Rapid Testing in COVID and Modified SIR Model

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Background: PCR Test vs Rapid Test

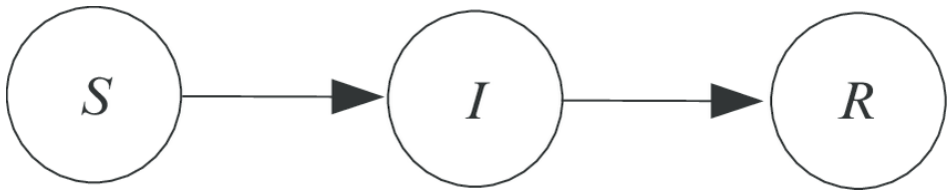
	PCR	Rapid Test
Technical Name	Polymerase Chain Reaction	Antigen
Test Target	RNA(genetic material)	Antigen(protein fragments specific to the Coronavirus)
Turnaround time	Generally in 2-3 day range but results can be in as little as 24 hours	Within 15 min
Accuracy	"Gold standard" for testing with high accuracy	Relative high accuracy but higher false positive rate

Research Question

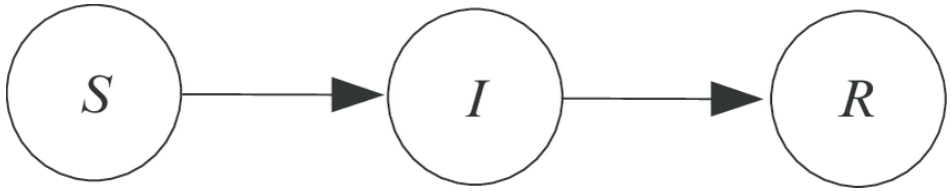
- How does popularization of rapid test influence the dynamics of the pandemic?
- We are trying to answer this question quantitatively by building a model with parameters which are able to characterize the effect of rapid test.
- We construct a numerical simulation to determine the potential effectiveness of rapid tests in the COVID pandemic

Simplest Model: SIR

- S: Susceptible
  Individuals those who are not immune to the disease of the population.
- I: Infected
  Individuals of the population who have been infected with the disease and are capable of spreading the disease to those in the susceptible category.
- R: Removed
  Individuals of the population who have been infected and then removed from the disease due to recovery. Those in this category are not able to be infected again or to transmit the infection to others.



Simplest Model: SIR



System of Ordinary Differential Equations:

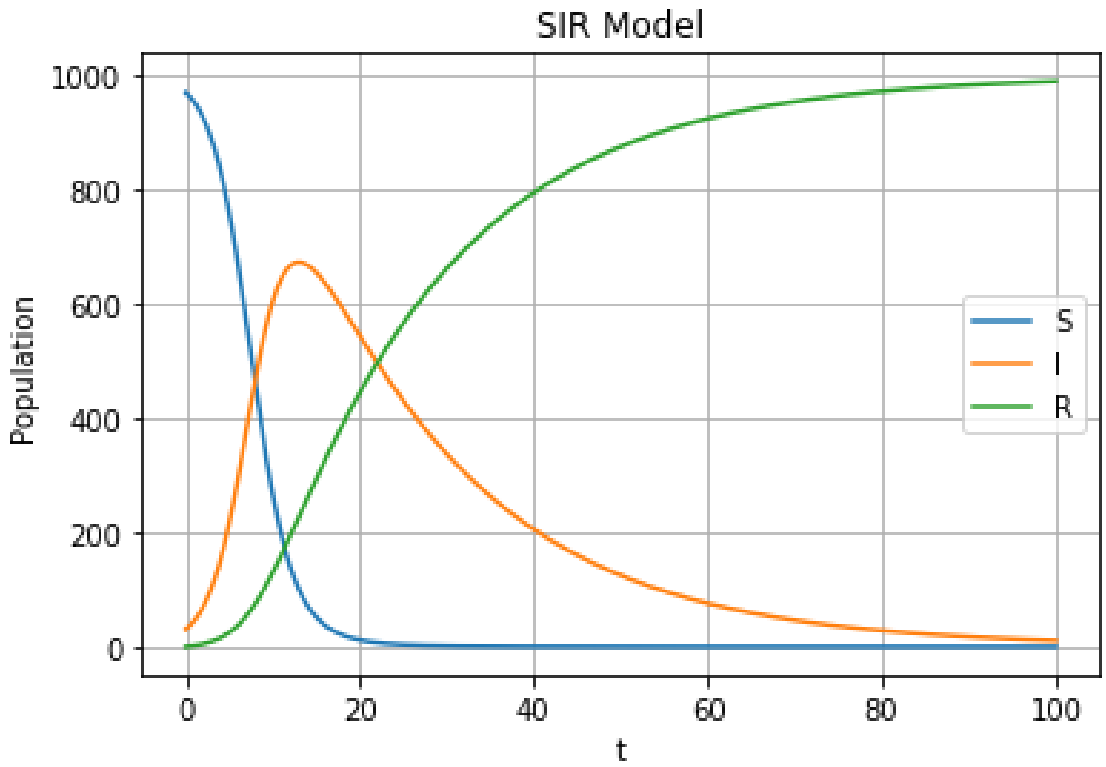
$$\frac{dS}{dt} = -\beta SI/N \tag{1}$$

$$\frac{dI}{dt} = \beta SI/N - \gamma I \tag{2}$$

$$\frac{dR}{dt} = \gamma I \tag{3}$$

- β: Probability of disease transmission per contact (dimensionless) times the number of contacts per unit time.
- N: Total population size
- γ: Recovery rate of infectious individuals (the reciprocal is the infectious period).

Numerical Solution of SIR Model



* Here we let $N = 1000$, $\beta = 0.5$, $\gamma = 0.05$, $S_0 = 970$, $I_0 = 30$, $R_0 = 0$

* Numerical Solution is found using **Python(scipy: odeint)** or equivalently using **MATLAB(ode45)**.

Advantages of SIR model

- Nice-looking analytical solution: Though explicit solution is hard to find, its implicit solution can be easily computed by hand through **separation of variables**.
- For equation involving only S and I, it is a special case of Lotka-Voterra (Predator Prey) system.

$$\begin{cases} \frac{dS}{dt} &= -\beta SI \\ \frac{dI}{dt} &= \beta SI - \gamma I \end{cases}$$

$$\begin{aligned} \Rightarrow \frac{dI}{dS} &= \frac{\beta SI - \gamma I}{-\beta SI} \\ &= -1 + \frac{\gamma}{\beta S} \\ \Rightarrow I &= -S + \frac{\gamma}{\beta} \log S + \text{Const.} \end{aligned}$$

Advantages of SIR model

$$I = -S + \frac{\gamma}{\beta} \log S + \text{Const.}$$

$$\Rightarrow I + S - \frac{\gamma}{\beta} \log S = \text{Const.}$$

In terms of the initial conditions,

$$I + S - \frac{\gamma}{\beta} \log S = I_0 + S_0 - \frac{\gamma}{\beta} \log S_0$$

Recall from last page, if $S = \gamma/\beta$, then

$$\frac{dI}{dS} = -1 + \frac{\gamma}{\beta S} = 0$$

$$\Rightarrow I_{crit} = I_0 + S_0 - \frac{\gamma}{\beta} \log \left(\frac{\gamma}{\beta} S_0\right) - \frac{\gamma}{\beta}$$

Limitations of SIR model

- If we add equation (1), (2), (3) together: we get

$$\frac{dN}{dt} = 0$$

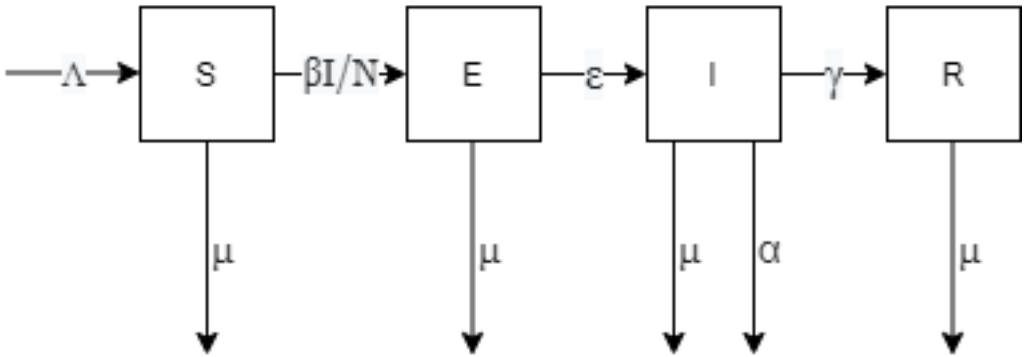
that is to say, **no net change** in total population

  - In other words, this model works best when the disease has low fatality rate
  - However, in the case of Covid, it may have fatality rate (for example covid)

- It is shown that for Covid, there is a **significant latency period** during which individuals have been infected but are not yet infectious themselves.

Introducing SEIR model (E stands for Exposed)!

Improved Traditional Model: SEIR

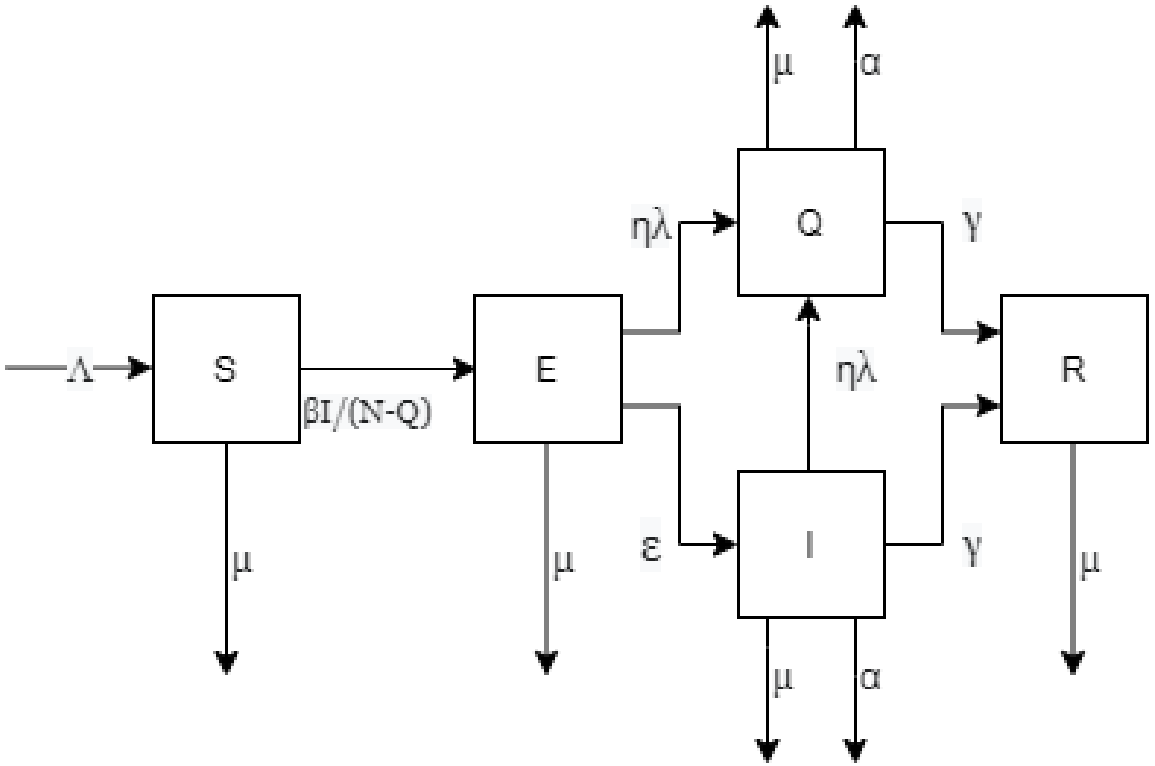


$$\dot{S} = \Lambda - \mu S - \beta SI/N$$
$$\dot{E} = \beta SI/N - (\mu + \epsilon)E$$
$$\dot{I} = \epsilon E - (\mu + \gamma + \alpha)$$
$$\dot{R} = \gamma I - \mu R$$

- Λ : Birth
- μ: Per-capita natural death rate.
- α: Virus-induced average fatality rate.
- ϵ: Rate of progression from exposed to infectious (the reciprocal is the incubation period).

Modified Model: SEIQR

- Q: Quarantined population



- λ: Percentage of the population that take rapid tests per day
- η: Sensitivity of the test $(0 \leq \eta \leq 1)$

We define **Effectivenss** $= \lambda \times \eta$ as the overall influence of rapid test.

Modified Model: SEIQR

$$\dot{S} = \Lambda - \mu S - \beta S \frac{I}{S + E + I + R}$$

$$\dot{E} = \beta S \frac{I}{S + E + I + R} - (\mu + \epsilon + \eta\lambda)E$$

$$\dot{I} = \epsilon E - (\mu + \gamma + \alpha + \eta\lambda)I$$

$$\dot{Q} = \eta\lambda(E + I) - (\mu + \gamma + \alpha)Q$$

$$\dot{R} = \gamma(Q + I) - \mu R$$

Numerical Analysis: Parameters and ICs

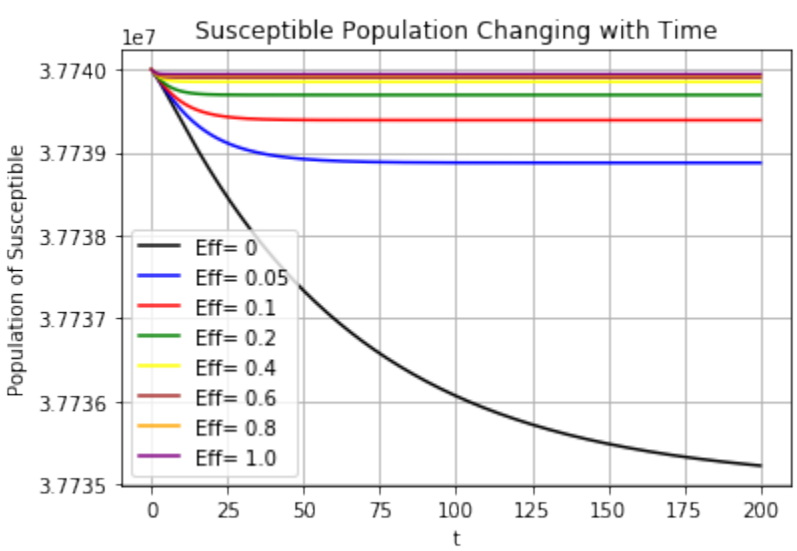
Parameters:

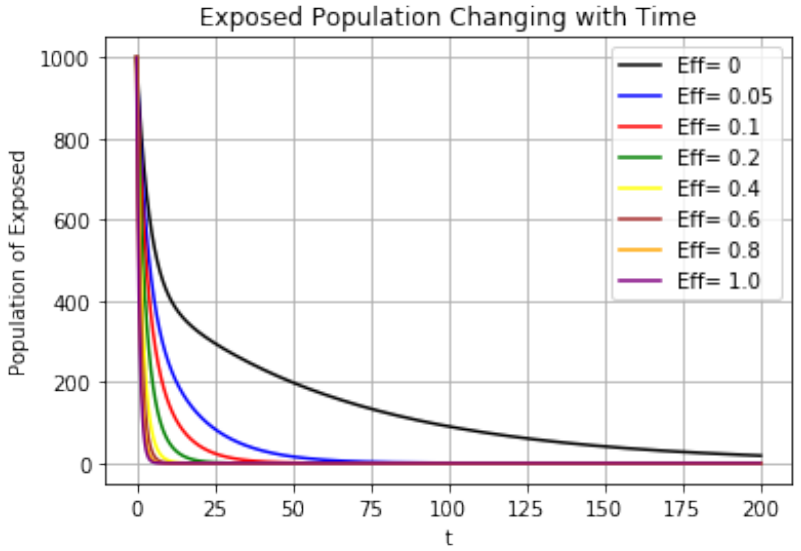
- fatality rate induced by COVID $\alpha = 0.01$
- incubation period: 5 days
- natural birth: 1034/day
- natural death: 775/day
- transmission rate: 0.07

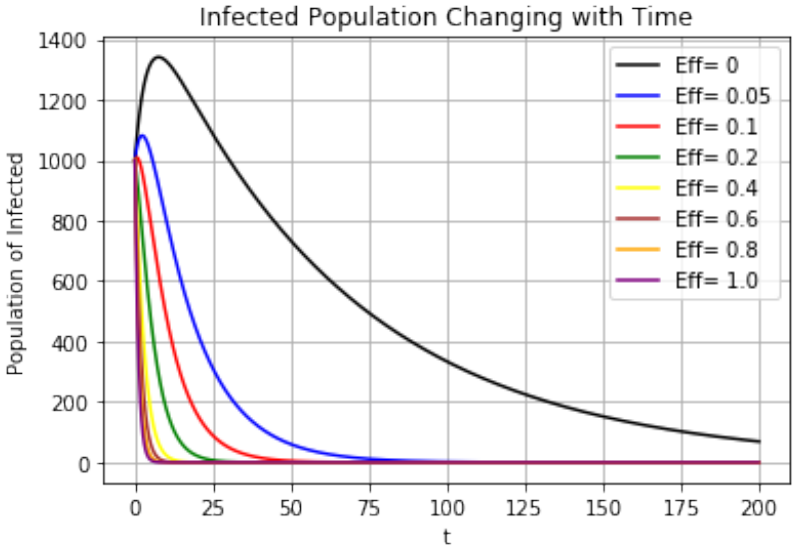
Initial Conditions:

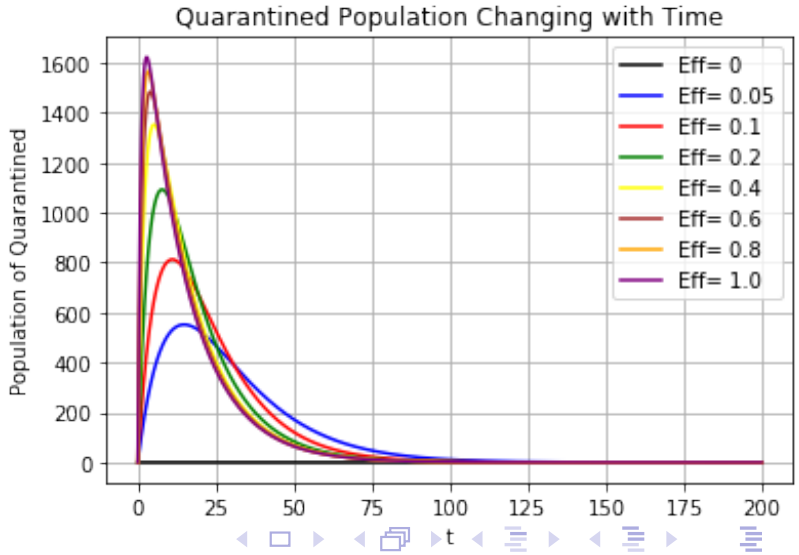
- $S_0 = 37,740,000$
- $I_0 = 1000$
- $Q_0 = 0$
- $R_0 = 0$

Numerical Analysis: Results



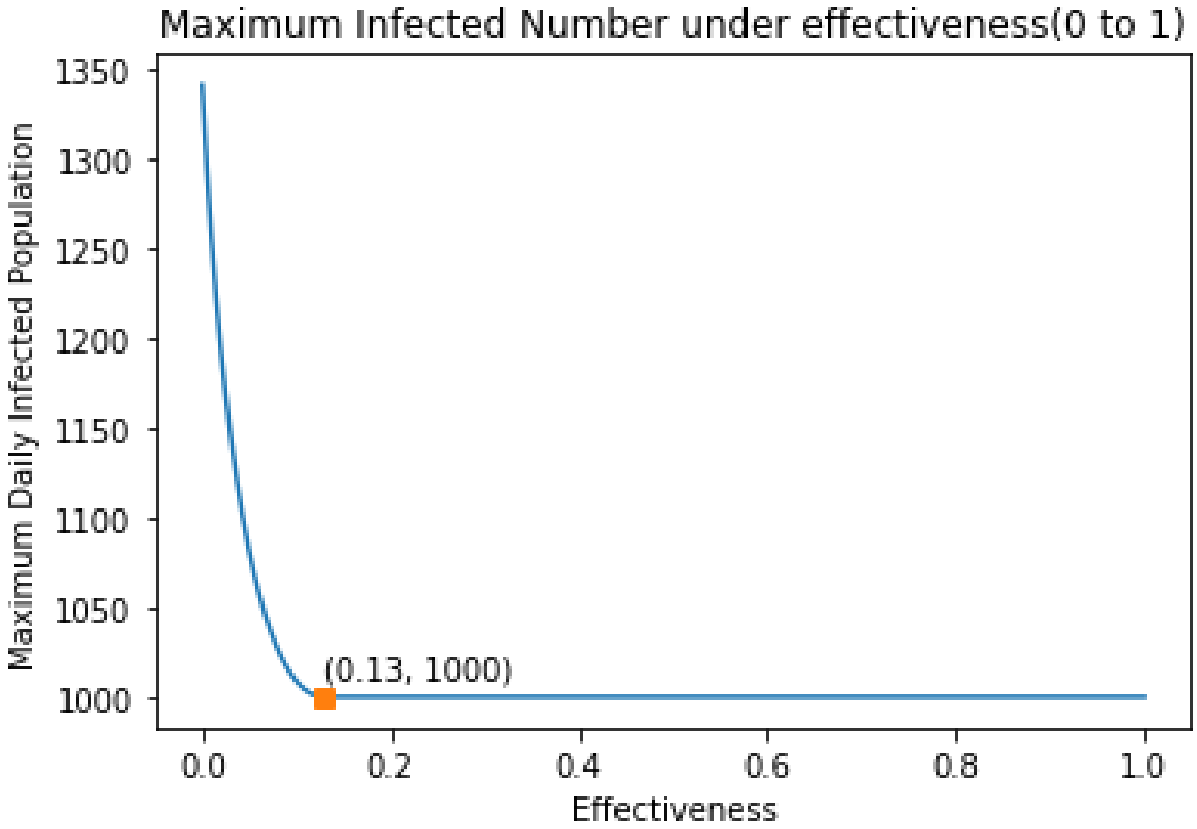






Numerical Analysis: Results

What is the minimum effectiveness we need to constrain number of infected people enough?



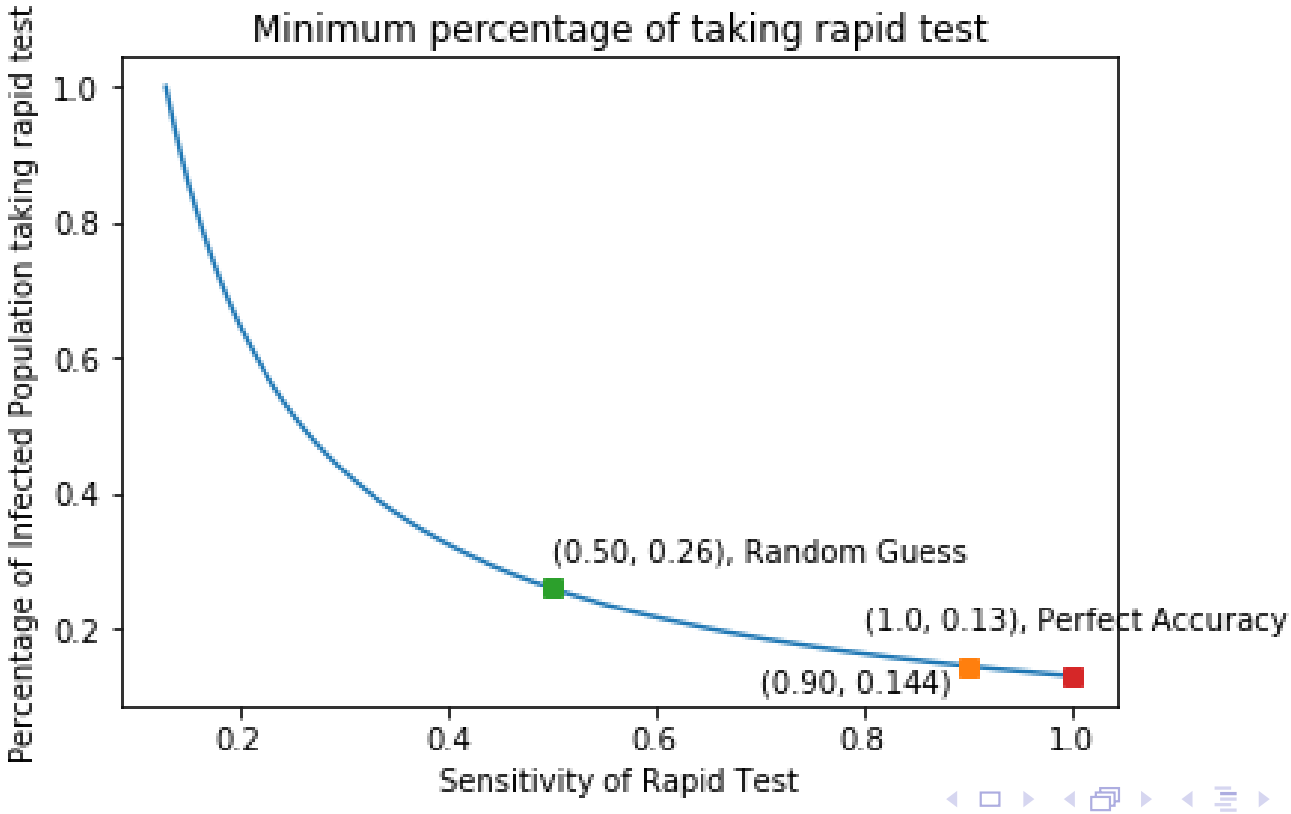
Effectiveness = 0.13 is enough!!

Numerical Analysis: Results

How to interpret 0.13?

Effectiveness = Sensitivity × Percentage of population taking rapid test

$$\textit{Minimum percentage of population taking rapid test} = \frac{0.13}{\textit{Sensitivity}}$$



Limitations and Next steps

- Hard to find analytical solution due to the existence of Exposed Period compared to simplest SIR model.

- Parameters are not always constant. For example, as time goes on, the transmission rate(β) may vary since more people are quarantined. May be in our next step, we can rewrite our parameters as a function of t.

- There are two many parameters within our model. It is hard to observe how each parameter may affect the dynamics of solution. It is worth trying to simplify this model.

References

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