Ray Tracing and the Light Transport Equation

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Overview

1. The Goal: Realistic Images
2. The What: Defining the Light Transport Equation
3. The How: Monte Carlo Integration and Ray Tracing

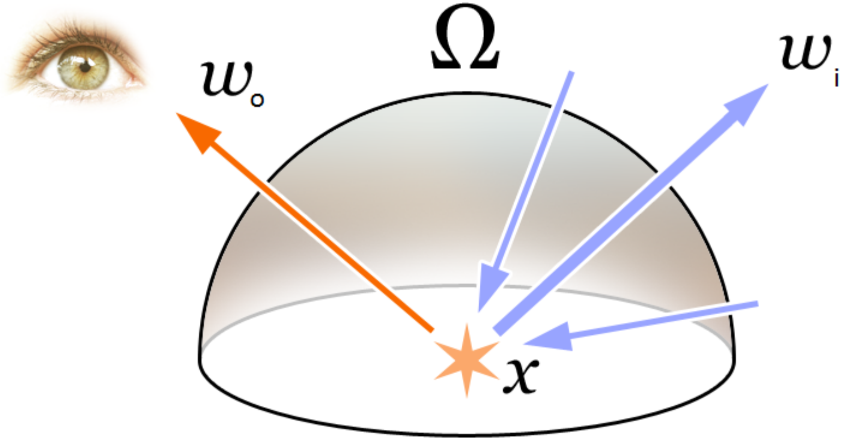
We Want ~~Pretty~~ Realistic Images!

- Why?
  - "CGI" in films
  - Realistic animation
  - Video games

We Want ~~Pretty~~ Realistic Images!

- Why?
  - "CGI" in films
  - Realistic animation
  - Video games
- How?
  - How do we generate realistic images?
  - How do we make it efficient?

Defining the Light Transport Equation



Building a model of how things look

What do we see? Light!

So, let's try to model how light works.

Wait... how *does* light work?

Building a model of how things look

Conservation of Energy

Light out - Light in = Light emitted - Light absorbed

Idea:

- If we can model the light coming out of a point, we can sample all the points we see and generate an image!

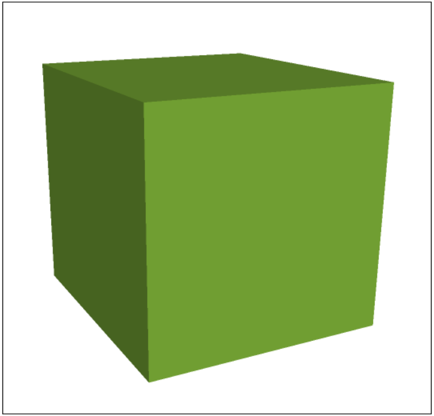
A naive first attempt

Paint all the things! Everything has a colour, so let's model that alone and see what we get.

First Light Model

$$L_o(x) = f(x)$$

A naive first attempt



Not bad! But we can do better.

Refining the model: Emitters

Light Model: with Emittance

$$L_o(x) = L_e(x) + f(x)$$

Refining the model: Perspective

Light doesn't just *exist* at a point, it depends on how you look at it!

Light Model: with Perspective

$$L_o(x, \omega_o) = L_e(x, \omega_o) + f(x, \omega_o)$$

So, have we got it now?

Refining the model: Perspective

Light doesn't just *exist* at a point, it depends on how you look at it!

Light Model: with Perspective

$$L_o(x, \omega_o) = L_e(x, \omega_o) + f(x, \omega_o)$$

So, have we got it now?
Not quite. What about the light coming *in*?

Refining the model: Reflection

Light Model: with Light in

$$L_o(x, \omega_o) = L_e(x, \omega_o) + \int_\Omega L_i(x, \omega_i) d\omega_i$$

Where did f go?

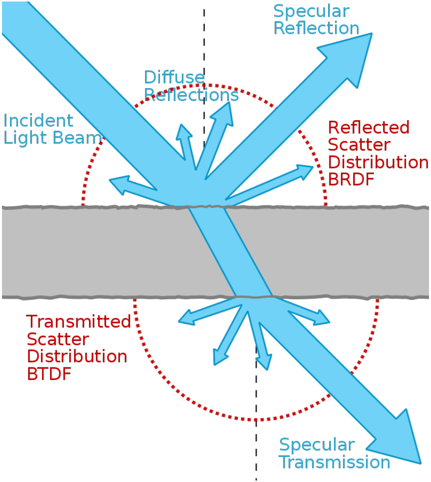
Refining the model: The Scattering Distribution Function

Light Model: with Reflectance

$$L_o(x, \omega_o) = L_e(x, \omega_o) + \int_\Omega f(x, \omega_o, \omega_i) L_i(x, \omega_i) d\omega_i$$

Let's re-purpose our friend f...

Refining the model: The Scattering Distribution Function



Refining the model: The Cosine Term

Light Model: with a Weakening Factor

$$L_o(x,\omega_o) = L_e(x,\omega_o) + \int_\Omega f(x,\omega_o,\omega_i)L_i(x,\omega_i)cos(\theta_i)d\omega_i$$

What the heck is *light flux*?

Refining the model: Colour and Motion

The Light Transport Equation

$$L_o(x, \omega_o, \lambda, t) = L_e(x, \omega_o, \lambda, t) + \int_\Omega f(x, \omega_o, \omega_i, \lambda, t) L_i(x, \omega_i, \lambda, t) cos(\theta_i) d\omega_i$$

Almost there... just need to cover wavelengths of light, and change over time.

The Complete Model (for today)

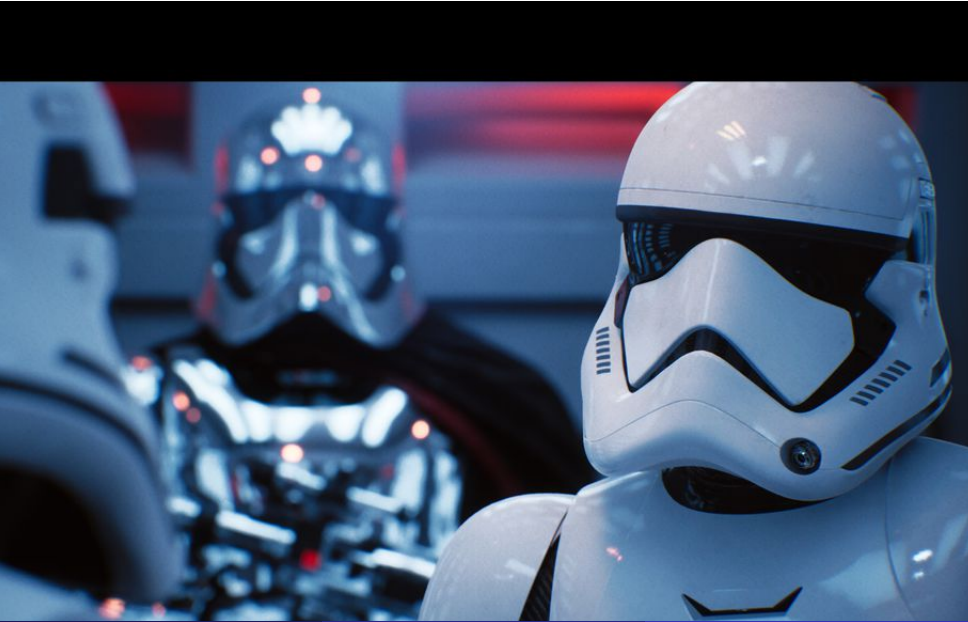
The Light Transport Equation

$$L_o(x, \omega_o, \lambda, t) = L_e(x, \omega_o, \lambda, t) + \int_\Omega f(x, \omega_o, \omega_i, \lambda, t) L_i(x, \omega_i, \lambda, t) cos(\theta_i) d\omega_i$$

More caveats we'll leave to the engineers:

- Polarization
- Interference and Fluorescence
- Various fun quantum effects

Evaluating the LTE: Ray Tracing



Monte Carlo Integration

So, we've got to evaluate a nasty integral. No analytic techniques will suffice, so we must turn to statistics to build an approximation.

Monte Carlo Integration

$$\int_{\Omega} F(x)dx \approx V\frac{1}{N}\sum_{i=1}^{N} F(x_i)$$

Monte Carlo Ray Tracing

Monte Carlo Ray Tracing

Take samples from our integral until L_o converges.

- How do we evaluate L_i?

Monte Carlo Ray Tracing

Monte Carlo Ray Tracing

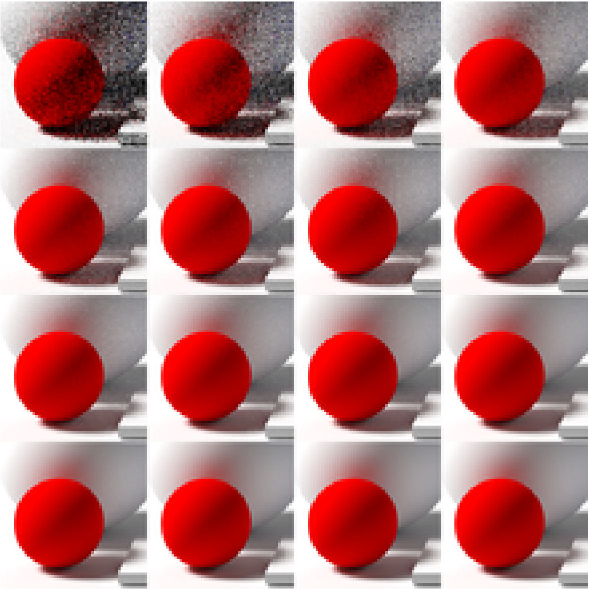
Take samples from our integral until L_o converges.

- How do we evaluate L_i?
- Recursively!

Monte Carlo Path Tracing

Continue to expand the sample of L_i until it goes to zero or escapes the scene.

Monte Carlo Path Tracing



Bias and Variance

The two main problems in applying these techniques in practice:

Bias

$$L_o(x, \omega_i, \lambda, t) + \beta(x, \omega_i, \lambda, t)$$

Variance

$$\delta Q_N = \sqrt{Var(Q_N)} = V \frac{\sigma_N}{\sqrt{N}}$$

War on Variance: Importance Sampling

Better samples = Faster convergence. What makes a good sample?

Importance Sampling

$$Q_N = \frac{1}{N} \sum_{i=1}^{N} \frac{F(x_i)}{p(x_i)}$$

War on Variance: Importance Sampling

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Importance Sampling

$$Q_N = \frac{1}{N} \sum_{i=1}^{N} \frac{F(x_i)}{p(x_i)}$$

$$F(x_i) = f(x_i, \omega_o, \omega_i, \lambda, t) * L_i(x_i, \omega_i, \lambda, t) * cos(\theta_i)$$

How can we choose a good distribution?

War on Variance: Importance Sampling

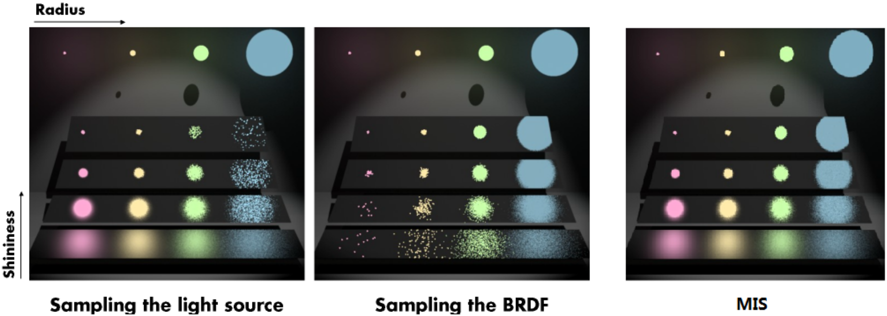
War on Variance: Multiple Importance Sampling

Multiple Importance Sampling

$$\int F(x)G(x)dx \approx \frac{1}{N_F}\sum_{i=1}^{N_F}\frac{F(x_i)G(x_i)w_F(x_i)}{p_F(x_i)} + \frac{1}{N_G}\sum_{i=1}^{N_G}\frac{F(x_i)G(x_i)w_G(x_i)}{p_G(x_i)}$$

Sample a product separately, by sampling each of the terms independently.
Provably good results!

War on Variance: Multiple Importance Sampling



A Note on Parallelism

Ray Tracing is part of a class of problems called *Embarrassingly Parallel* tasks.

How do we take advantage of the parallel nature of this problem?

A Note on Real-Time

It is not feasible to take hundreds of samples each frame for real-time applications. How can we take advantage of the Ray Tracing framework without spending so much time?