daniel"
E-Mail: d.calderon@mail.utoronto.ca
Website: https://sites.google.com/view/daniel-calderon-math

✯ Parker Glynn-Adey (he/him)
Lectures: LEC 02 and LEC 03
Preferred Names: "Parker" or "Professor Parker"
E-Mail: parker.glynn.adey@utoronto.ca
Website: https://pgadey.ca/
Office: IC 344

Professor Parker is the course coordinator. He is responsible for the administration of the course.

Office Hours

Parker holds office hours on Wednesdays and Fridays 10-11:00am in IC 404. Daniel holds office hours on Tuesdays 10:15-11:15am in IC 404. Office hours are a dedicated time that Parker and Daniel are available to answer your questions, discuss course content, and generally be of support. If you would like help in the course but have a scheduling conflict that prevents you from attending office hours, please email us to schedule an appointment.

Textbooks

- Damiano and Little. A course in linear algebra. *Dover*, 2011.
- Axler. Linear algebra done right. *Springer*, 1997.

Prerequisite / Exclusions

Prerequisites:
Grade 12 Calculus and Vectors or [Grade 12 Advanced Functions and Introductory Calculus and Geometry and Discrete Mathematics]
Exclusions:
MATA23H3, MAT223H, MAT240H
Course Description:
"A conceptual and rigorous approach to introductory linear algebra that focuses on mathematical proofs, the logical development of fundamental structures, and essential computational techniques. This course covers complex numbers, vectors in Euclidean n-space, systems of linear equations, matrices and matrix algebra, Gaussian reduction, structure theorems for solutions of linear systems, dependence and independence, rank equation, linear transformations of Euclidean n-space, determinants, Cramer's rule, eigenvalues and eigenvectors, characteristic polynomial, and diagonalization."

Your Professor's Message

Hi! I'm Parker Glynn-Adey, the professor for MAT A22. This is one of my favourite courses at the University of Toronto. It is an introduction to the study of linear algebra. At a deeper level, it is really an introduction to the study of modern algebra. Algebra is incredible elegant and this course is full of interesting proofs. We will develop the whole theory of linear algebra starting from the axioms. This will be a huge boost to your mathematical maturity and help prepart you for courses like: MAT B24: Linear Algebra II, MAT C01: Groups and Symmetry, and MAT D01: Fields and Groups.

Course Outline

- Vector Spaces
- Linear Transformations
- The Determinant Function
- Eigenvalues and Eigenvectors

Student Learning Outcomes

By the end of the course, students will be able to:

- Prove theorems about algebraic structures.
- Critically assess the validity of arguments in mathematics.
- State definitions and theorems precisely.
- Solve equations involving matrices and vectors.
- Find the eigenvalues and eigenvectors of a linear transformation.

Grading Scheme

Exam	$1 \times 40\% =$	40%
Test	$1 \times 20\% =$	20%
Homeworks	$(11 - 1) \times 3\% =$	30%
Quizzes	$5 \times 2\% =$	10%

Grading Policy

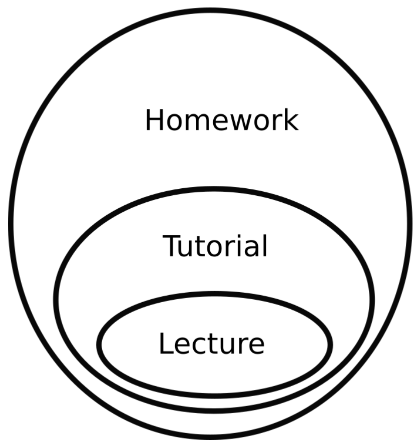
The grading scale for this course is based on a points system. Therefore, grades will NOT be rounded up or down. In general, grades are only changed due to a miscalculation. If you have concerns about your grade on an homework, or term test, you have five days after the grade is posted on Crowdmark to contact the TA who graded the question. Therefore, do NOT wait until the end of the academic term if you have questions about your grade. (For details about end of term issues, see the Section below on the Day of Small Things.) For a list of who graded which questions, please see the Quercus page.

All grades will be distributed via Crowdmark. You will not see your grades on Quercus.

The Day of Small Things

On Wednesday April 12th, the first Wednesday after Week 12 of Term, Parker will hold office hours 10-12:00 and 13-15:00 in IC 344. These special office hours are called “The Day of Small Things”. If you have any issues during the term that cannot be handle, in a simple manner, then the issue will be handled on that day. You will have to come to campus and discuss the matter with Parker in order to get it resolved.

The Course Content Egg



In this course, we will have the following structure to our lectures, tutorials, and homeworks.

$$\text{Lecture} \subsetneq \text{Tutorial} \subsetneq \text{Homework}$$

That is to say, there will be material covered in the tutorials is not covered in lecture. Similarly, there will be material covered in the homework which is not covered in tutorial or lecture. This is an intentional design choice. We want to give you a broad exposure to linear algebra.

Homeworks

Goal: these homeworks give you the opportunity to deepen your understanding of topics covered in this course, and to practice. We use these homeworks to determine if you can solve problems slowly, without time constraints.

Procedure: we will be using Crowdmark to grade homework submissions. You will get a personalized submission link sent to your UToronto email address. Do NOT share this link with other students.

Due Dates: The due date for each homework is Friday in the early afternoon at 13:59 (EST). If you submit after the due date, even if you are one minute late, your homework will receive a grade of zero. Please see the FAQ on Errors While Submitting below.

Evaluation Criteria: The TAs will only grade two questions. This policy is called *subset grading*. Present your solutions in a logical and clear manner. Detailed solutions will be made available shortly after the zero date. Please pay attention to the following when writing homeworks:

Format: solutions are neatly and correctly assembled and have a professional style. The graders should not struggle to read your work.

Completeness: all steps are clearly and accurately explained.

Content: the written solutions demonstrate mastery and fluency with the content of the course.

tl;dr: Do good work and submit it via Crowdmark as early as possible.

Advice on Submitting Homeworks

- Photograph all your pages early.
- Set aside at least a half an hour to upload your homework.
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2. to use or possess an unauthorized aid or aids or obtain unauthorized assistance in any academic examination or term test or in connection with any other form of academic work;

3. to personate another person, or to have another person personate, at any academic examination or term test or in connection with any other form of academic work;

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Helpful Resources

- The Math Help Room (IC404) is always helpful. The TAs and Parker hold office hours there.
- The Centre for Teaching and Learning has numeracy workshops.

Communication Policy

Please follow the following order of communication:

1. In-person during lecture or tutorial
2. In-person during office hours
3. Piazza
4. E-mail

That is, please reach out in-person first, use Piazzem, and only use e-mail as a last resort. Before sending an e-mail, be sure to post on Piazza. If you have a question, many other people have it too.

E-Mail Communication

All e-mail must be from an official University of Toronto account. You must include [MAT A22] in the subject line, or your e-mail might get lost. Please include your name and student number in every e-mail that you send. Be sure to include the precise question, and the problem or difficulty.

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I need help with this question:  Find a vector orthogonal to v =
[1,0].  My problem is this:  I don't know what the word ‘‘vector’’
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No. We will not replace a low term test grade with a higher final exam grade.

MAT A22 Schedule (Winter 2023) Please print this page for reference throughout the course.

Week / Dates	**Textbook material to be covered**	**Homework etc.**
Week 1 Mon. Jan. 10 Fri. Jan. 13	§1.1 Vector Spaces §1.2 Subspaces	Welcome!
Week 2 Mon. Jan. 16 Fri. Jan. 20	§1.3 Linear Combinations §1.4 Linear Dependence and Linear Independence	Tutorials start Homework #1
Week 3 Mon. Jan. 23 Fri. Jan. 27	§1.5 Interlude on Solving Systems of Linear Equations	Homework #2 Quiz #1
Week 4 Mon. Jan. 30 Fri. Feb. 3	§1.6 Bases and Dimension	Homework #3
Week 5 Mon. Feb. 6 Fri. Feb. 10	§2.1 Linear Transformations §2.2 Linear Transformations between Finite-Dimensional Vector Spaces	Homework #4 Quiz #2
Week 6 Mon. Feb. 13 Fri. Feb. 17	§2.3 Kernel and Image	Homework #5
	READING WEEK	
Week 7 Mon. Feb. 27 Fri. Mar. 3	§2.4 Applications of the Dimension Theorem	Homework #6 Quiz #3
Week 8 Mon. Mar. 6 Fri. Mar. 10	§2.5 Composition of Linear Transformations §2.6 The Inverse of a Linear Transformation	Homework #7
Week 9 Mon. Mar. 13 Fri. Mar. 17	§2.7 Change of Basis §3.1 The Determinant as Area §3.2 The Determinant of an $n \times n$ Matrix	Homework #8 Quiz #4
Week 10 Mon. Mar. 20 Fri. Mar. 24	§4.1 Eigenvalues and Eigenvectors	Homework #9
Week 11 Mon. Mar. 27 Fri. Mar. 31	§4.2 Diagonalizability	Homework #10 Quiz #5 DROP DEADLINE
Week 12 Mon. Apr. 3 Fri. Apr. 7	Review	Homework #11

Reading Week ...... Saturday February 18th – Sunday February 26th
Drop deadline .... Monday March 27th

A.12 Bibliography

Bibliography

[1] S. Freeman, S. L. Eddy, M. McDonough, M. K. Smith, N. Okoroafor, H. Jordt, and M. P. Wenderoth. Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the national academy of sciences*, 111(23):8410–8415, 2014.

[2] T. Fukawa-Connelly, K. Weber, and J. P. Mejía-Ramos. Informal content and student note-taking in advanced mathematics classes. *Journal for Research in Mathematics Education*, 48(5):567–579, 2017.

[3] L. Kramer, E. Fuller, C. Watson, A. Castillo, P. D. Oliva, and G. Potvin. Establishing a new standard of care for calculus using trials with randomized student allocation. *Science*, 381(6661):995–998, 2023.

[4] L. Ludwig, M. Abell, H. Soto-Johnson, L. Braddy, and D. Ensley. Guide to evidence-based instructional practices in undergraduate mathematics. *Washington, DC: Mathematical Association of America*, 2018.

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