

# Virtual Ring Routing

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- 1 The Set-Up
- 2 Constructing The Ring
- 3 Routing Table
- 4 The Routing Algorithm
- 5 Benefits and Limitations of VRR
- 6 Current Research
- 7 When Physical Network is Also a Ring
- 8 Limitations and Future Directions

# The Set-Up

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Main Concerns When Routing:

- Memory capacity constraints
- Over-reliance on specific nodes in the network
- Scalability
- Length of routing path

# The Premise of VRR

Utilizing two different networks, the physical network and the virtual network, VRR determines routing paths between nodes independent of their location.

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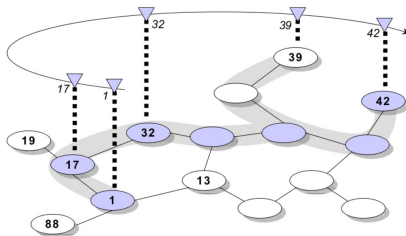
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**Figure:** The underlying physical network and the virtual overlay (image from Wikipedia)



# The Physical Network

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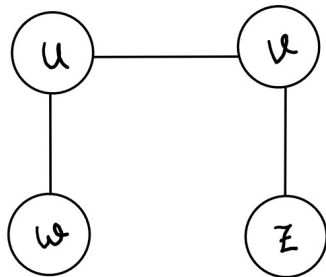
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$(u, v) \in E$  is a physical link

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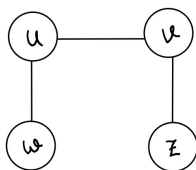
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$\langle u, z \rangle \in E_v$  is a virtual link, where  $\langle u, z \rangle = vl((u, v), (v, z))$

# Identifier Space

To set up routing, we need an identifier space  $\Omega$ :

- well-ordered set
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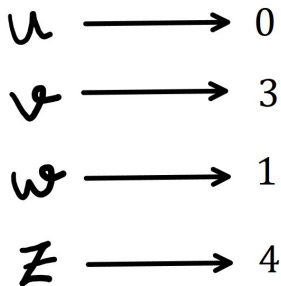
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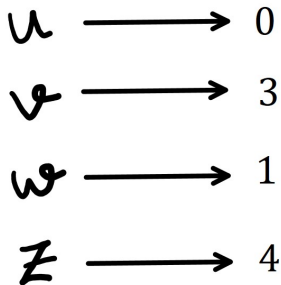
In VRR, we always take  $\Omega \subseteq \mathbb{N}$  and assume the function assigns  $id$  randomly

# Node to Id Mapping

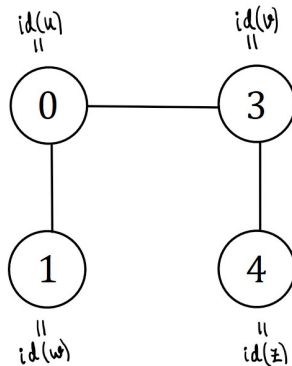


Node to Id Mapping

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Node to Id Mapping



We often use  $v$  or  $id(v)$  to refer to the same node



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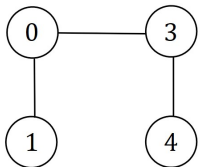
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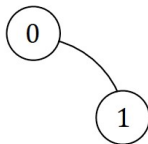
If  $id(v)$  is the largest, then  $v_+$  is the node in  $V$  with the smallest  $id$

Note: It is always possible to create a virtual link between any two nodes in  $V$  since we require  $G$  to be connected

# For Example...

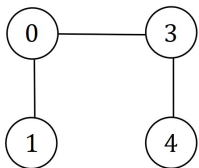


Physical Network

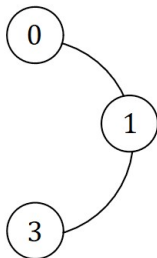


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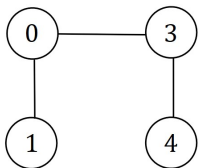
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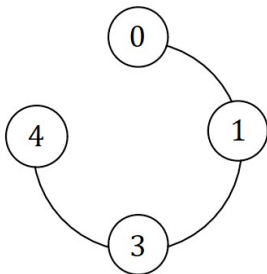
$$\langle 0, 1 \rangle = v/(0, 1)$$

$$\langle 1, 3 \rangle = v/((1, 0), (0, 3))$$

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Physical Network



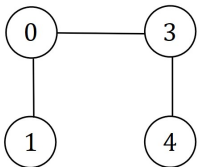
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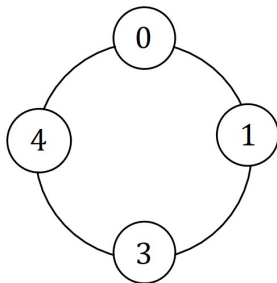
$$\langle 3, 4 \rangle = v/(3, 4)$$



# For Example...



Physical Network



Virtual Network

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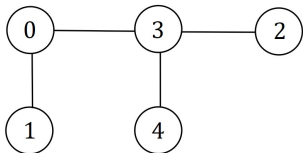
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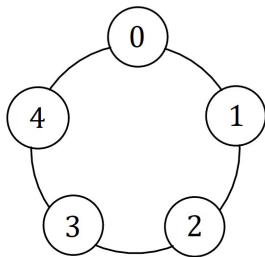
$$\langle 4, 0 \rangle = v/((4, 3), (3, 0))$$

# Another Node?

If we add another node then



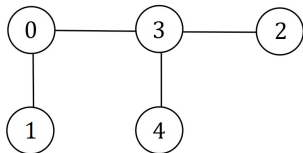
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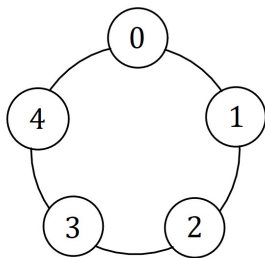
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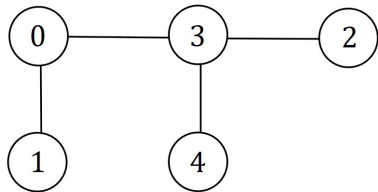


Virtual Network

$$\langle 1, 2 \rangle = vl(\langle 1, 3 \rangle, (3, 2))$$

$$\langle 2, 3 \rangle = vl(2, 3)$$

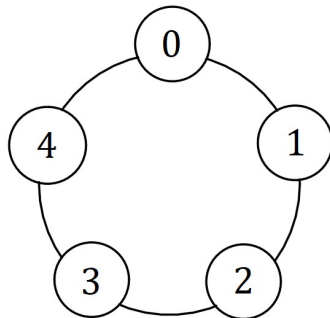
# Physical and Virtual Neighbours



Physical Network

$$P_3 = \{0, 4, 2\}$$

$$V_3 = \{4, 2\}$$



Virtual Network

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## Definition 2

The **routing table** at node  $u \in V$  is a collection of 4-tuples (from, to, next hop, prev. hop) from  $V \times V \times V \times V$  of routing paths that go through node  $u$ .

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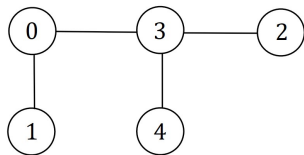
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- $End_u$  denotes the set of all nodes in the "to" or "from" coordinate in an entry on the routing table of  $u$  other than  $u$  itself  
i.e. all the nodes we know how to travel to from  $u$

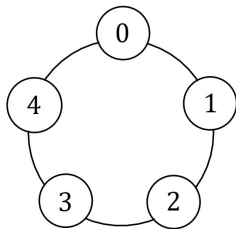
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Physical Network

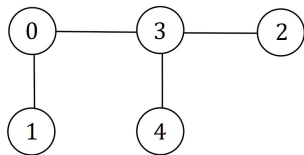
Routing Table at 3

from	to	next hop	prev. hop
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4	3	-	4
0	3	-	0
4	0	0	4
1	2	2	0

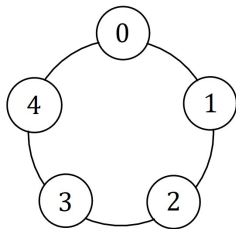


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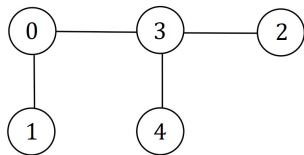
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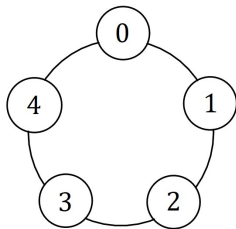
Routing Table at 2

from	to	next hop	prev. hop
2	3	3	-
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Virtual Network

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Once the physical network is fixed, the routing table is fixed

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What should  $\langle 2, 0 \rangle$  be?

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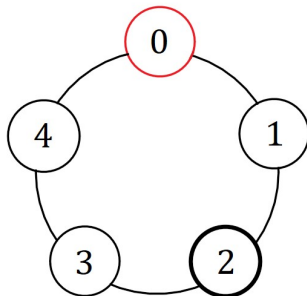
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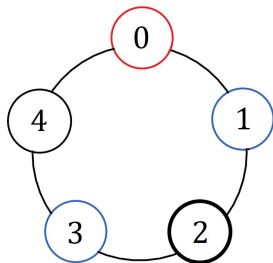
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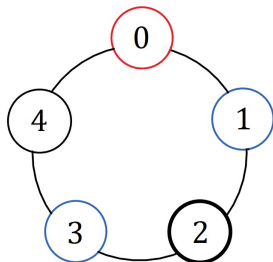
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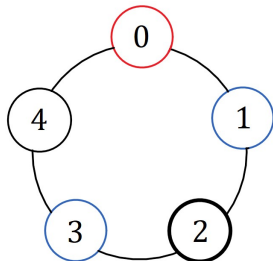
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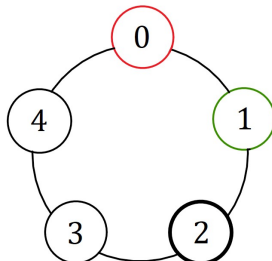
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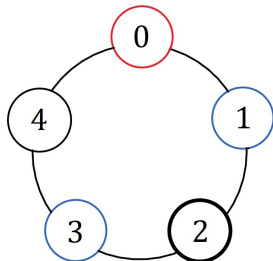


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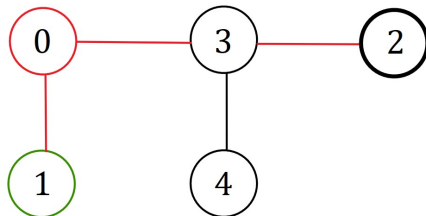
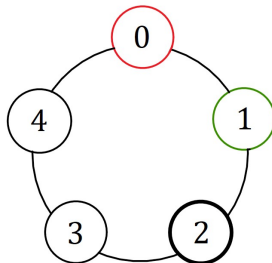


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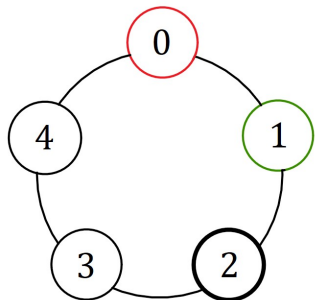
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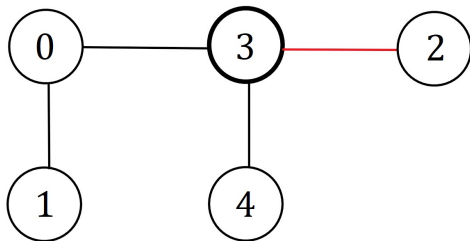
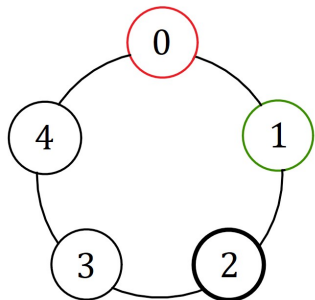
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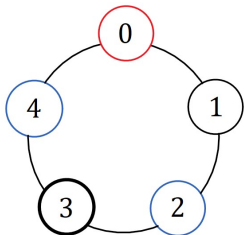
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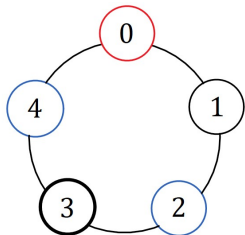
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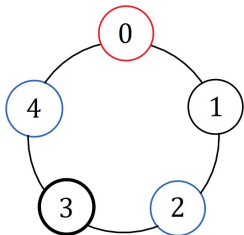
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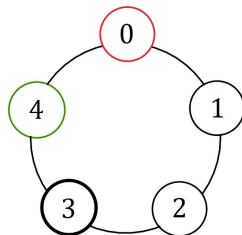
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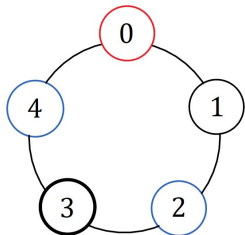


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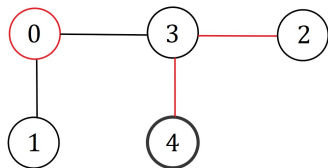
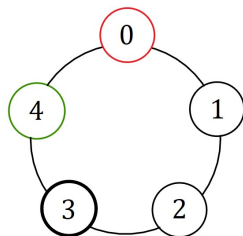


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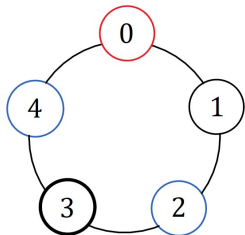


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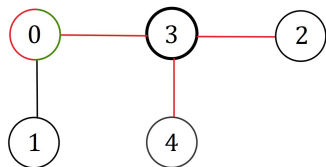
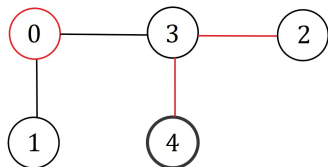
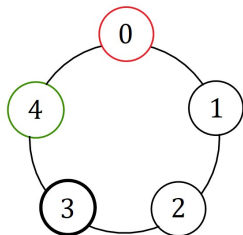


# Comparing at Each Node

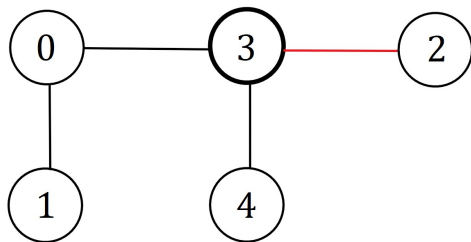
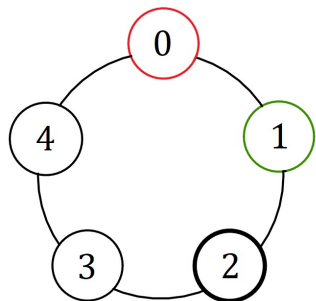
$$V_3 = \{4, 1\}$$



$$d(4, 0) < d(2, 0) \implies T' = 4$$



# Third (And Hopefully Last) Attempt



# Using the Routing Table

Routing Table at 3

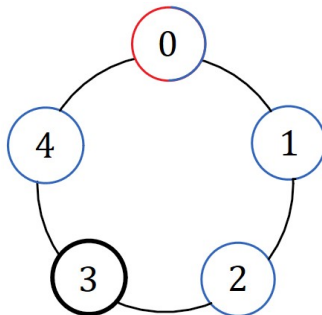
from	to	next hop	prev. hop
2	3	-	2
4	3	-	4
0	3	-	0
4	0	0	4
1	2	2	0

# Using the Routing Table

Routing Table at 3

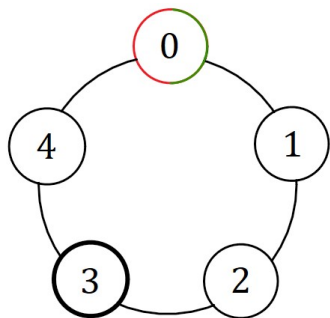
from	to	next hop	prev. hop
2	3	-	2
4	3	-	4
0	3	-	0
4	0	0	4
1	2	2	0

$$End_3 = \{0, 1, 2, 4\}$$





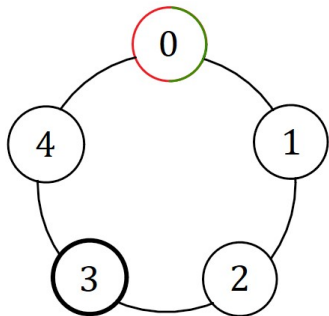
# The Actual Routing Path



Routing Table Entry for 0

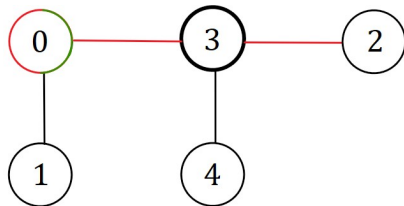
from	to	next hop	prev. hop
4	0	0	4

# The Actual Routing Path



Routing Table Entry for 0

from	to	next hop	prev. hop
4	0	0	4



# Greedy Transition Vs. Non-Greedy Transition

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Routing Table at 3

from	to	next hop	prev. hop
2	3	-	2
4	3	-	4
0	3	-	0
4	0	0	4
1	2	2	0

$$V_3 = \{4, 2\}, T' = 0$$

# Greedy Transition Vs. Non-Greedy Transition

Routing Table at 3

from	to	next hop	prev. hop
2	3	-	2
4	3	-	4
0	3	-	0
4	0	0	4
1	2	2	0

$$V_3 = \{4, 2\}, T' = 0$$

Routing Table at 2

from	to	next hop	prev. hop
2	3	3	-
2	1	3	-

$$V_2 = \{1, 3\}, T' = 1$$

# Greedy Transition Vs. Non-Greedy Transition

Routing Table at 3

from	to	next hop	prev. hop
2	3	-	2
4	3	-	4
0	3	-	0
4	0	0	4
1	2	2	0

$$V_3 = \{4, 2\}, T' = 0$$

$T' \notin V_3$   
Greedy Transition

Routing Table at 2

from	to	next hop	prev. hop
2	3	3	-
2	1	3	-

$$V_2 = \{1, 3\}, T' = 1$$

$T' \in V_2$   
Non-Greedy Transition

# Summarizing The Routing Algorithm

Given a source-target pair  $s$  and  $t$ , we can summarize the routing algorithm as follows:

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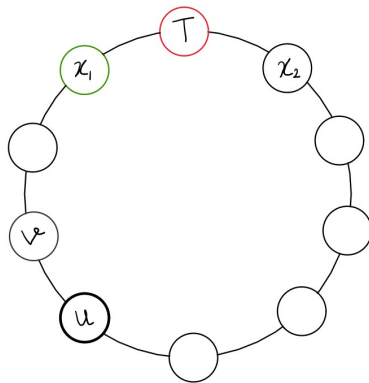
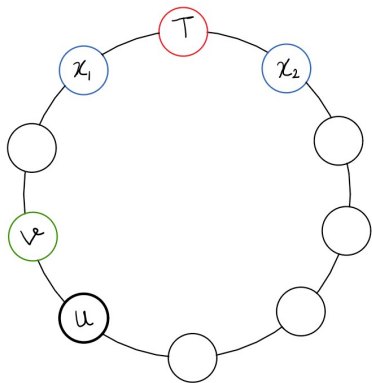
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- If no such  $x_2$  exists in  $End_u$ , then we forward the message along to the next hop node towards  $T' = x_1$
- Once we reach an intermediate target  $x_i$ , we set the intermediate target from  $End_{x_i}$

# What exactly is a better node?

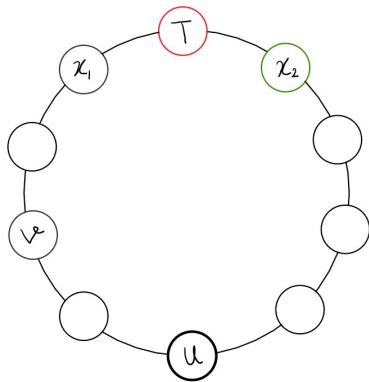
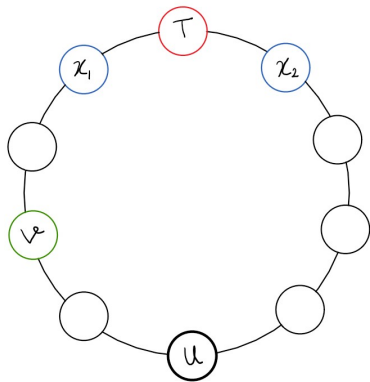
- $x$  is a better node if  $d(x, T) < d(T', T)$ , where  $T$  is the target,  $T'$  is the intermediate target

Note: We want to choose the “best” possible  $x$ ,  
i.e. the one that gets us closest to the target

# Some Edge Cases



# Some Edge Cases



# Remarks About the Intermediate Target(s)

- Each new intermediate target  $T'$  is always closer to the target  $T$  than the previous one.
- At any node  $u$ , the current intermediate target is closer to the target than  $u$  or equidistant
- Once we set  $T' = T$ , there are no more new intermediate targets

# Properties of This Routing Algorithm

## Proposition 1

VRR schemes using greedy routing exhibit no loops

Proof: If we have a loop, we must visit the same node twice, say  $v$ .

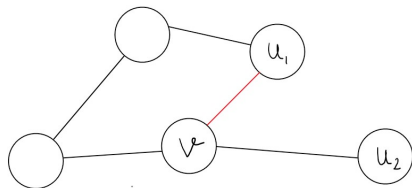


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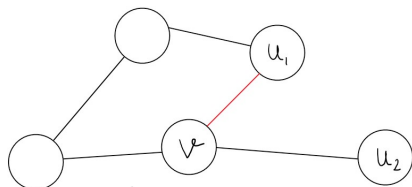
At time 1

# Properties of This Routing Algorithm

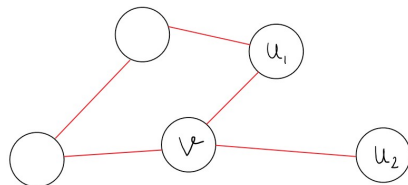
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At time 1



At time 2

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# Benefits of VRR

- Each node only stores local info; less memory used relative to increase in size of physical network
- Flexible since adding/ removing a node only affects nodes that had that node in their routing table

# Limitations of VRR

- Routing path may not be the shortest path

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## Definition 3

We define **stretch** as the ratio between the actual routing path and the shortest possible path on the physical network

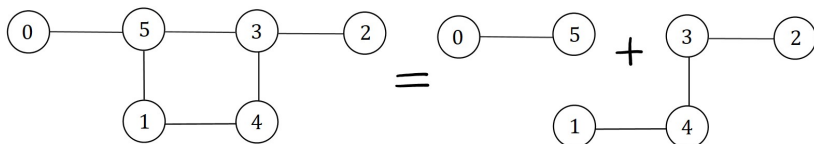
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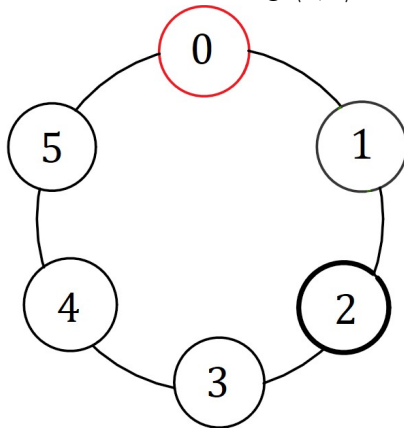
We define **stretch** as the ratio between the actual routing path and the shortest possible path on the physical network

Consider the following example



# Stretch

Suppose again that we are routing  $\langle 2, 0 \rangle$ . Our ring:

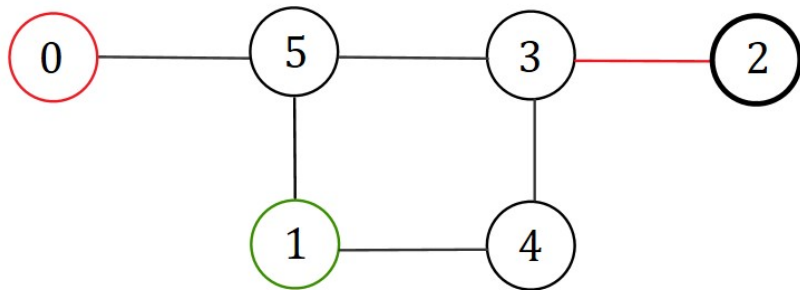




# The Actual Routing Path

from	to	next hop	prev. hop
2	3	3	-
2	1	3	-

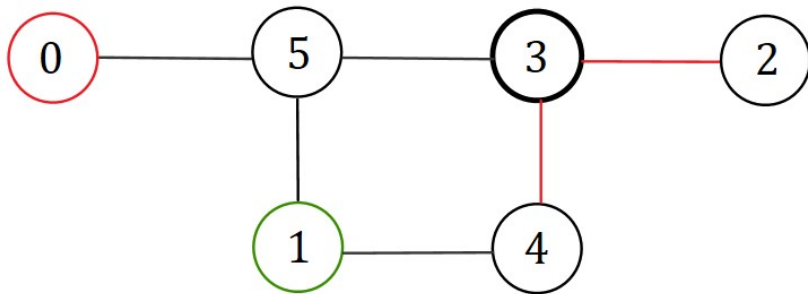
$$d(1,0) < d(3,0) \\ \implies T' = 1$$



# The Actual Routing Path

from	to	next hop	prev. hop
3	5	5	-
3	4	4	-
3	2	2	-
2	1	4	2
4	5	5	4

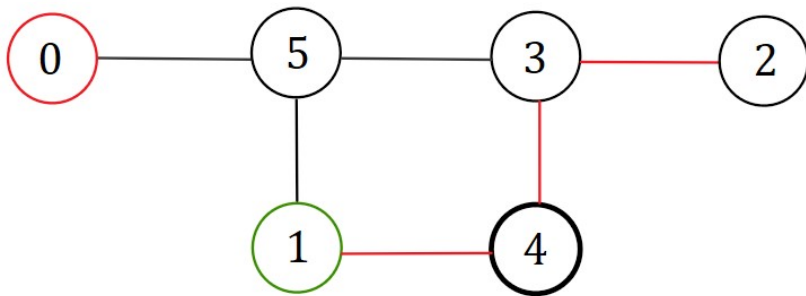
$$d(5,0) \neq d(1,0)$$
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# The Actual Routing Path

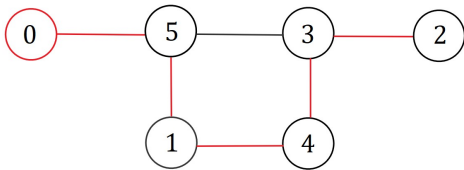
from	to	next hop	prev. hop
4	3	3	-
4	1	2	-
4	5	3	-

$$d(5,0) \neq d(1,0)$$
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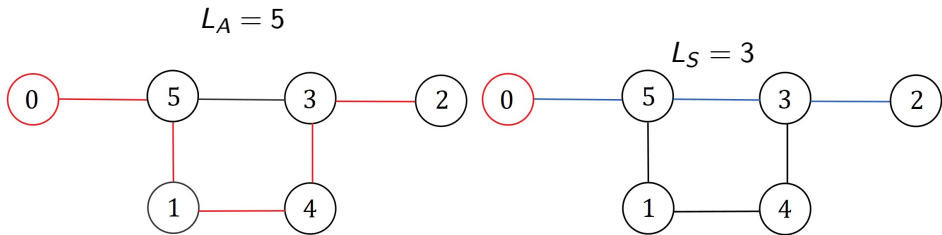


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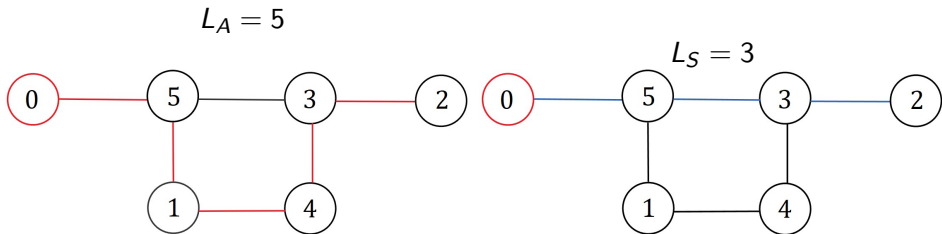
$$L_A = 5$$



# Stretch



# Stretch



Stretch is 5:3

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The questions:

- How can we study stretch?



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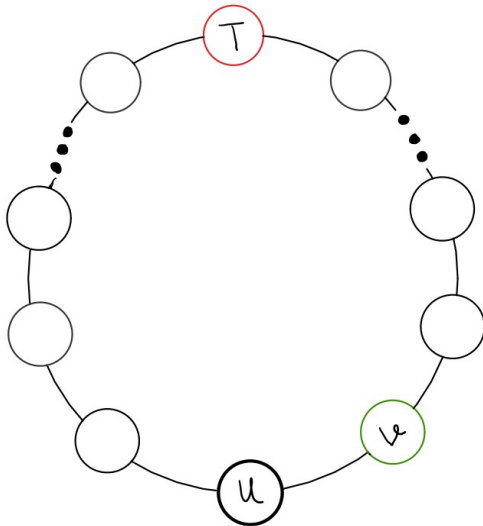
# Current Research: Careful Study of Stretch

The questions:

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The issues:

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The virtual network for large N

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# Physical Network as a Ring

- When the physical network is also a ring, we call it a physical ring.
- There are only two physical neighbours
- We can consider the physical ring a permutation of the virtual ring

# Stretch on a Ring

## Proposition 2

If the physical network is a ring of  $N > 3$  nodes, then the maximum stretch for any routing path is  $\frac{N-2}{2}$

## Proof of Prop'n 2

Proof: Consider source-target pair  $S$  and  $T$ . Suppose that, on the physical ring,  $d(S, T) = k \geq 2$

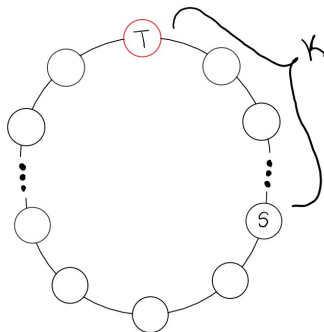


Figure: The Physical Ring

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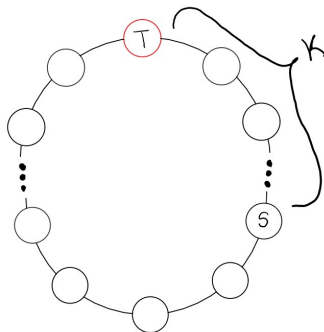


Figure: The Physical Ring

By prop'n 1, the actual routing path either travels the minor arc or the major arc.

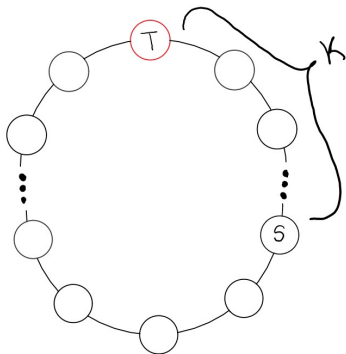


Figure: The Physical Ring

If we travel the minor arc then

$$\frac{L_A}{L_S} = \frac{k}{k} = 1$$

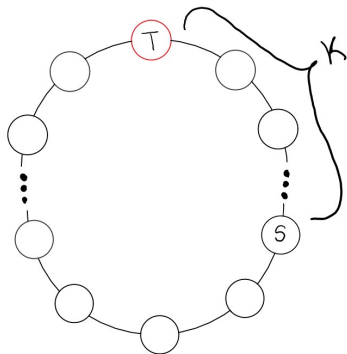


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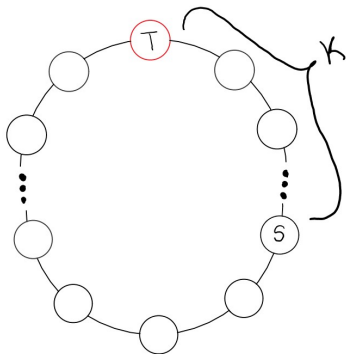


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This ratio is maximized when  
 $k = 2$

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# Limitations of the Physical Ring

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- Not practical since relatively poor connectivity
- Even the shortest paths are long

- Explore other physical network models

- Explore other physical network models
- Trying to formulate routing algorithm into Markov Chain process or as (sub)martingales and apply related theorems

Thanks for listening!  
Any questions?

## **To better understand the mechanics of VRR:**

M. Caesar, M. Castro, E. B. Nightingale, G. O'Shea, and A. Rowstron. Virtual ring routing: Network routing inspired by DHTs. In *ACM annual conference of the Special Interest Group on Data Communication (SIGCOMM)*, pages 351–362, 2006.

## **To read about other proofs related to VRR:**

Malkhi, D., Sen, S., Talwar, K., Werneck, R. F., and Wieder, U. (2009). Virtual ring routing trends. In *Proceedings of the 23rd international conference on distributed computing*, Berlin, Heidelberg.