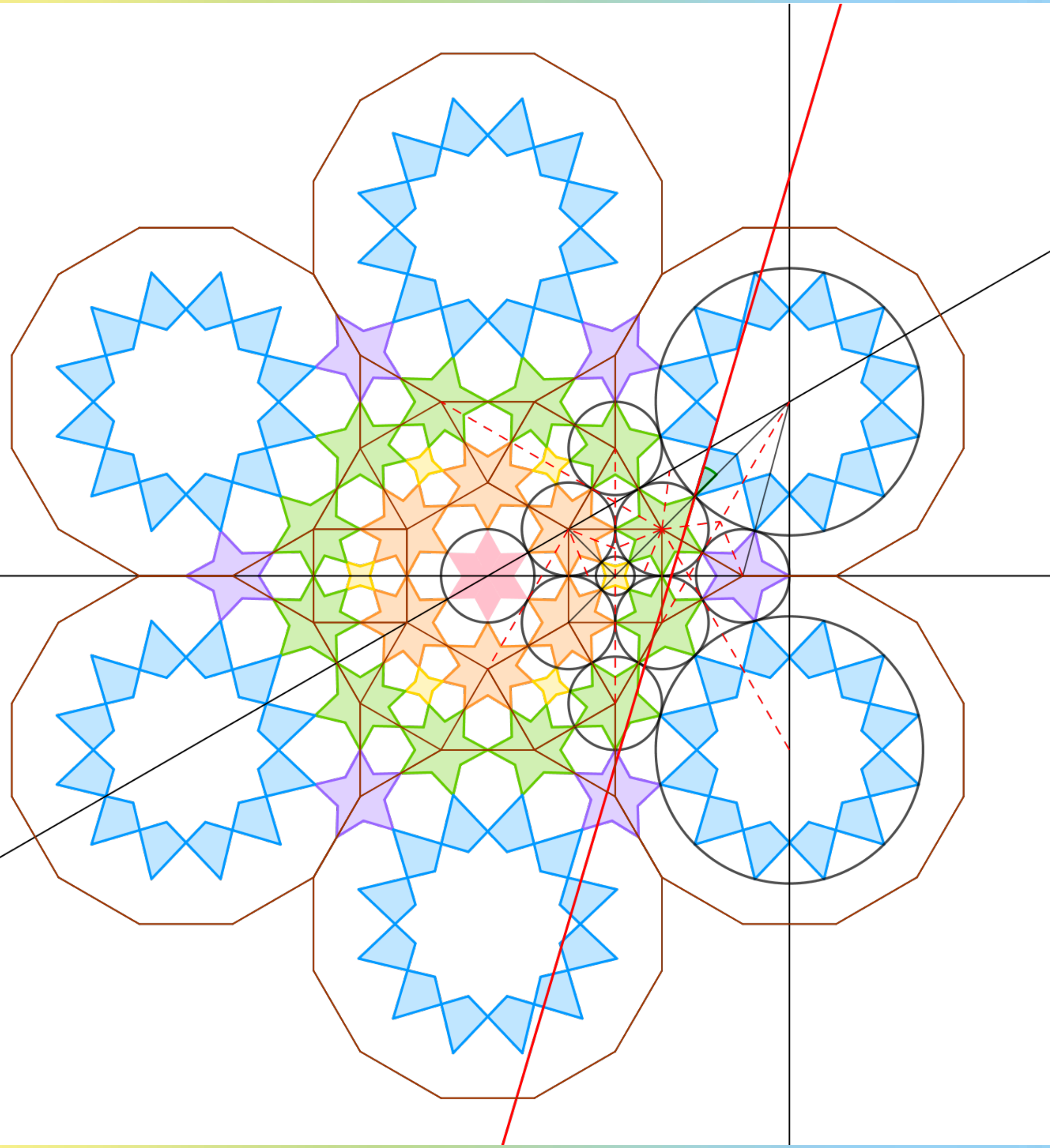
# Star rosettes in GeoGebra

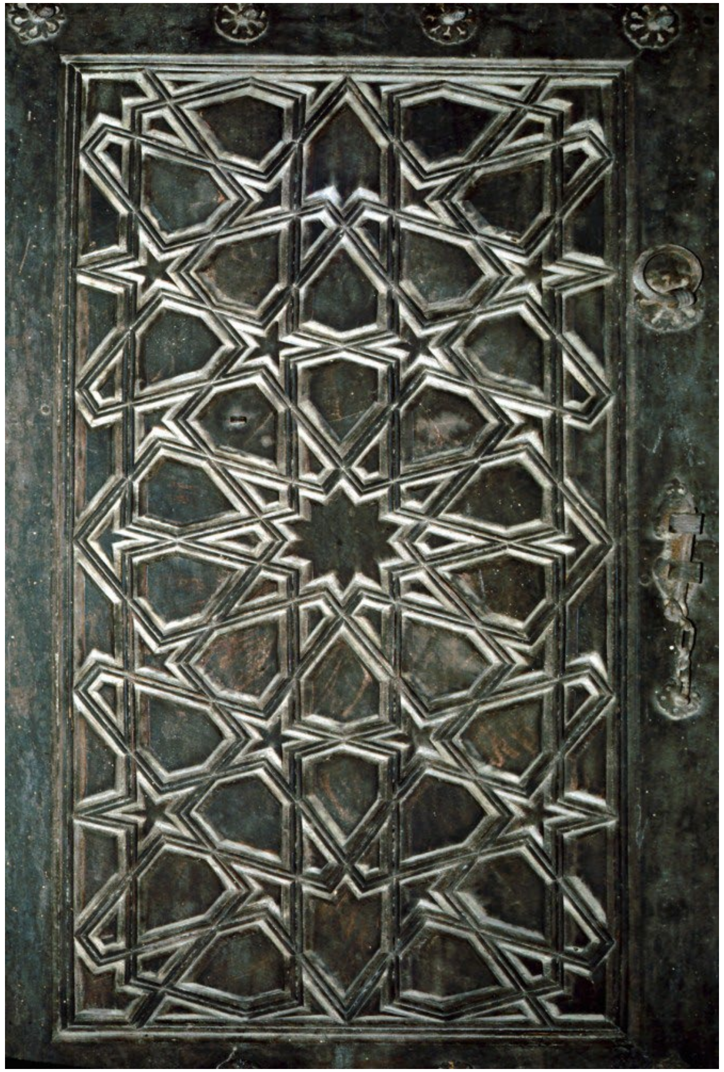
constructing traditional patterns with contemporary technologies



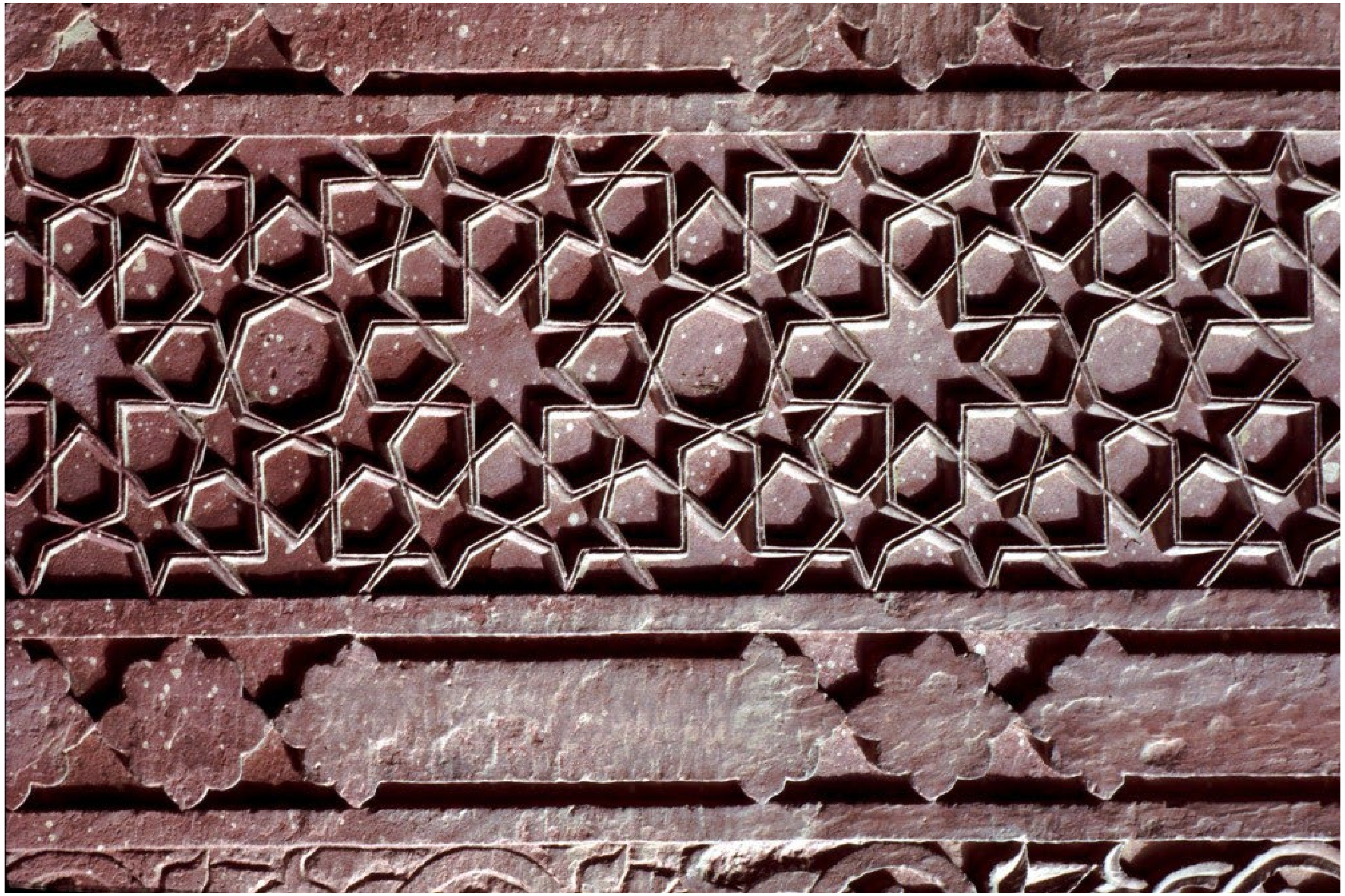
Sarah Brewer

University of Toronto Undergraduate Seminar
26 October 2022

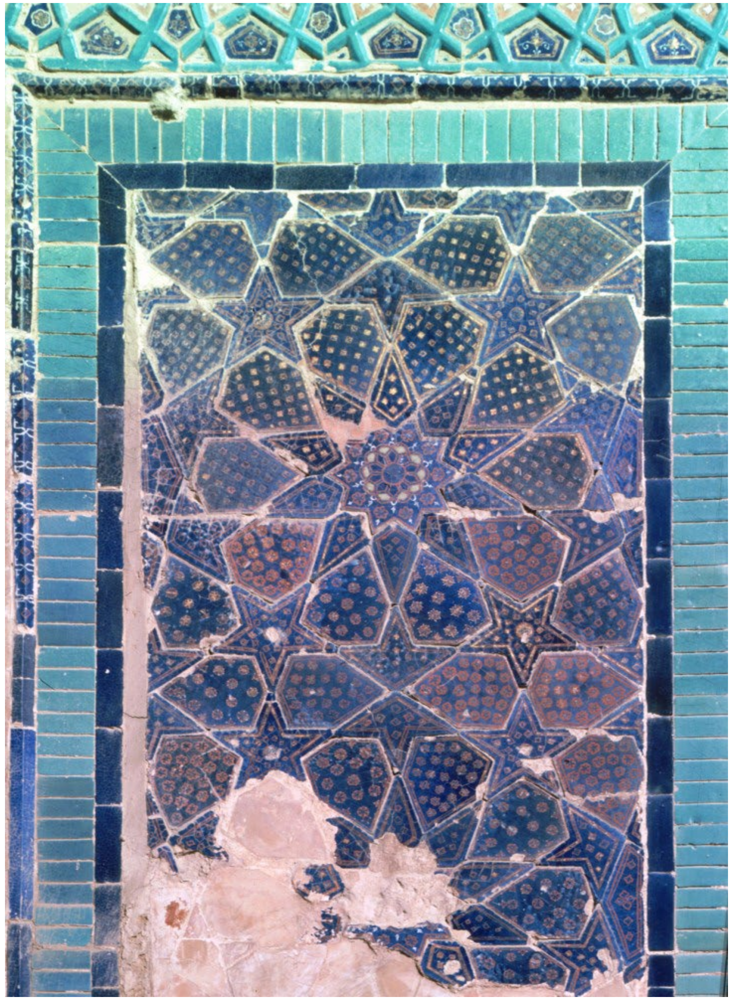
reworked from a talk “**Generating Families of Islamic Star Rosette Patterns Based on k-Uniform Tilings**“ given at Bridges Aalto, 01 August 2022



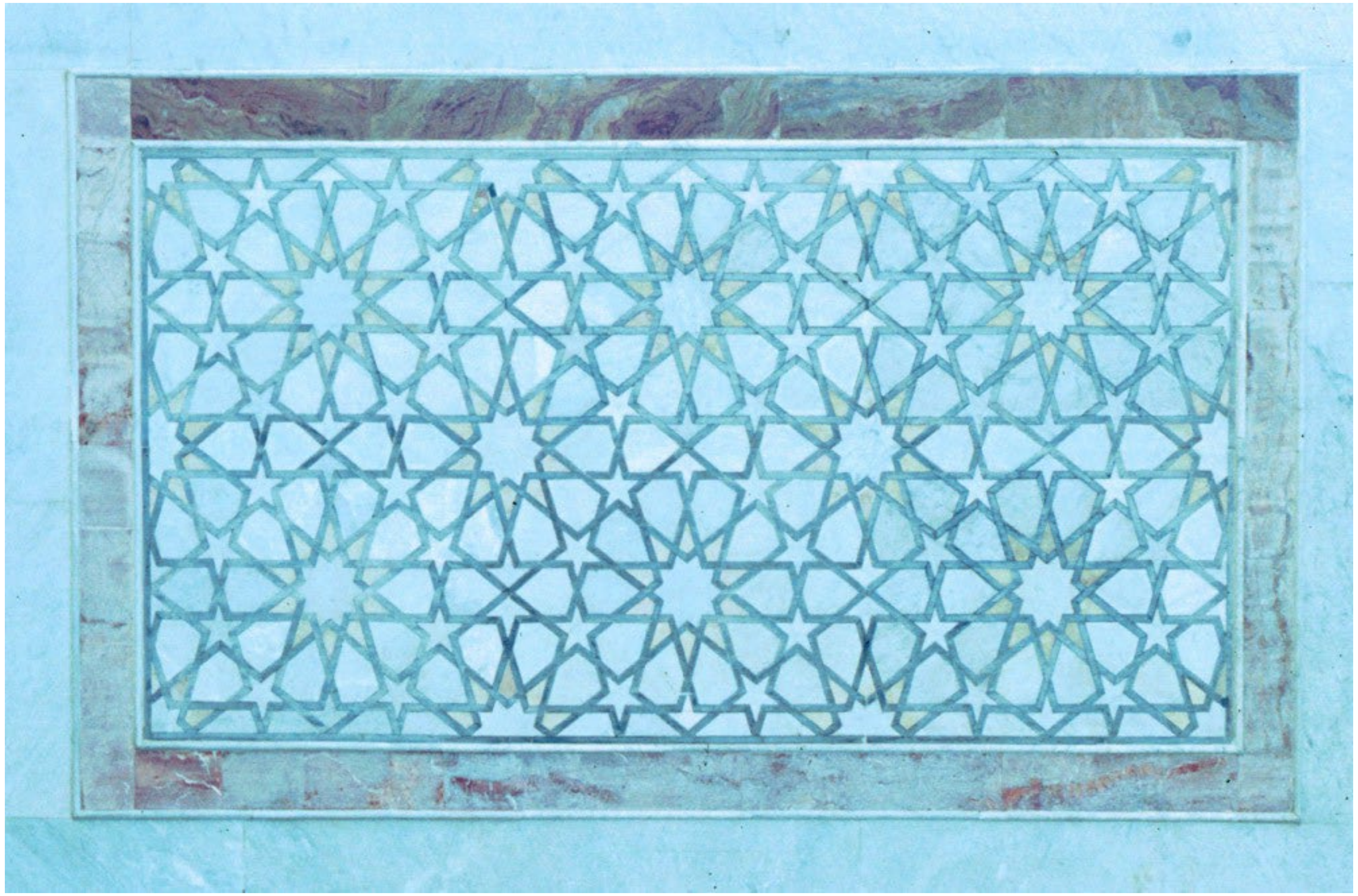
Wooden Door, Eski Mosque, Edirne, Turkey.
Wade Photo Archive TUR 0124.



Stone Panel, Agra Fort, Agra, India.
Wade Photo Archive IND 0326.

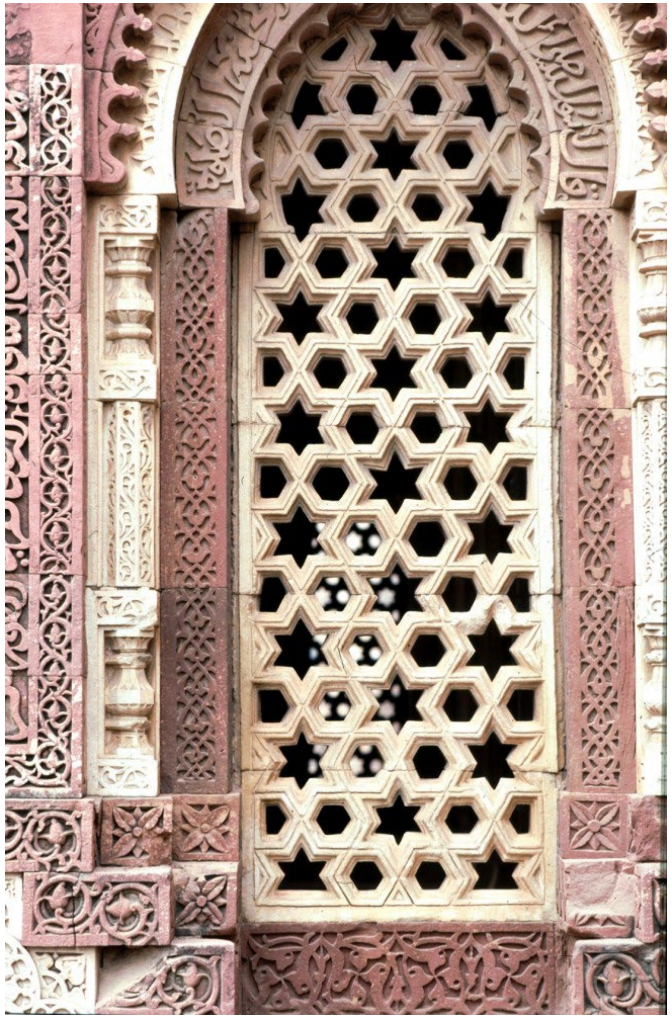


Ceramic Panel, Shakh-i-Zindeh complex, Samarkand, Uzbekistan.
Wade Photo Archive [TRA 0205].

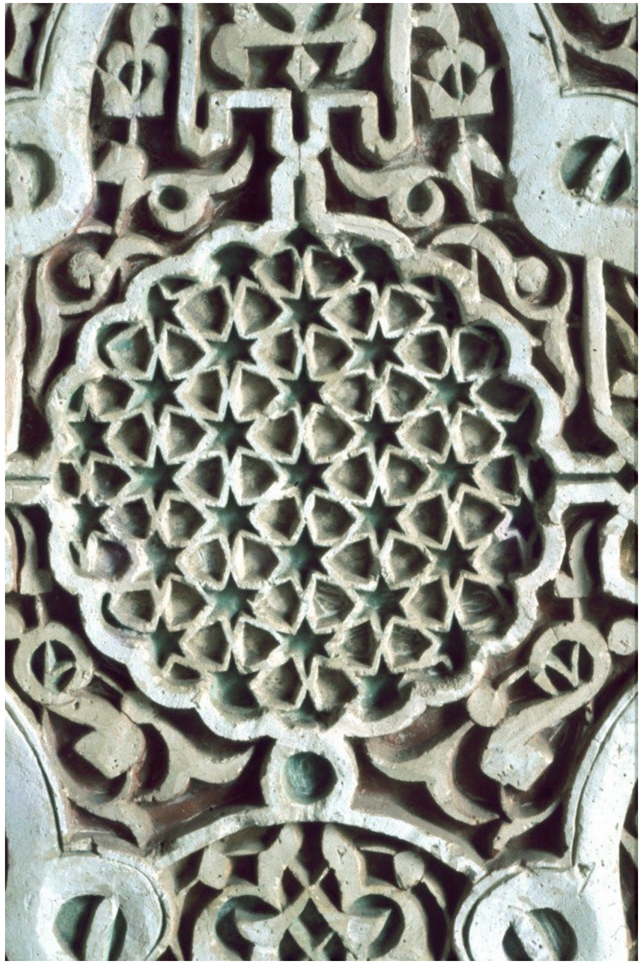


Stone Inlay Panel, Great Mosque, Damascus, Syria.
Wade Photo Archive [SYR 0126].

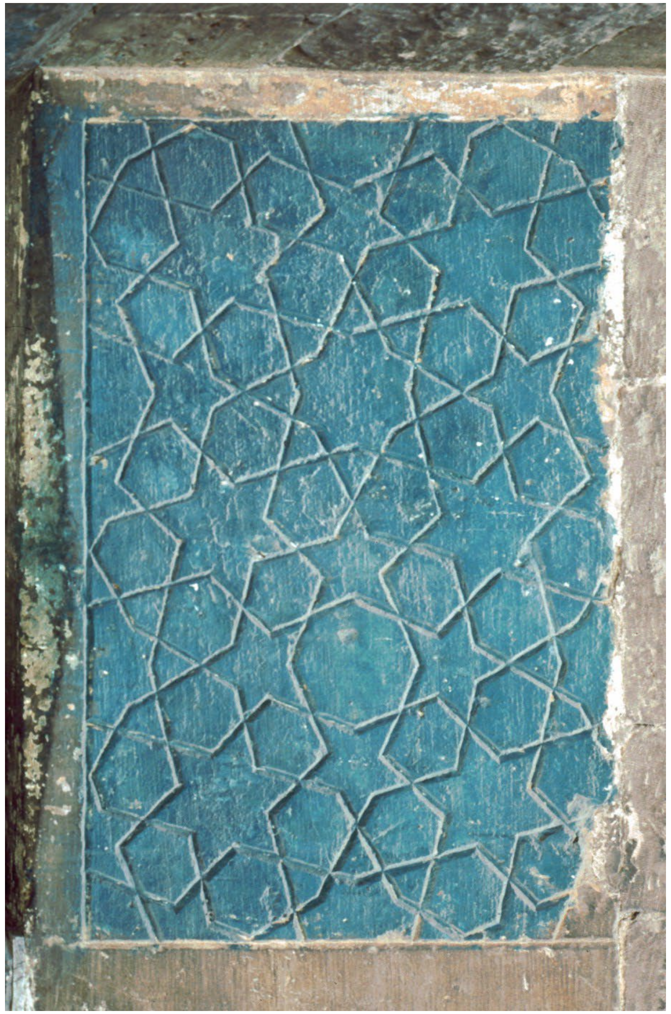
Stone Inlay Panel, Bibi Khanum,
Samarkand, Uzbekistan.
Wade Photo Archive [TRA 0305].



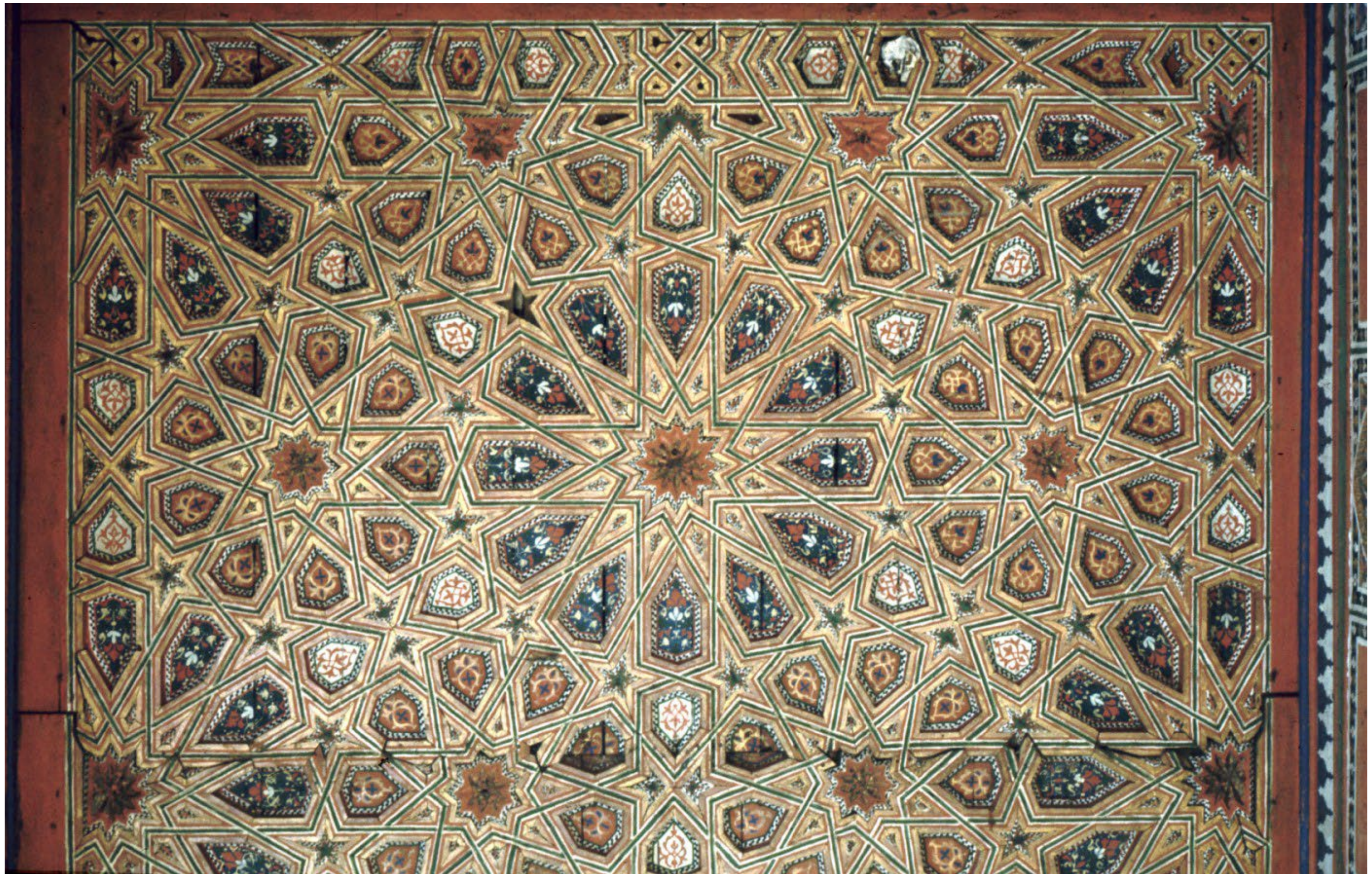
Stone Screen, Alai Darwaza, Delhi, India.
Wade Photo Archive [IND 0106].



Stucco Fountain, Sultan’s Palace,
Tangier, Morocco.
Wade Photo Archive [MOR 1508].

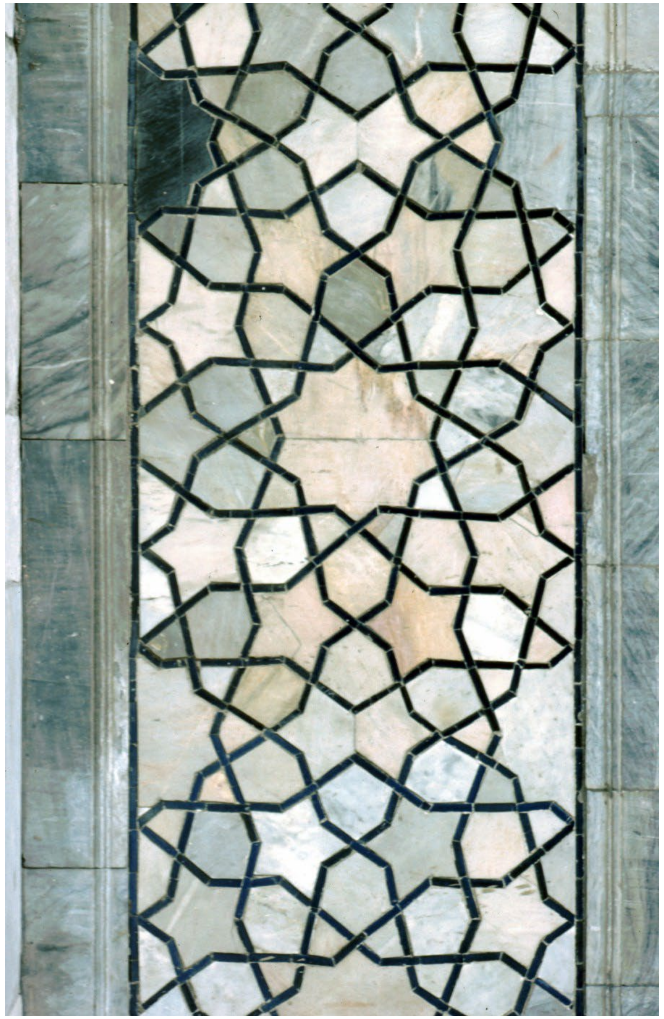


Carved Stone Panel, Altun Bogha Mosque, Aleppo, Syria.
Wade Photo Archive [SYR 0426](SYR 0426).

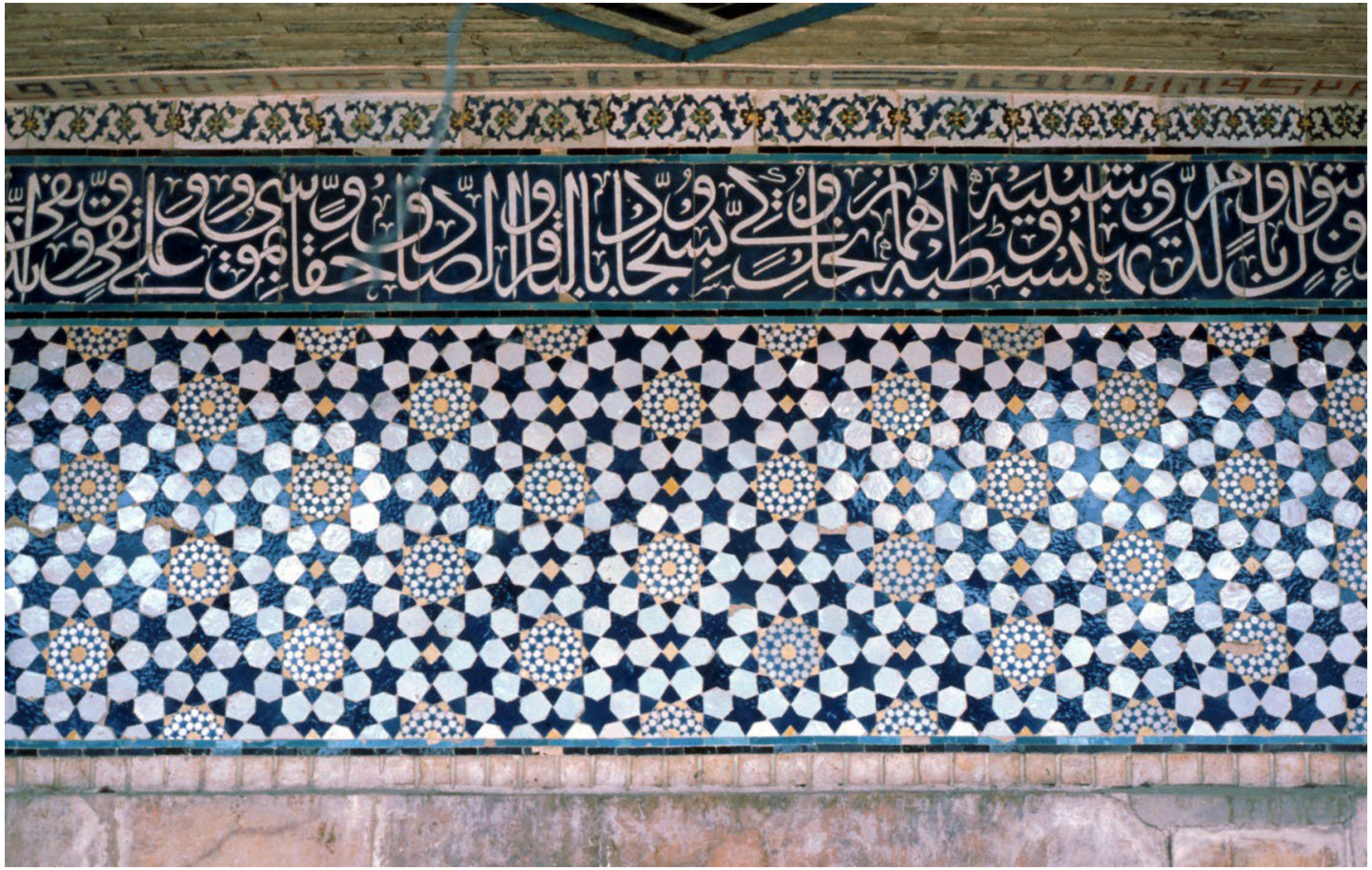


Wooden Ceiling, Tomb of Moulay Ishmael, Meknes, Morocco.
Wade Photo Archive MOR 0833.

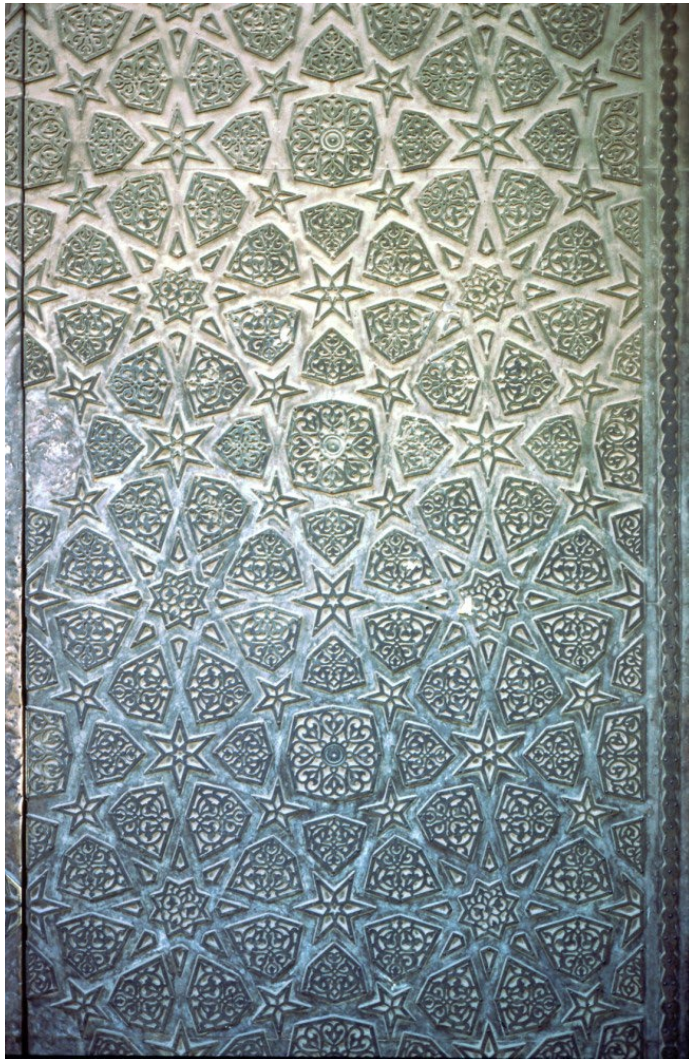
# Star Rosettes “in the Wild”



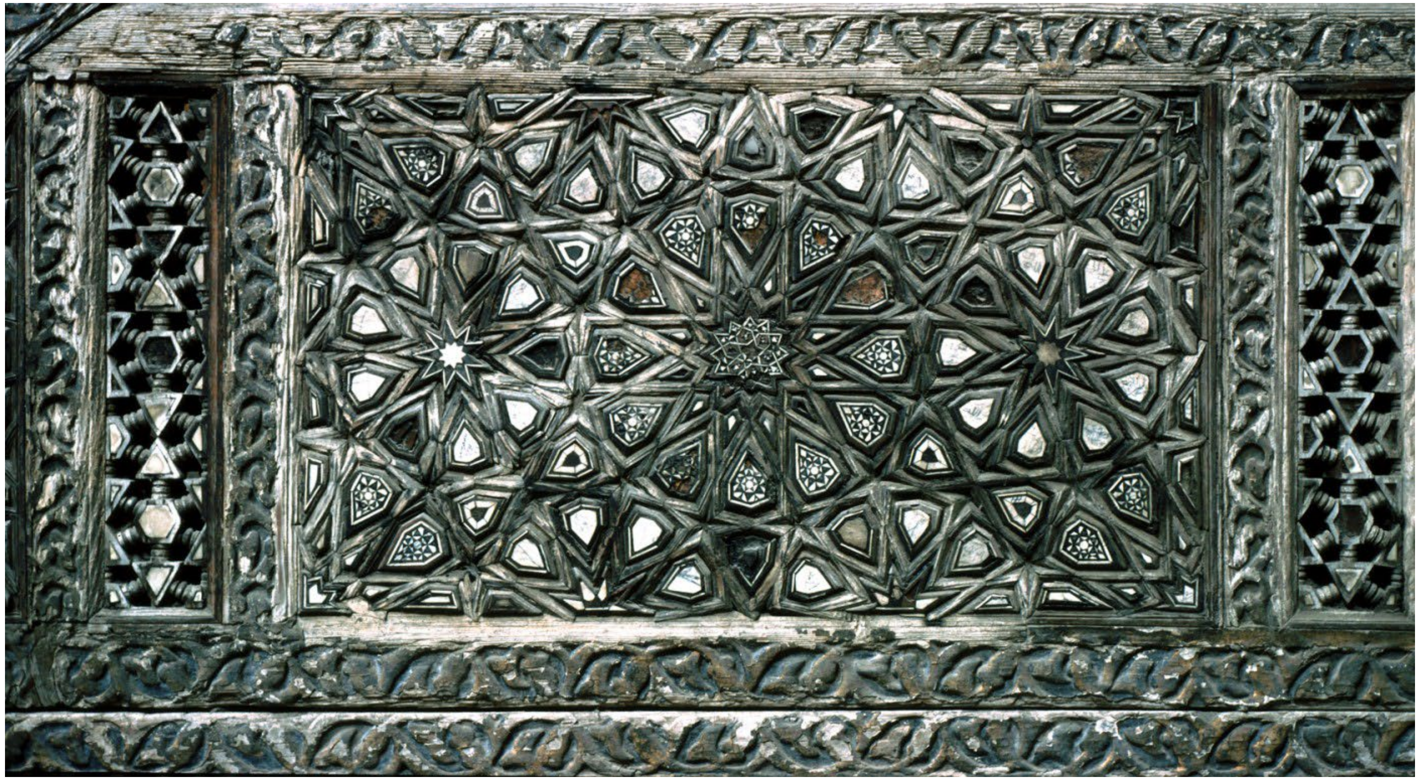
Stone Inlay Panel, Shirdar Madrassa, Samarkand, Uzbekistan.
Wade Photo Archive TRA 0335.



Ceramic Panel, Masjid-i-Jami, Isfahan, Iran.
Wade Photo Archive IRA 0517.

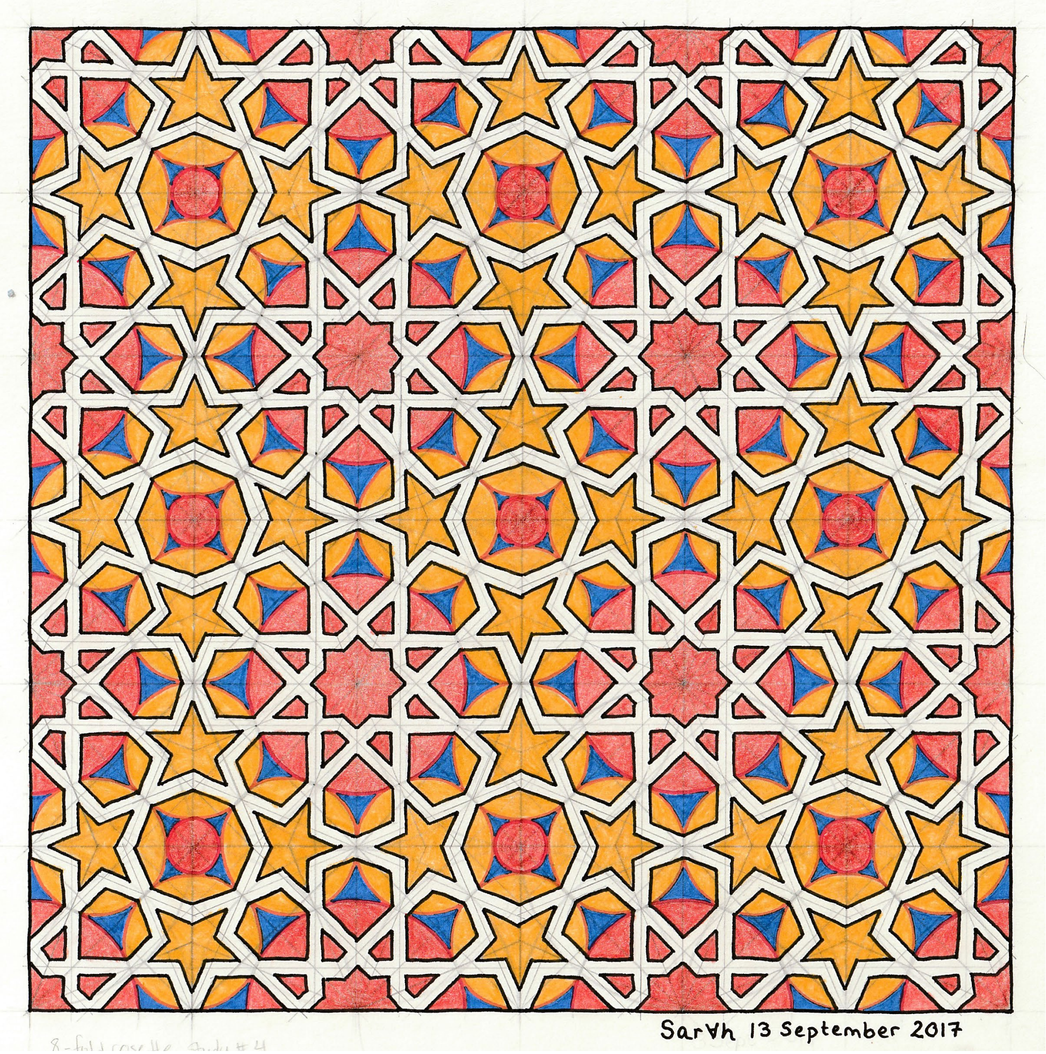


Metalwork Door, Mosque of al-Mu'ayyad, Cairo, Egypt.
Wade Photo Archive EGY 1514.

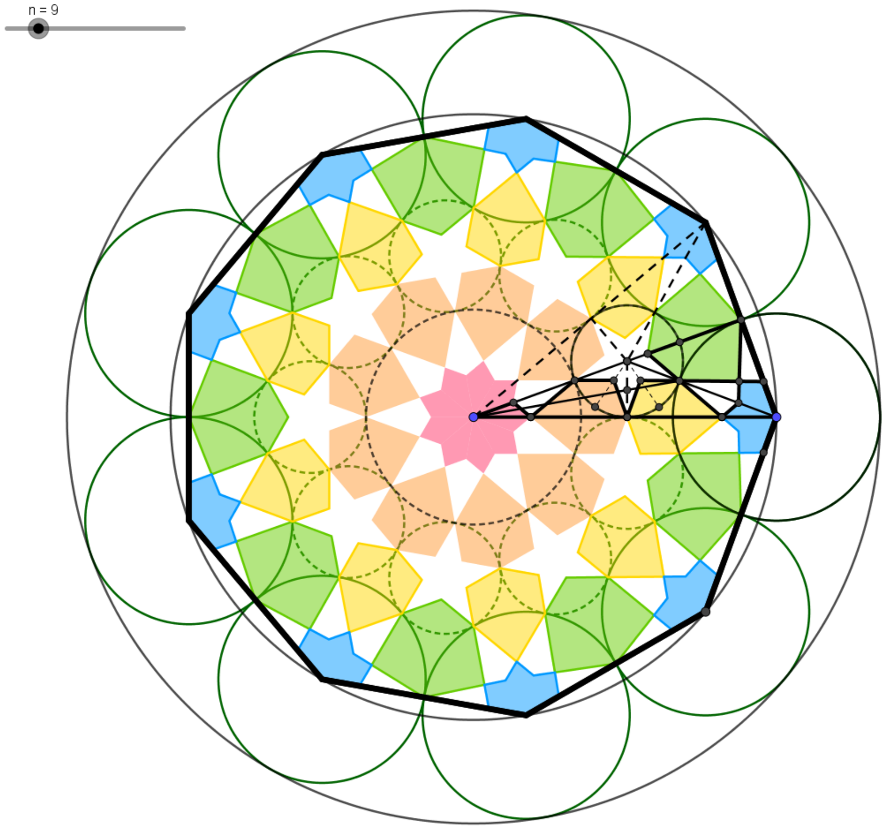


Wooden Minbar, Mosque of al-Mu'ayyad, Cairo, Egypt.
Wade Photo Archive EGY 1216.

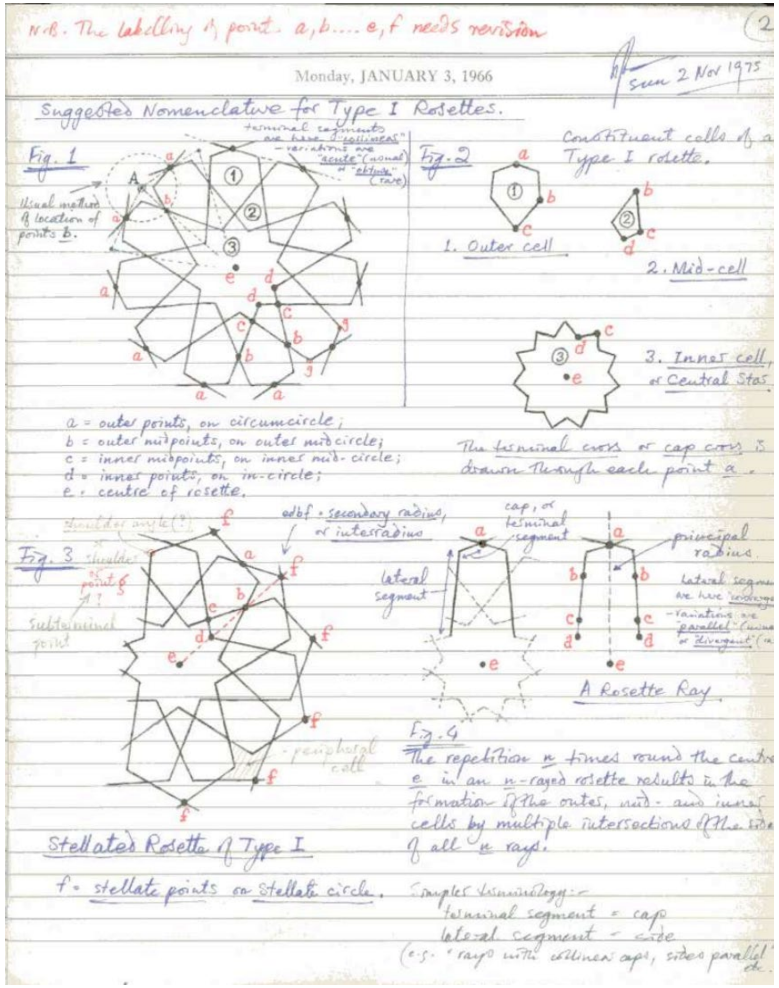
Anatomy of a Star Rosette

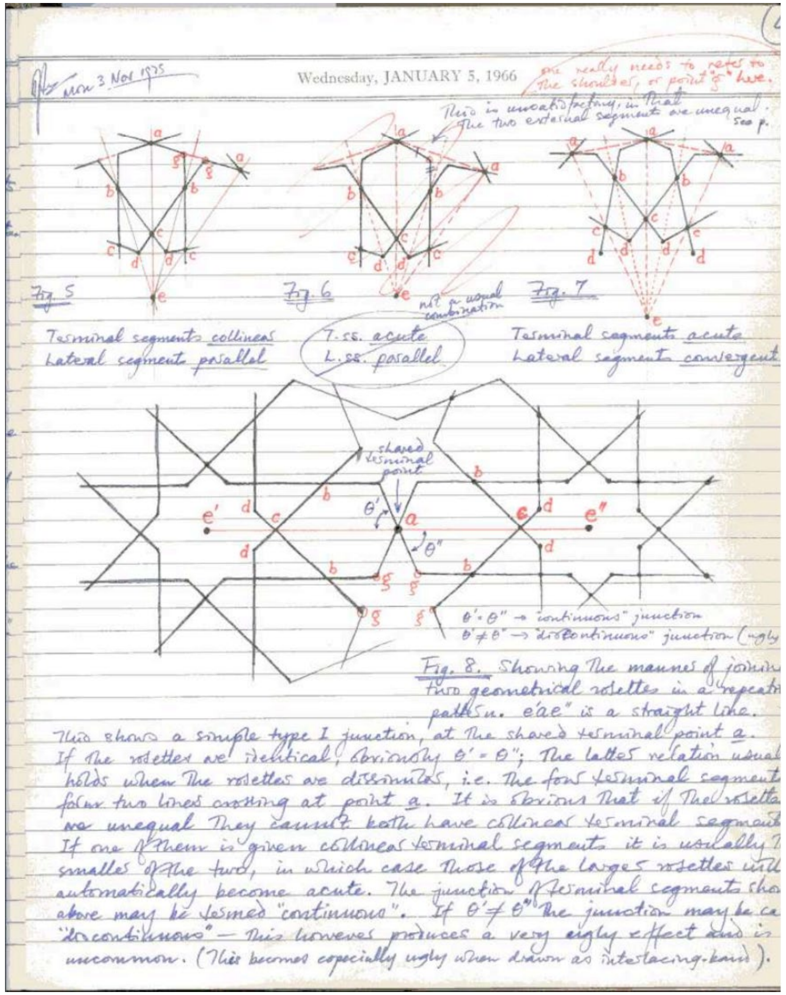


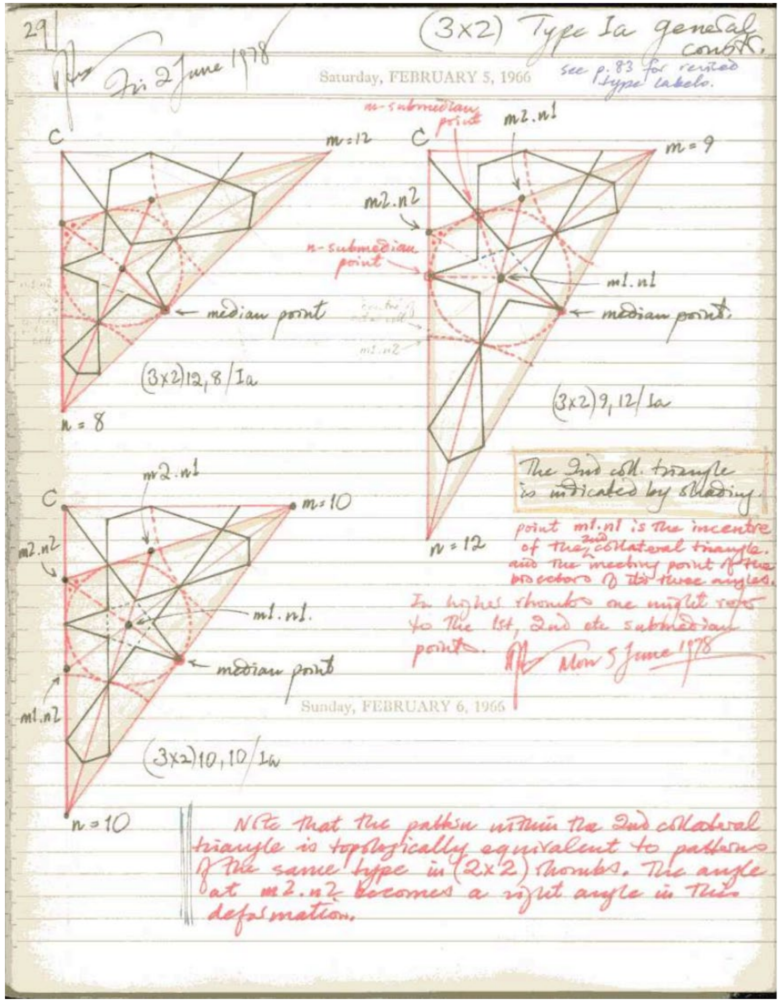

Ideal or Canonical 8-fold Rosette Pattern, drawn after Samira Mian.




9-fold Parallel Rosette pattern based on concentric circle packing

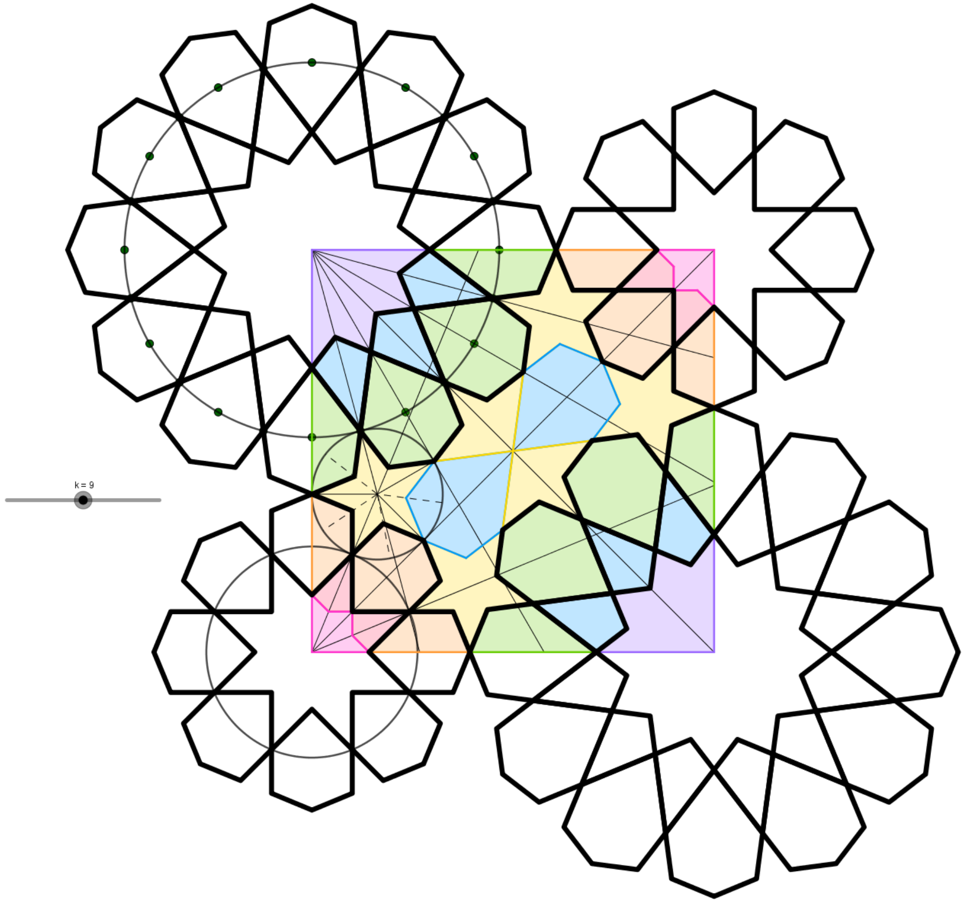


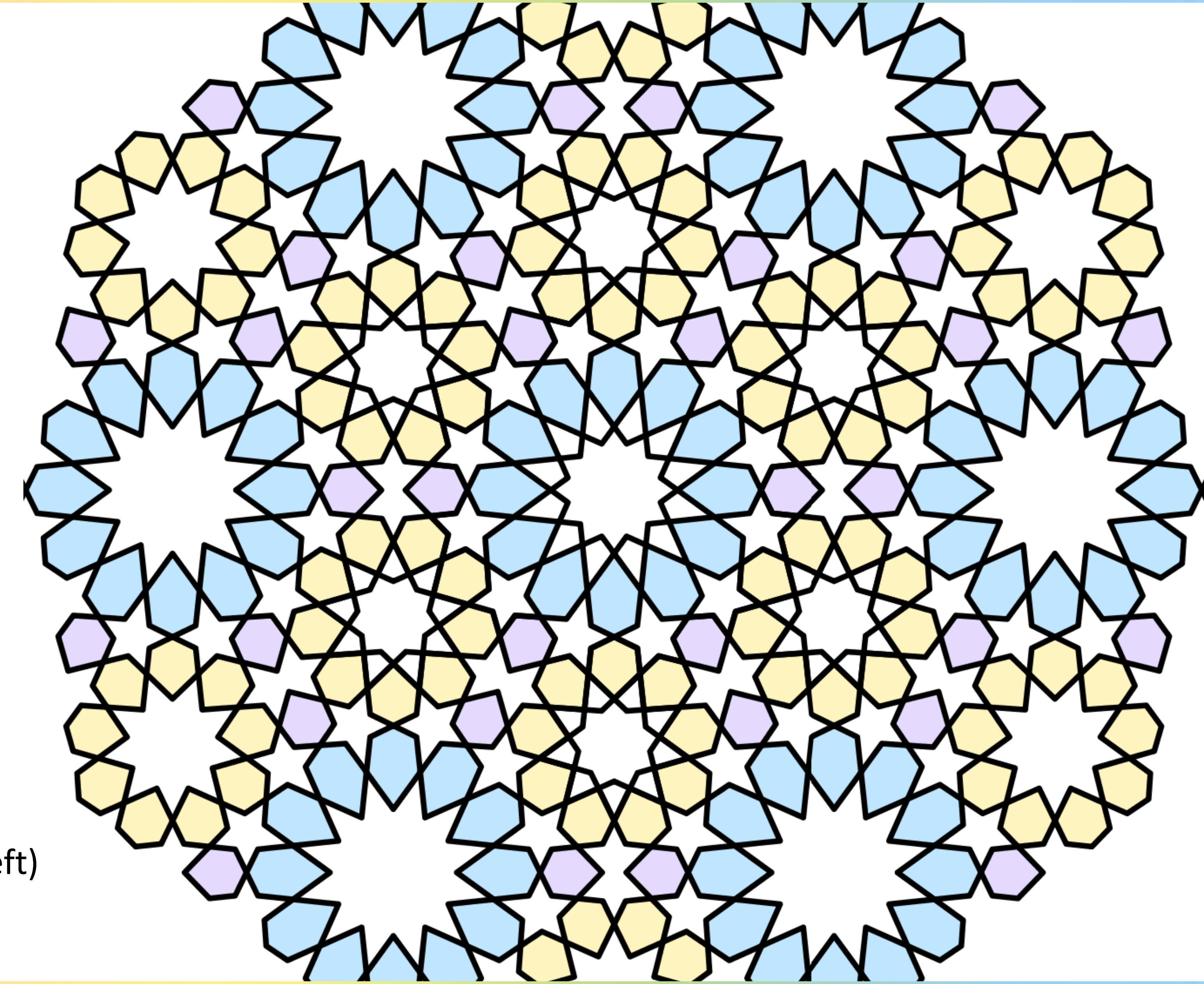




Pages 7, 9, and 33 from Part I of A.J. Lee’s Notes on Islamic Star Patterns, http://www.tilingsearch.org/tony/

# Anatomy of a Star Rosette

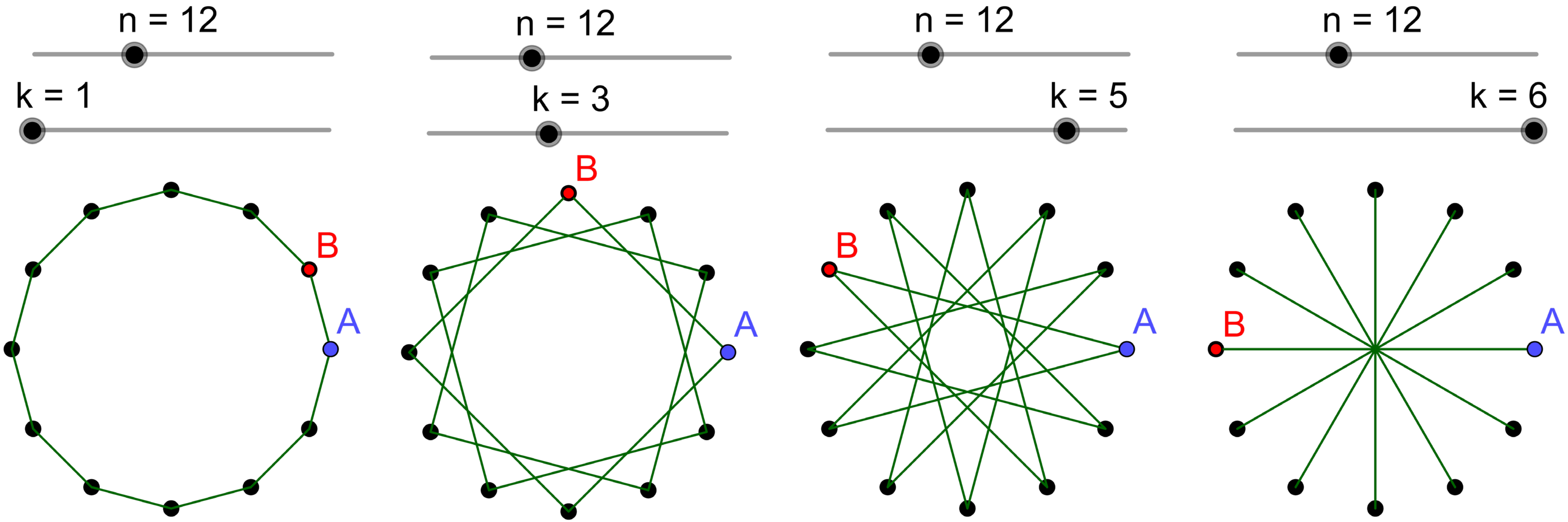


12 & 8 Star Rosettes via a {24/9} star polygon (left)

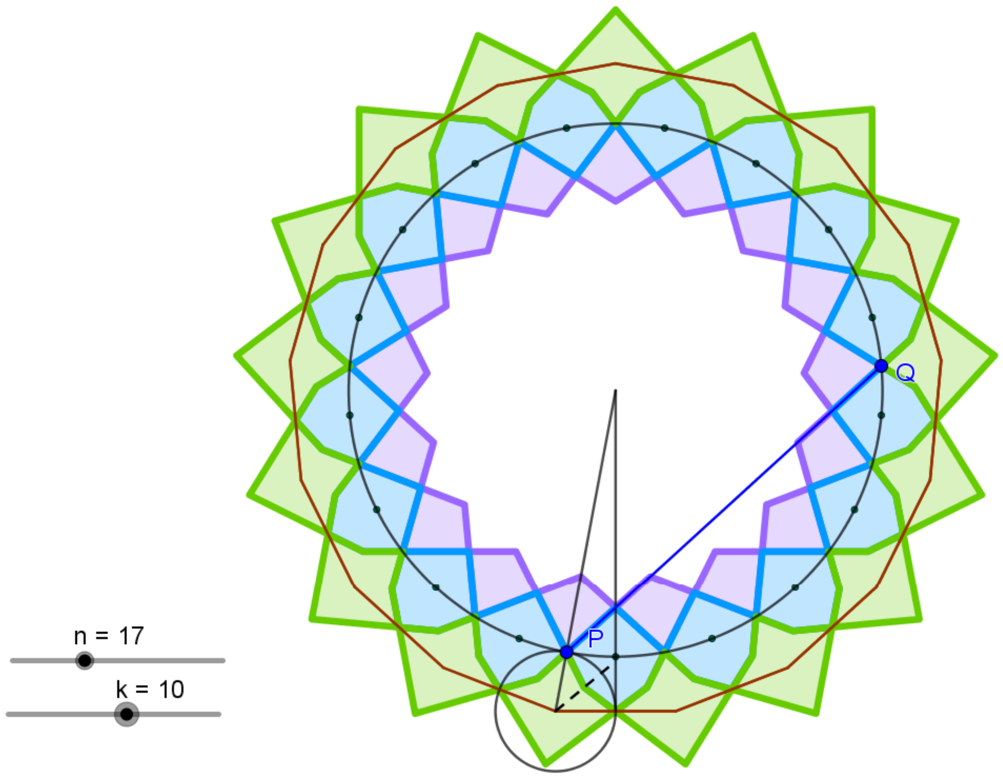
12&9 Star Rosettes at a 26° angle (right)

Anatomy of a Star Rosette

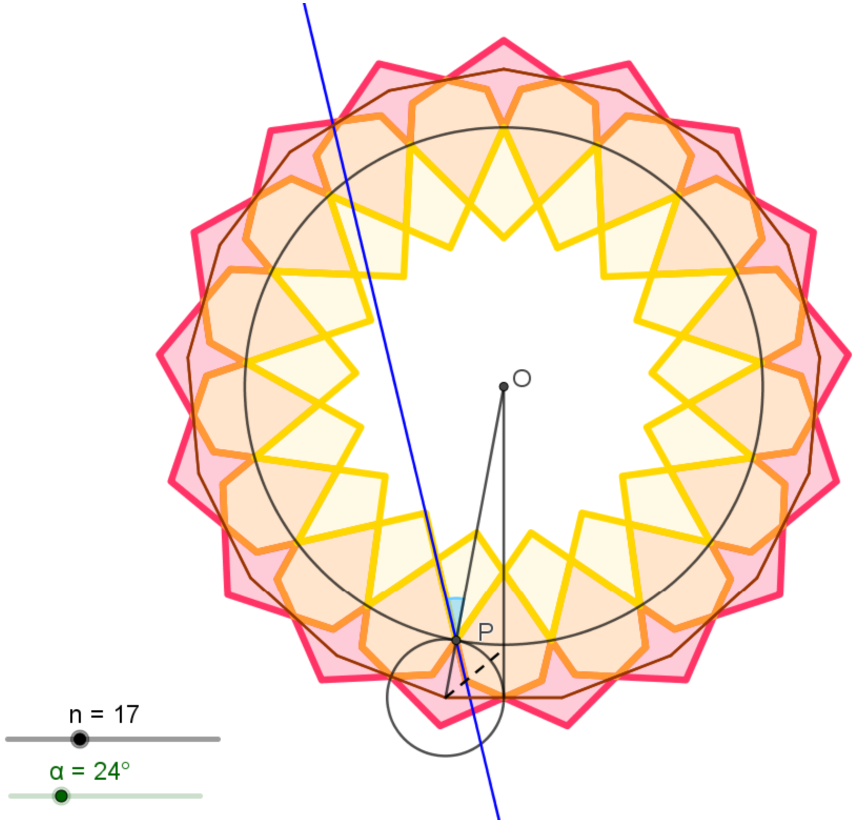



An {n / 1} star polygon is just a polygon; for n even, an {n / $\frac{n}{2}$} star polygon is an asterisk;
if k is a factor of n, then the {n / k} star polygon is a compound of smaller overlapping polygons;
if n and k are relatively prime, the {n / k} star polygon can be drawn without picking up your pencil.

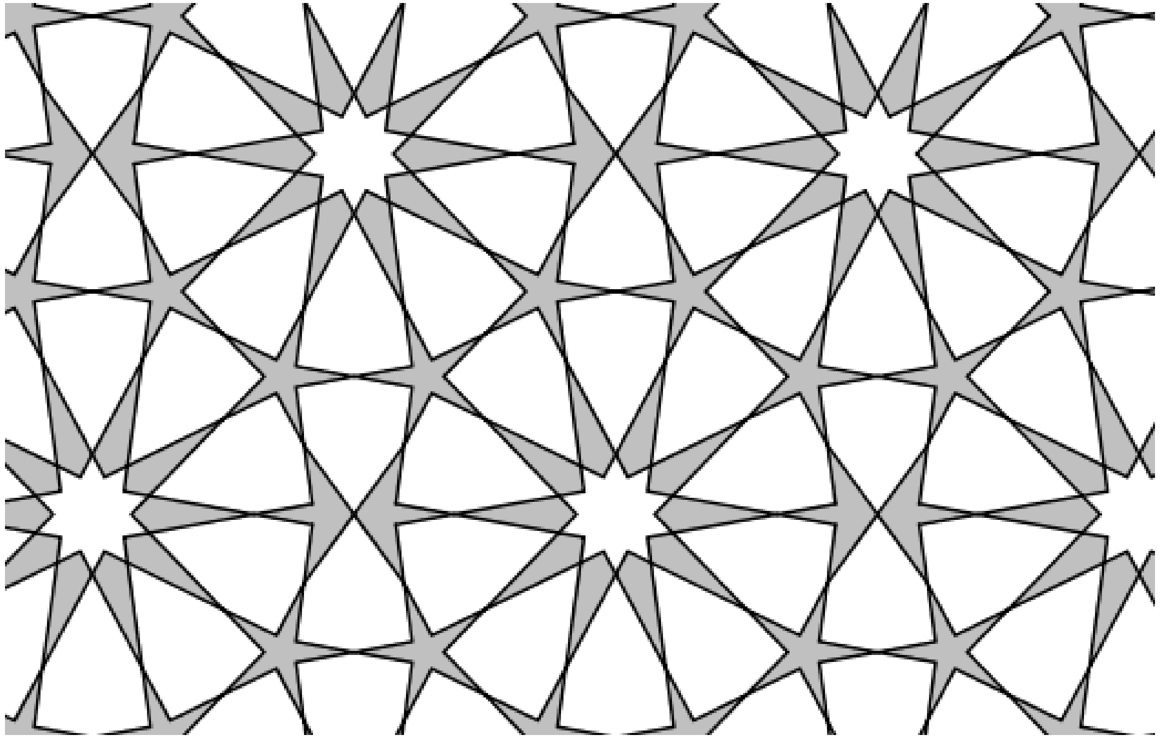
# Anatomy of a Star Rosette

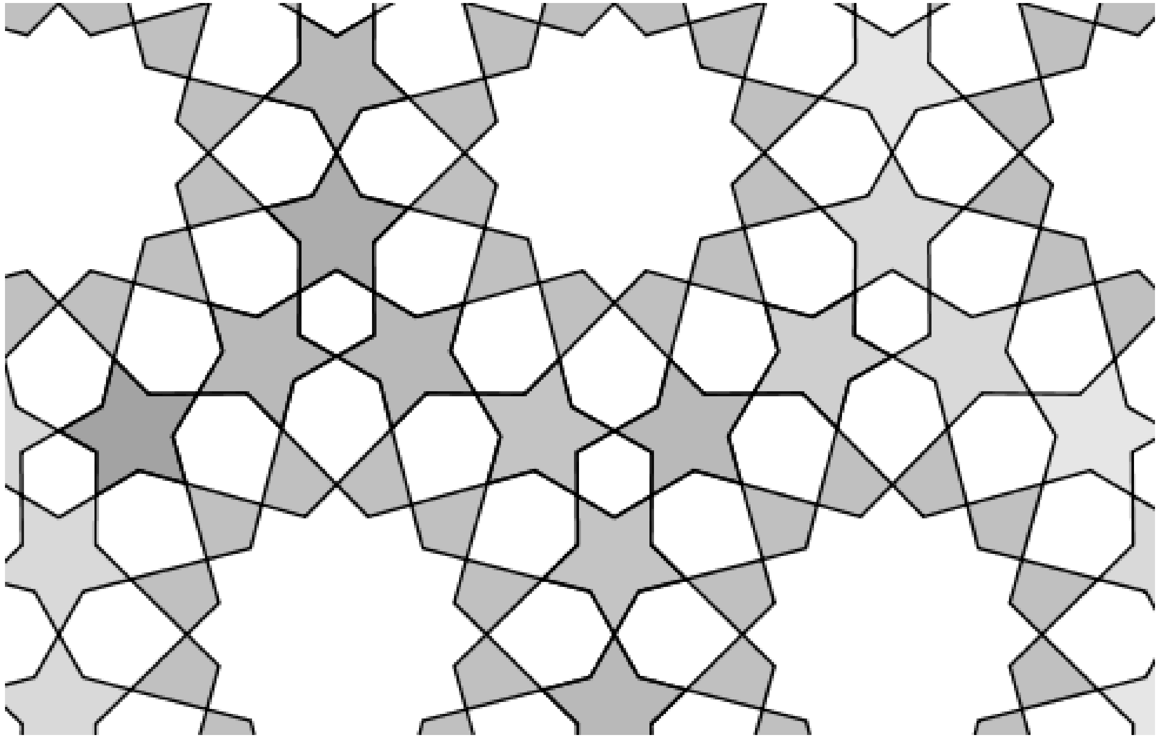


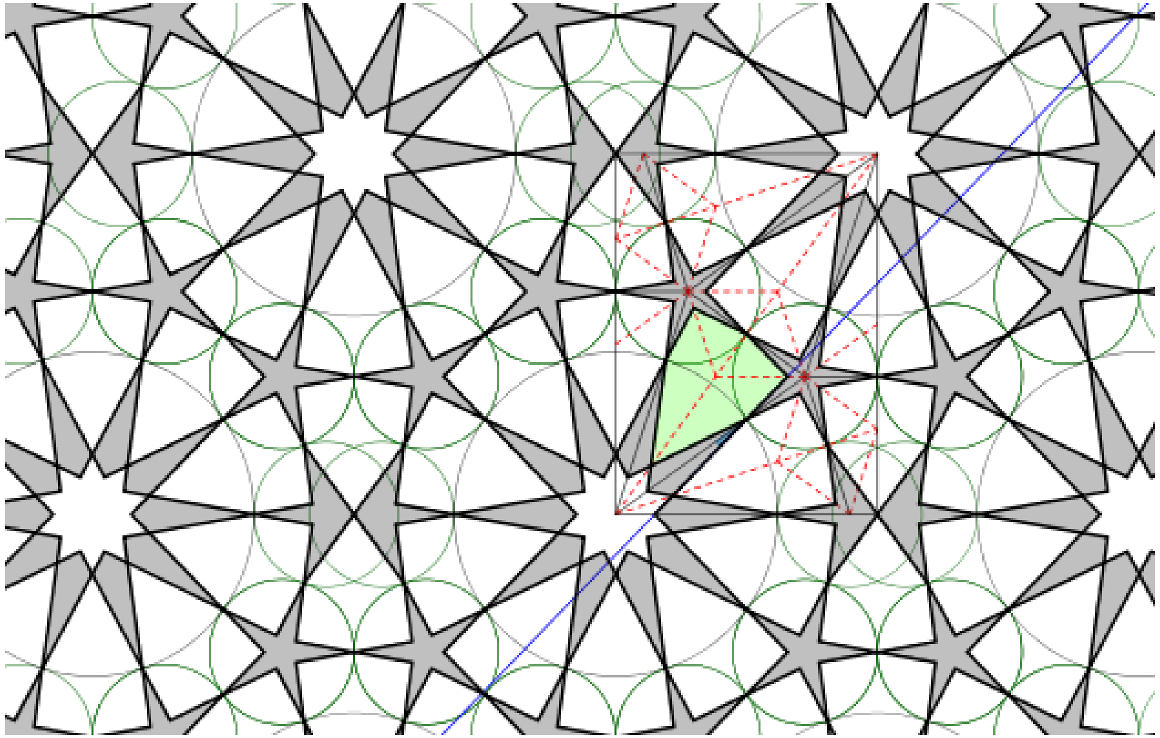

17-fold Star Rosette based on a { 34 / 10 } star polygon.

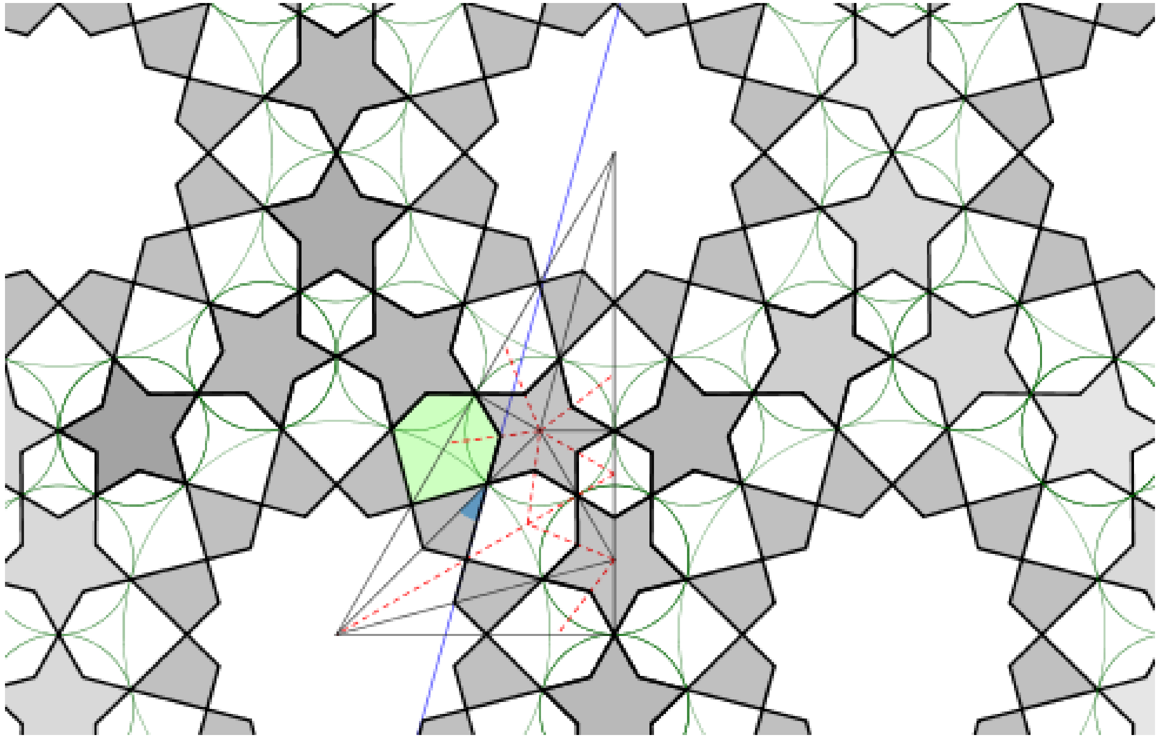


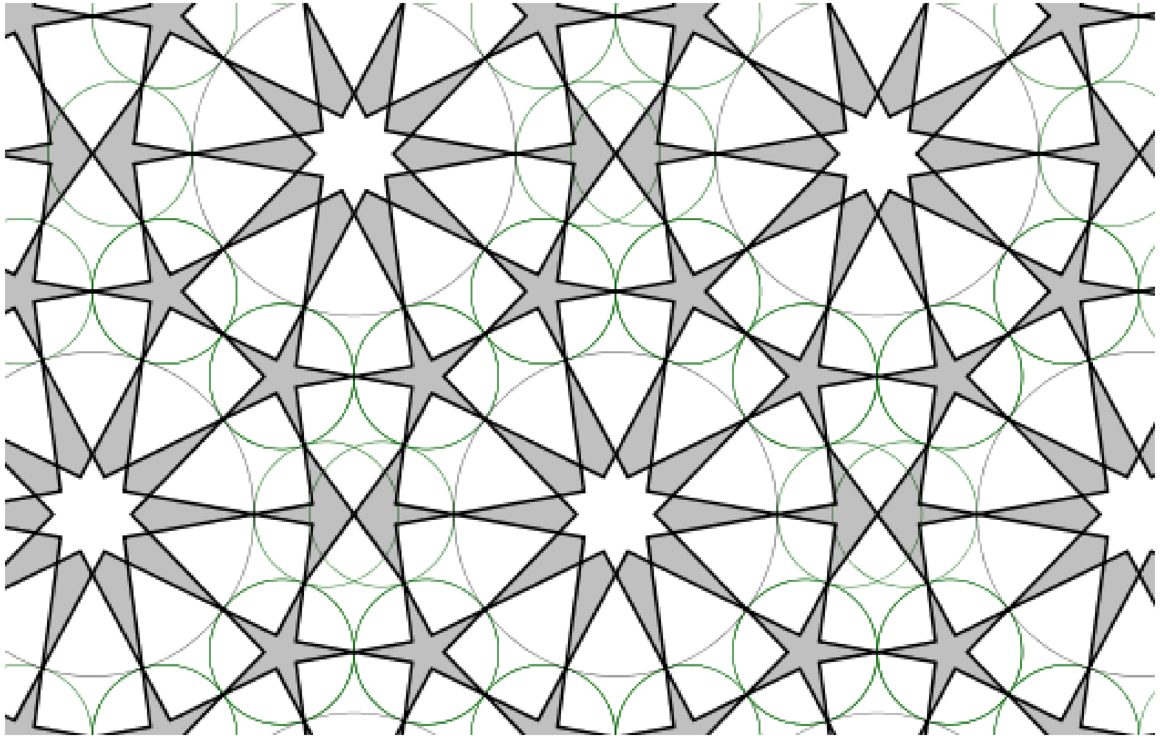

17-fold Star Rosette based on an angle of 24°

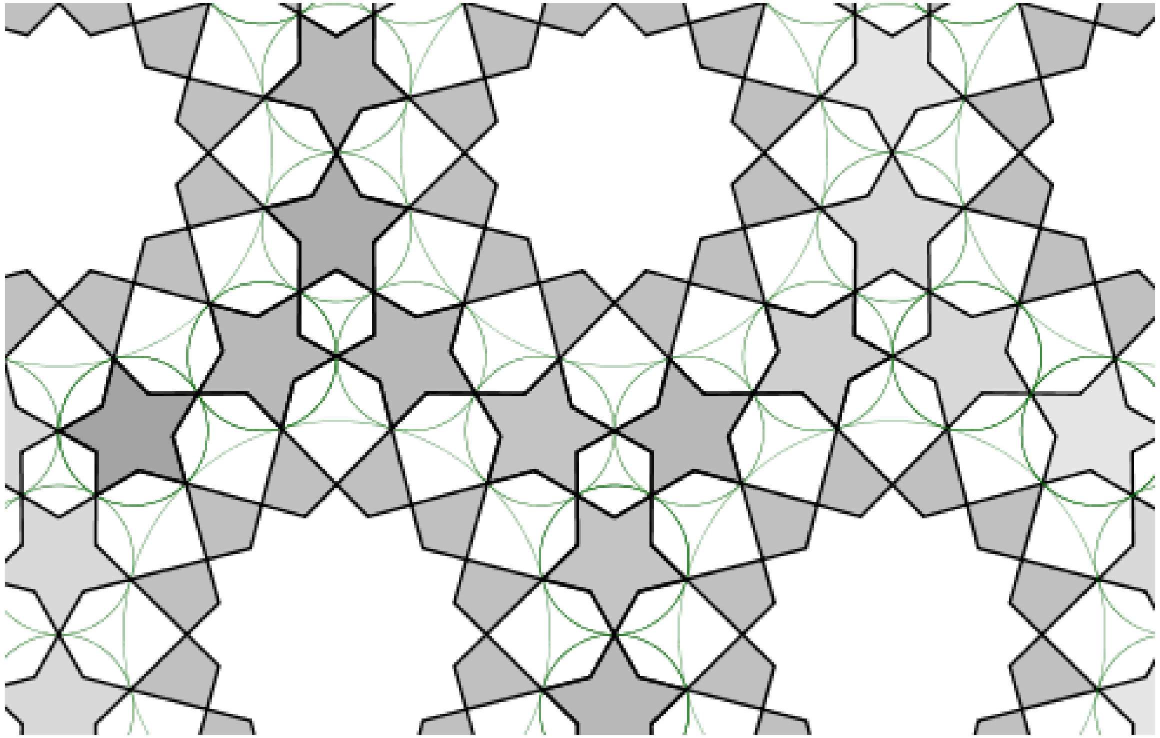


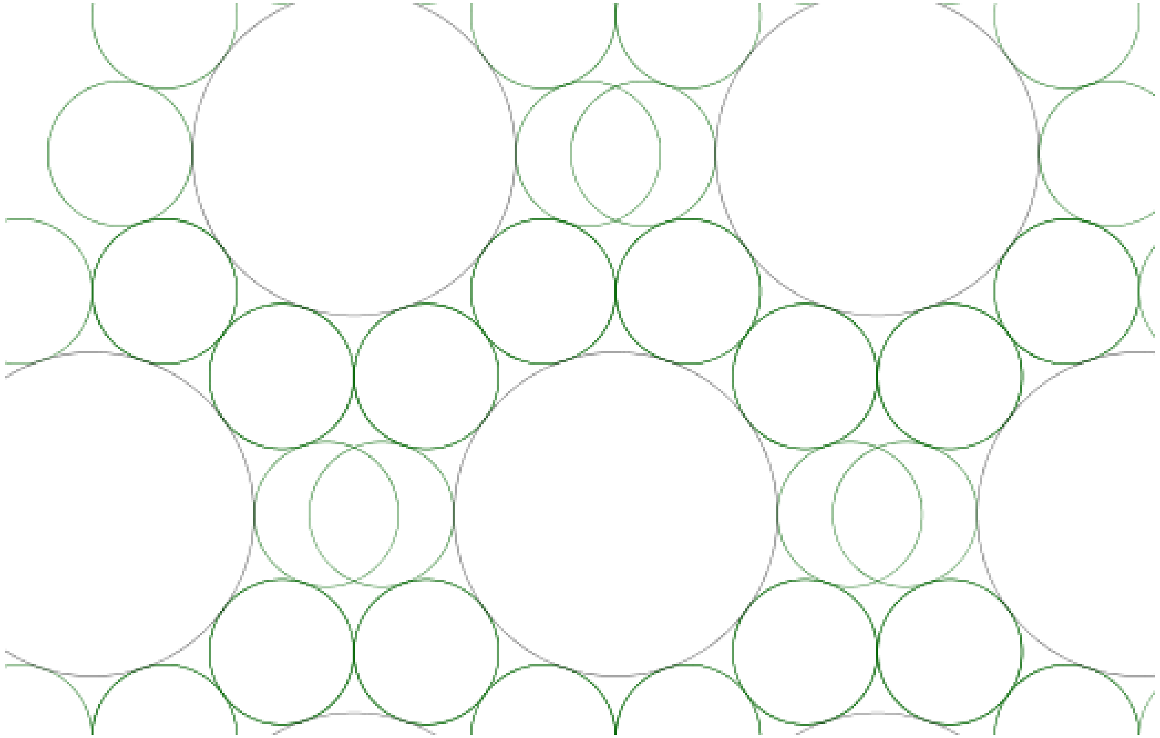


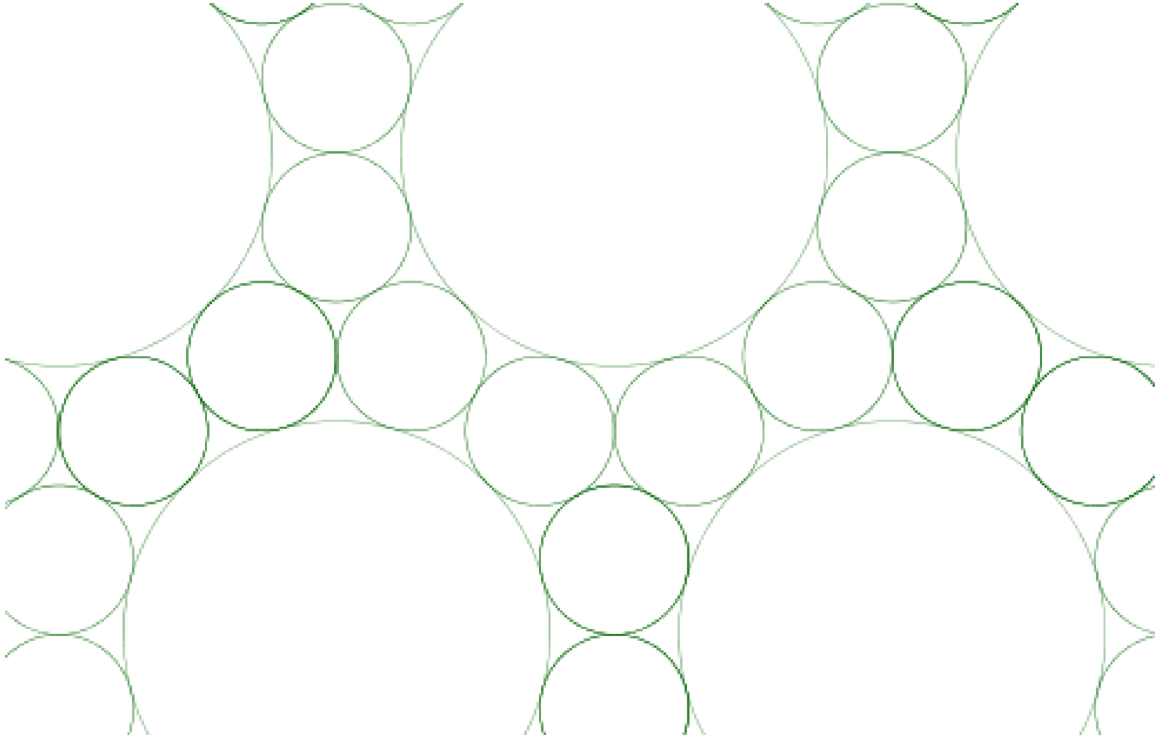


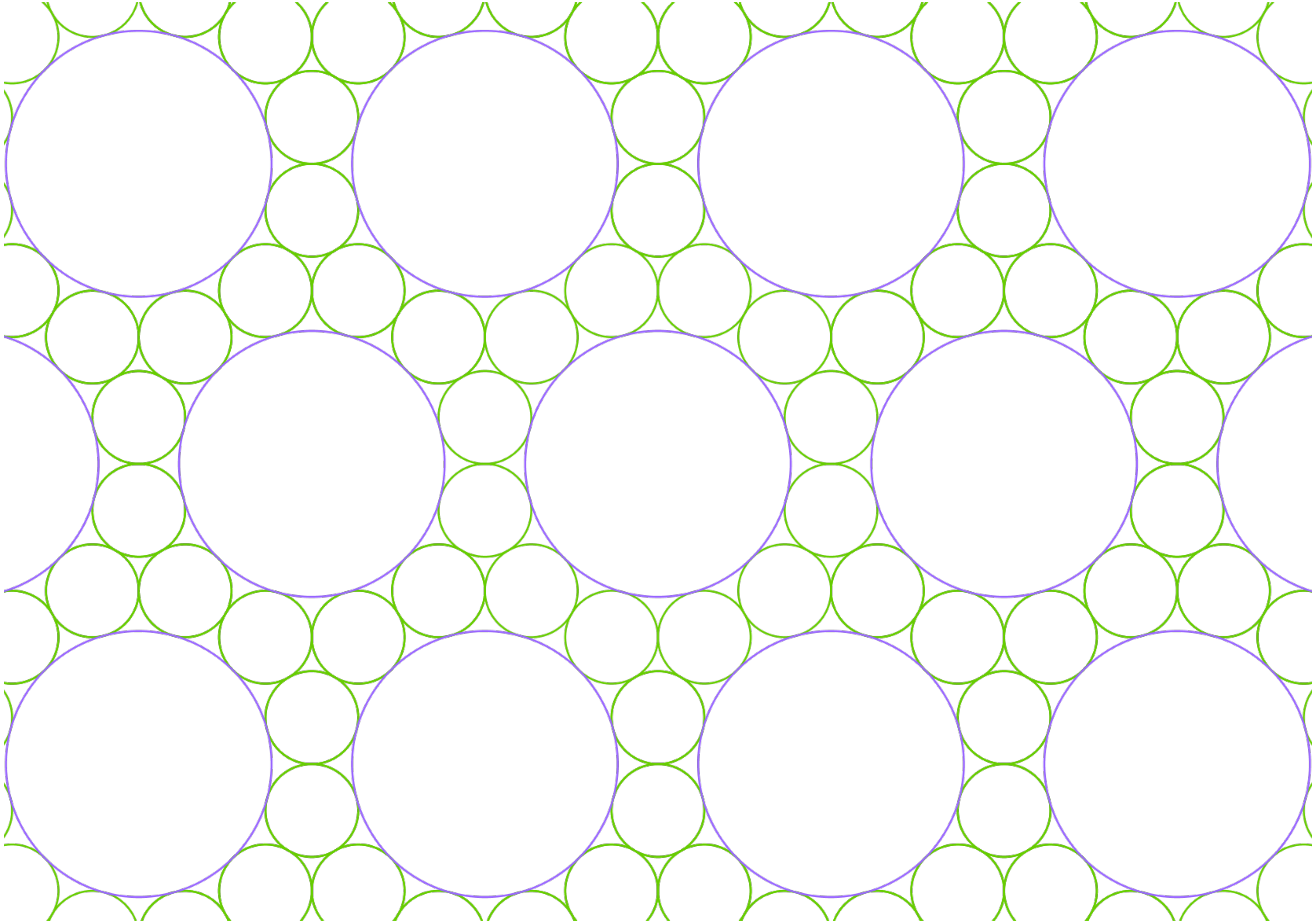


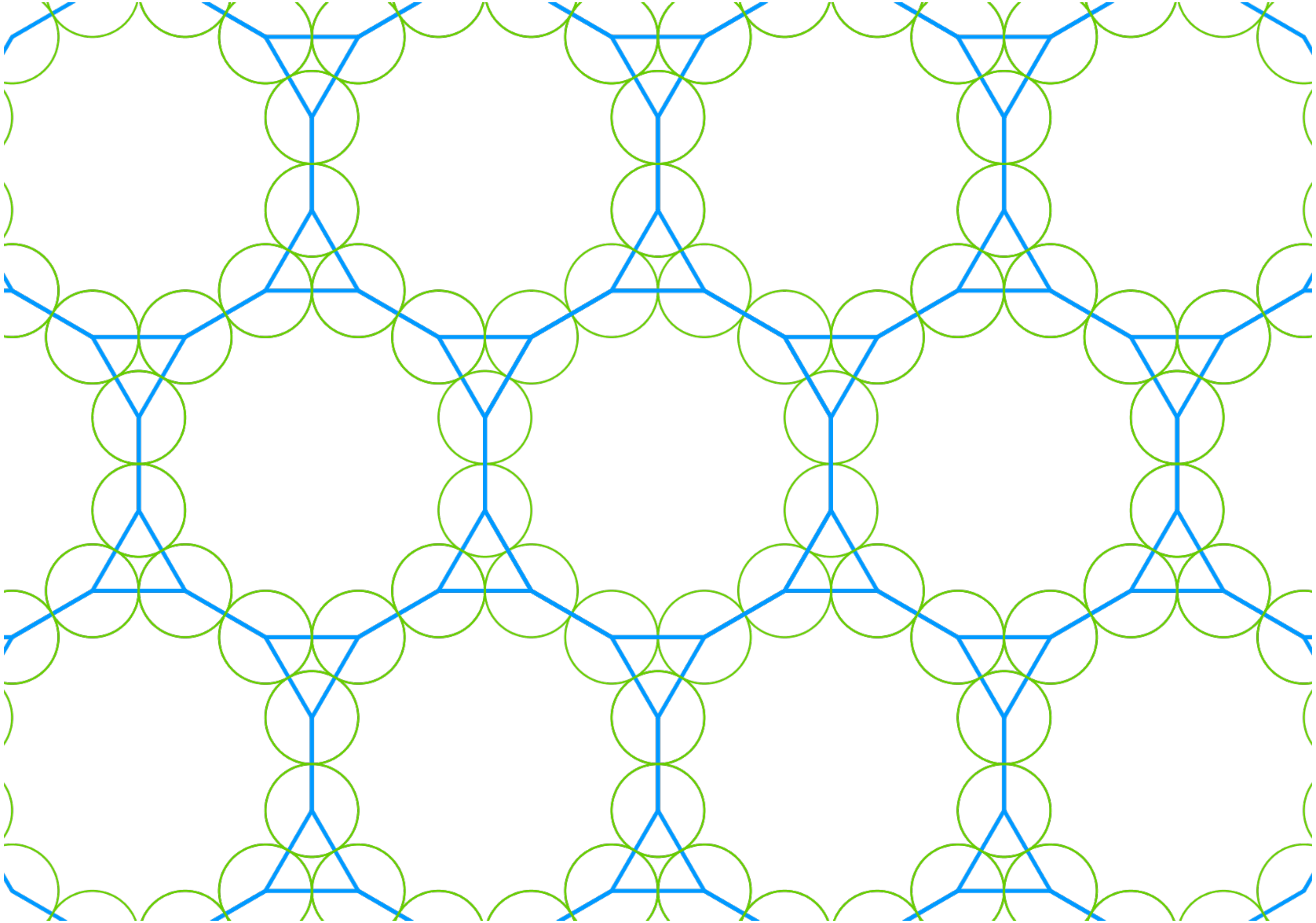


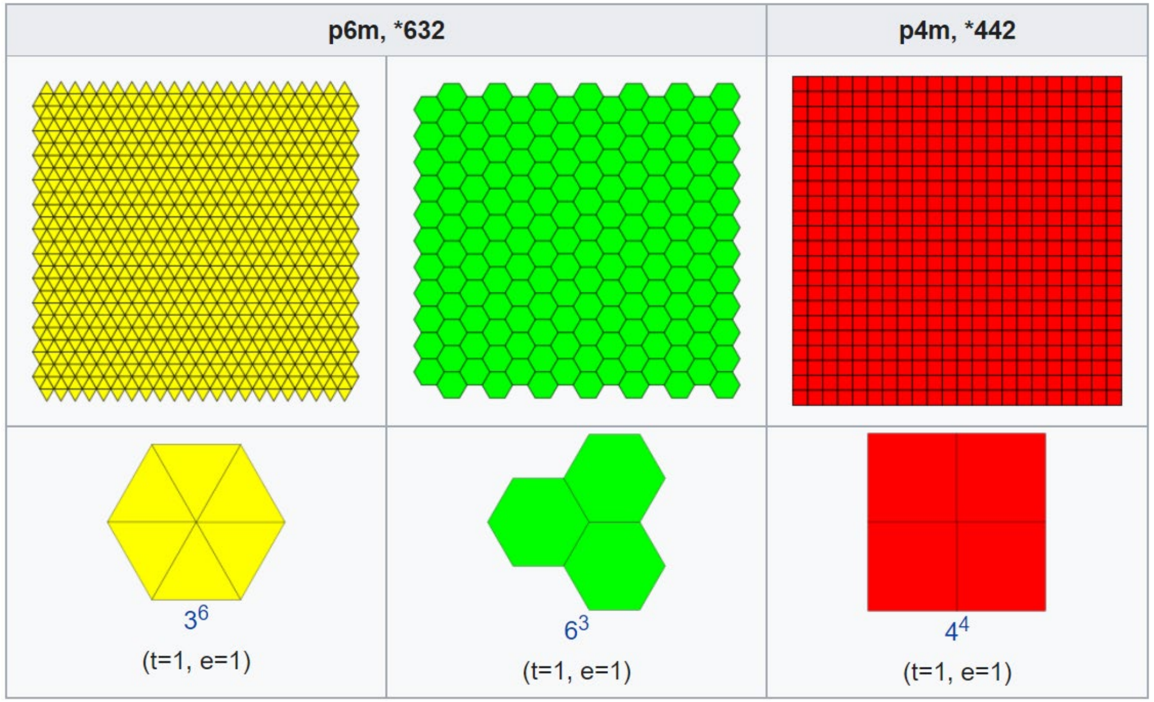


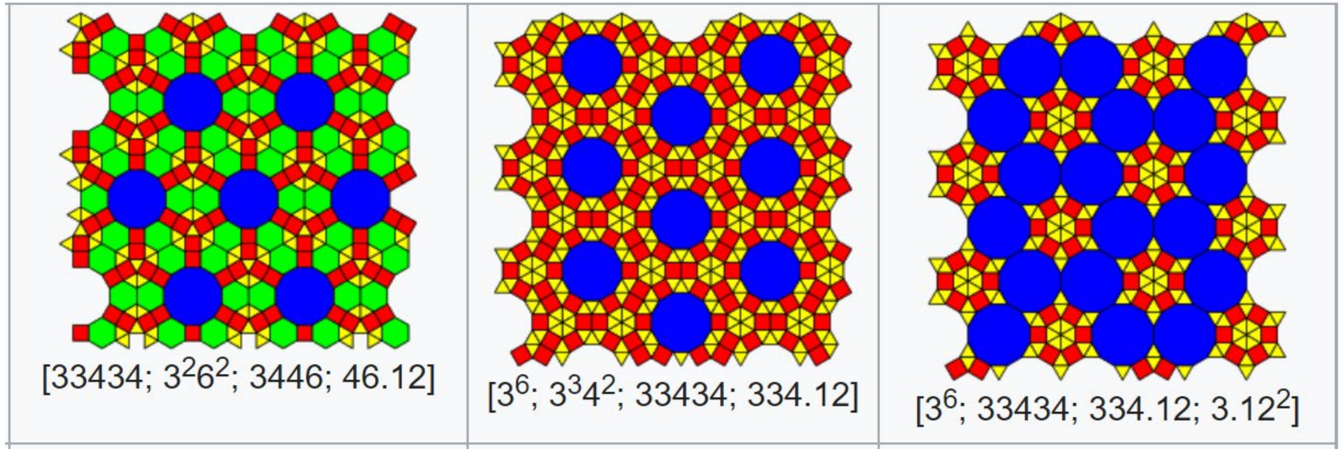


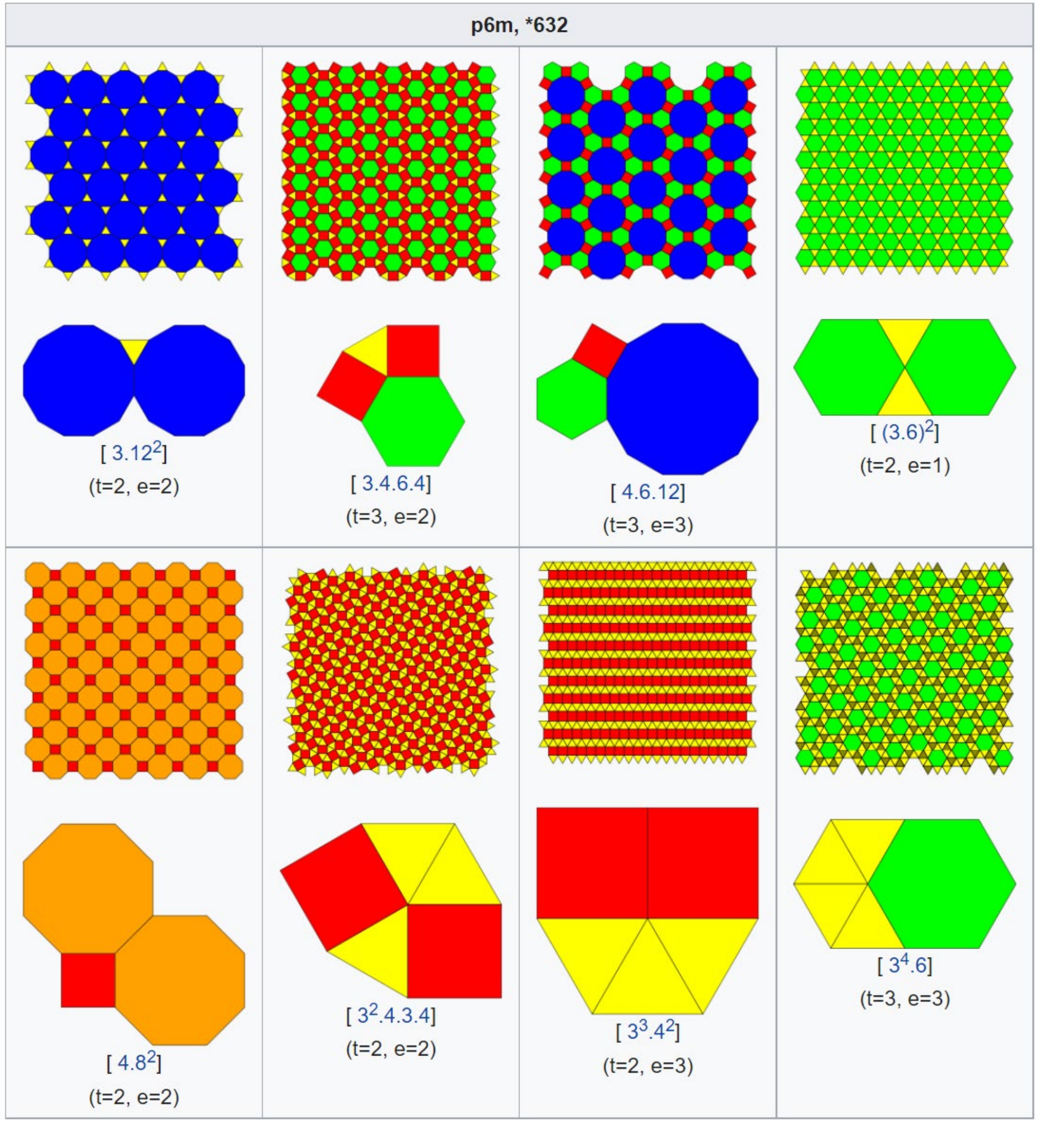


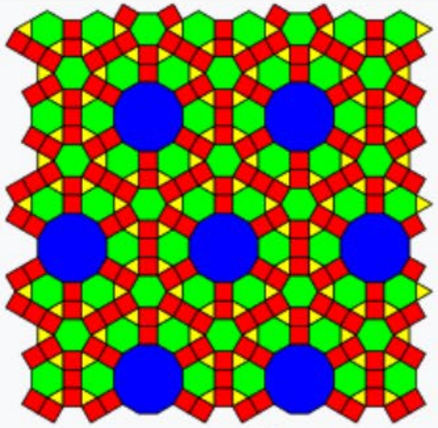




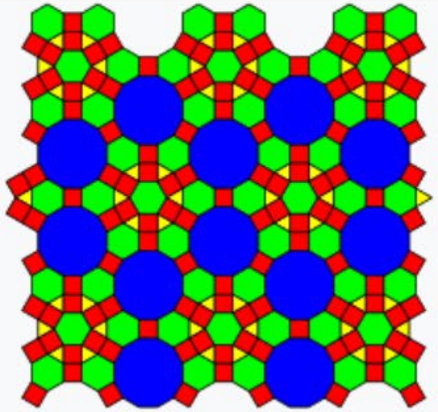

p6m, *632
p4m, *442
$3^6$
(t=1, e=1)
$6^3$
(t=1, e=1)
$4^4$
(t=1, e=1)


[33434; $3^2 6^2$; 3446; 46.12]
[$3^6$; $3^3 4^2$; 33434; 334.12]
[$3^6$; 33434; 334.12; $3.12^2$]

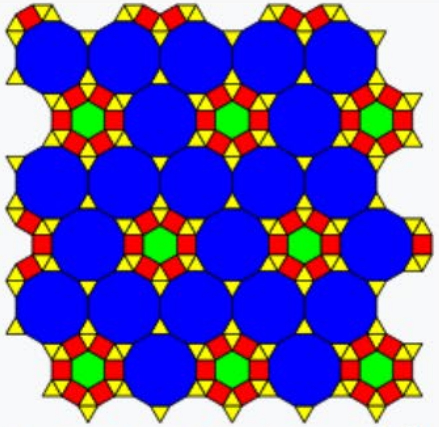

p6m, *632
[ $3.12^2$]
(t=2, e=2)
[ 3.4.6.4]
(t=3, e=2)
[ 4.6.12]
(t=3, e=3)
[ $(3.6)^2$]
(t=2, e=1)
[ $4.8^2$]
(t=2, e=2)
[ $3^2.4.3.4$]
(t=2, e=2)
[ $3^3.4^2$]
(t=2, e=3)
[ $3^4.6$]
(t=3, e=3)



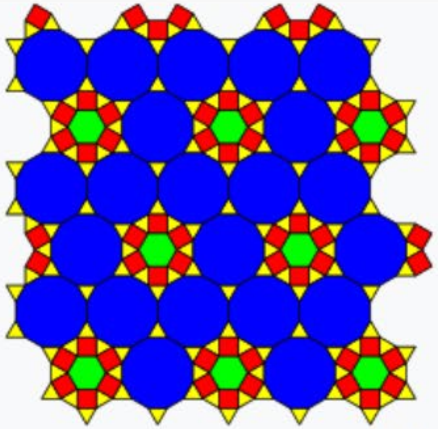
[3464; $(3446)^2$; 46.12]



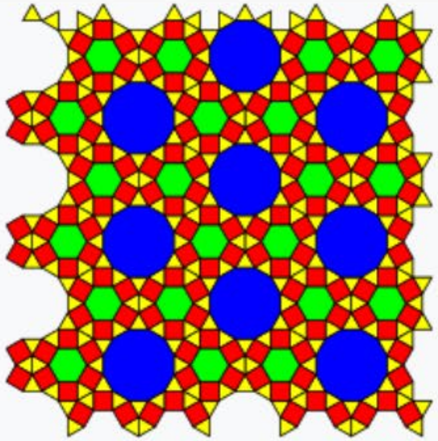
[3464; 3446; $(46.12)^2$]



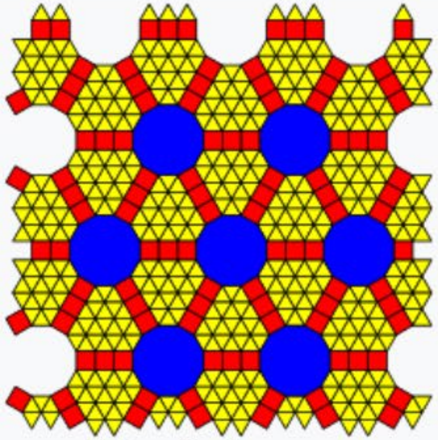
[334.12; 3464; $(3.12^2)^2$]



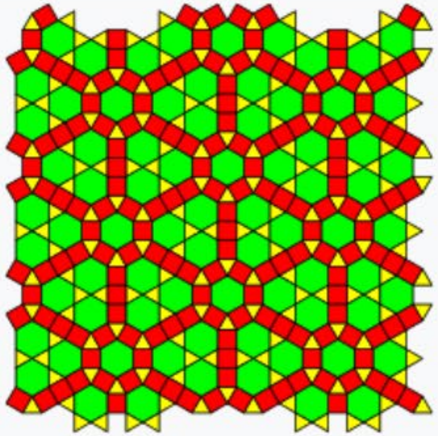
[343.12; 3464; $(3.12^2)^2$]



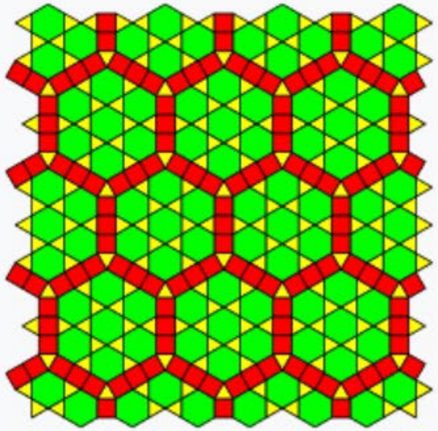
[33434; 343.12; $(3464)^2$]



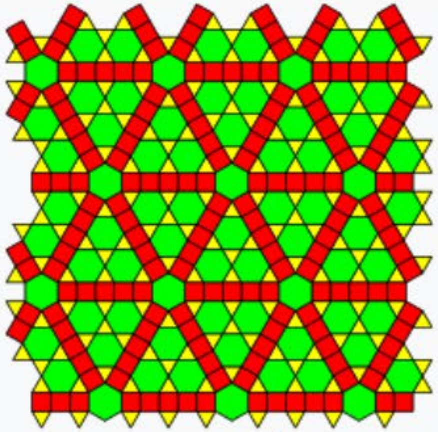
[$(3^6)^2$; $3^3 4^2$; 334.12]



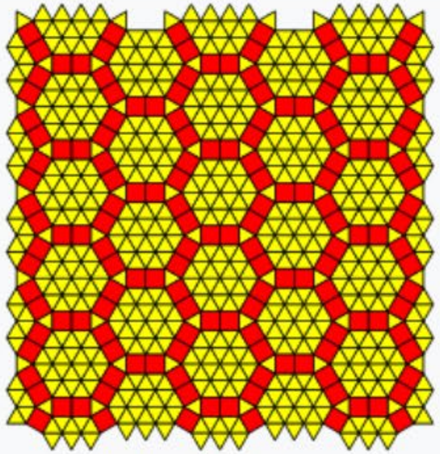
[$(3464)^2$; 3446; 3636]



[3464; 3446; $(3636)^2$]

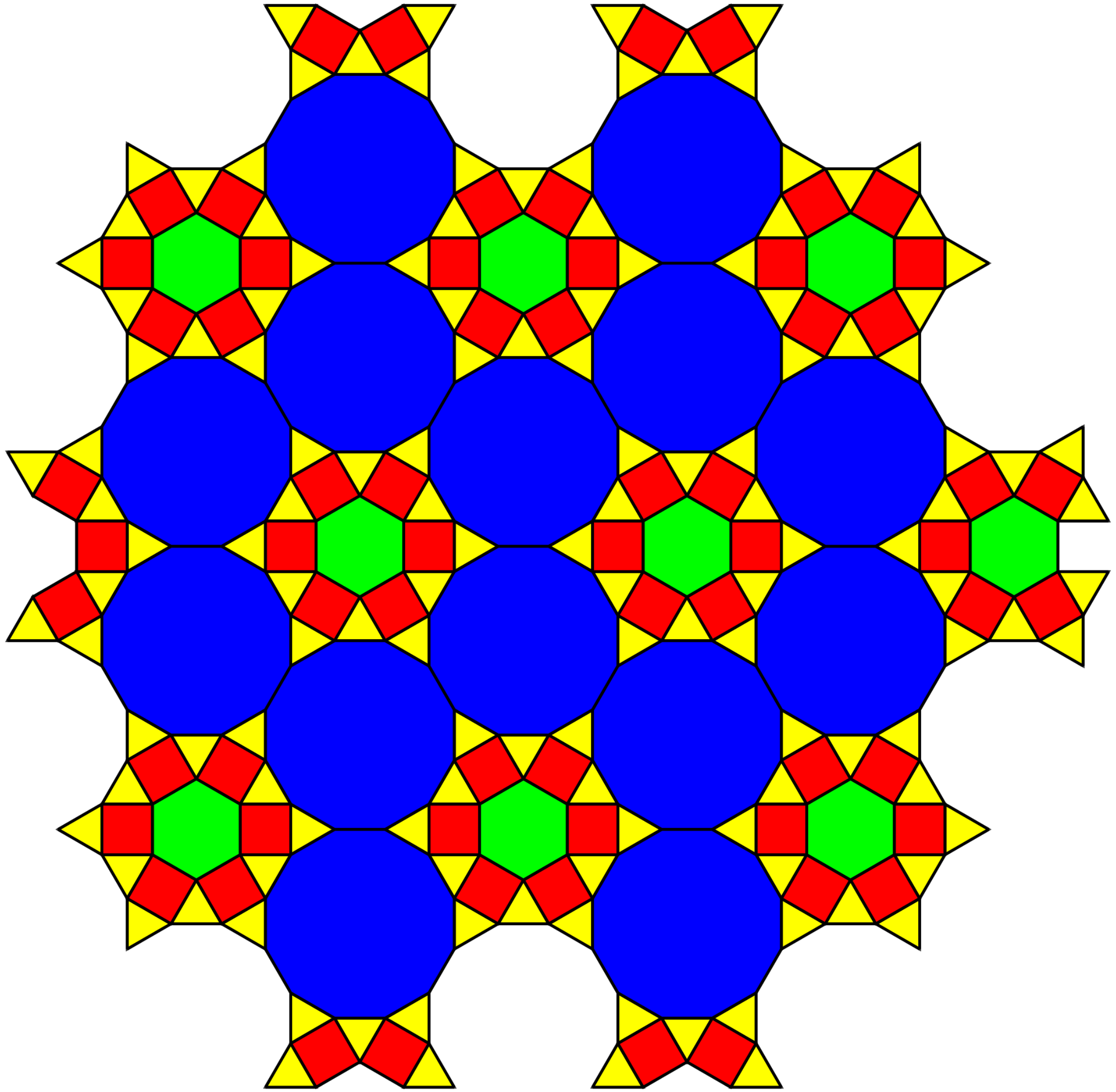


[3464; $(3446)^2$; 3636]



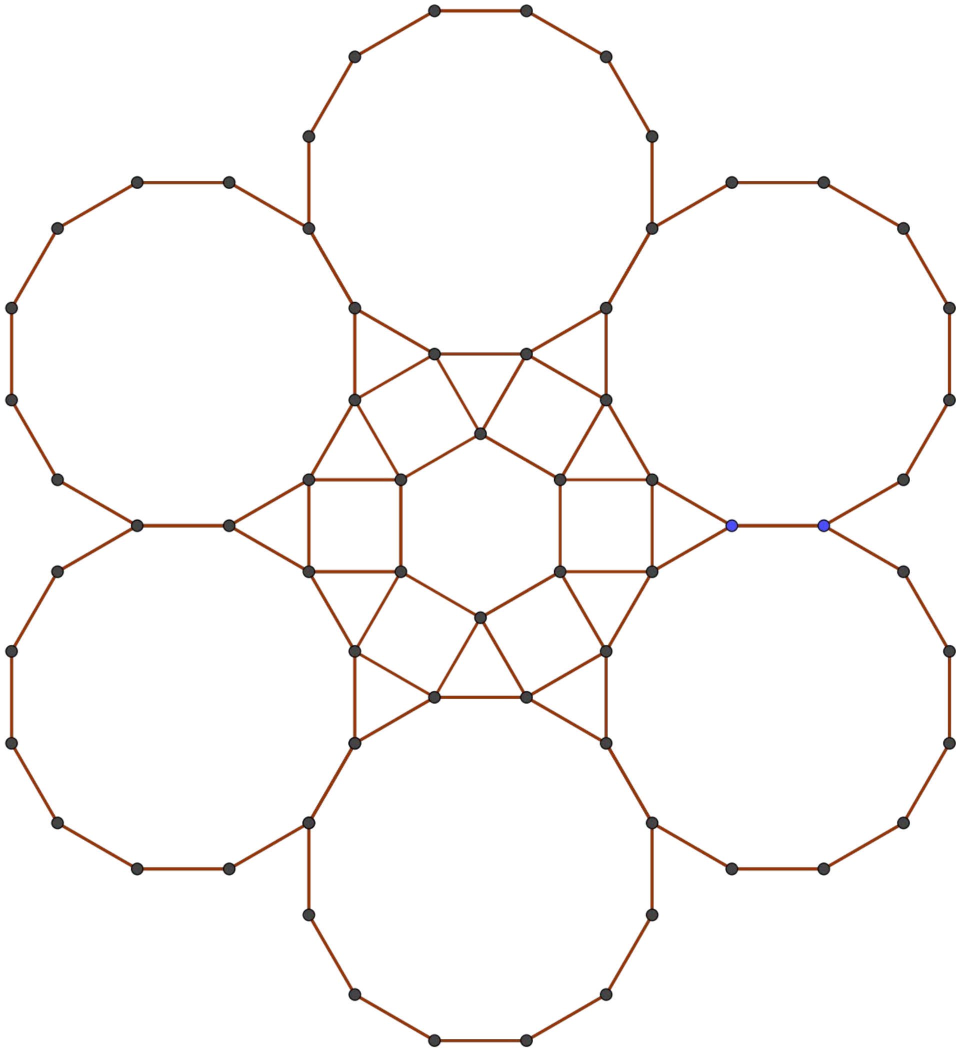
[$(3^6)^2$; $3^3 4^2$; 33434]

From Polygons to Pattern



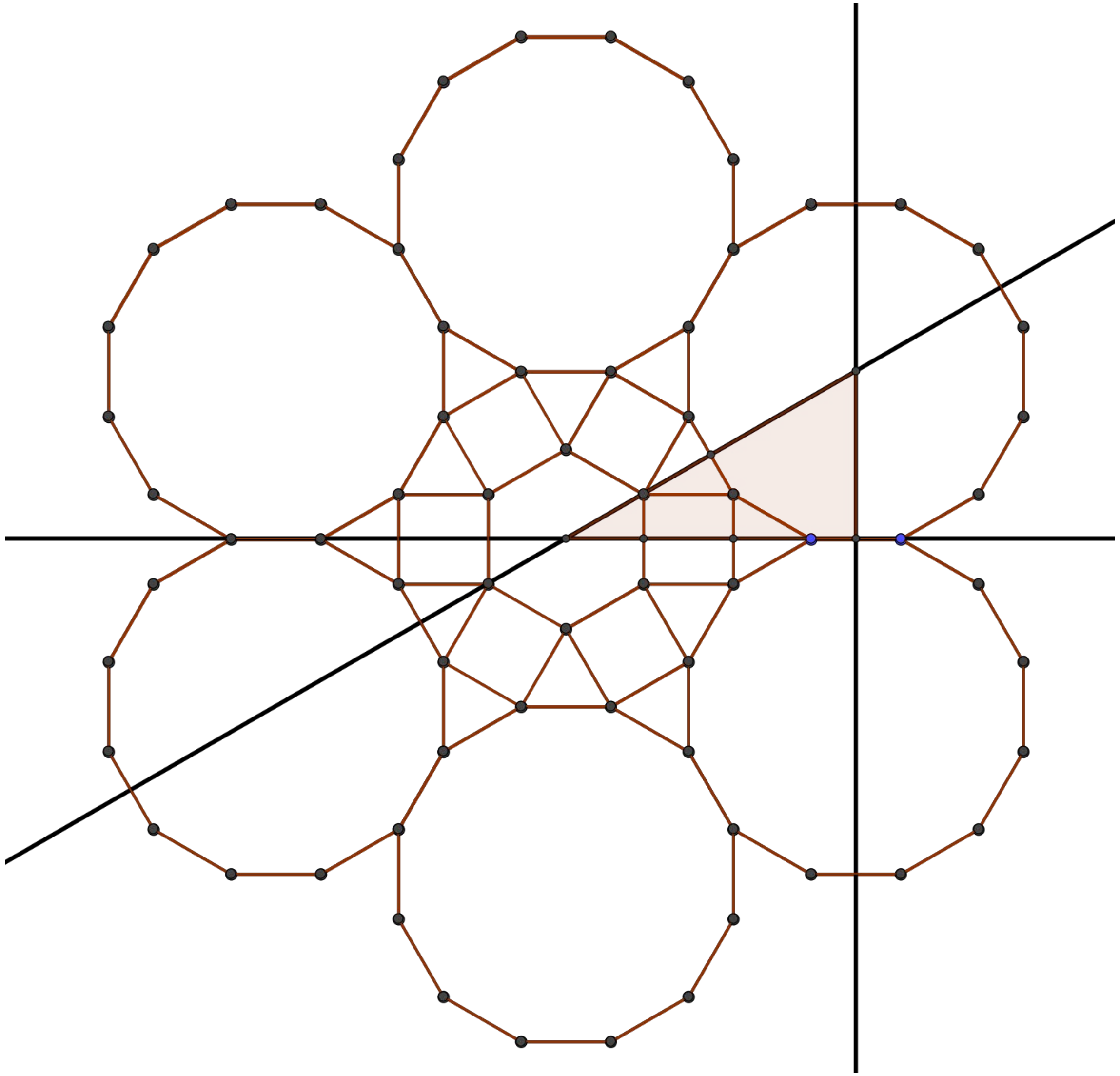
[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling of the plane.
Image by Tom Ruen

From Polygons to Pattern



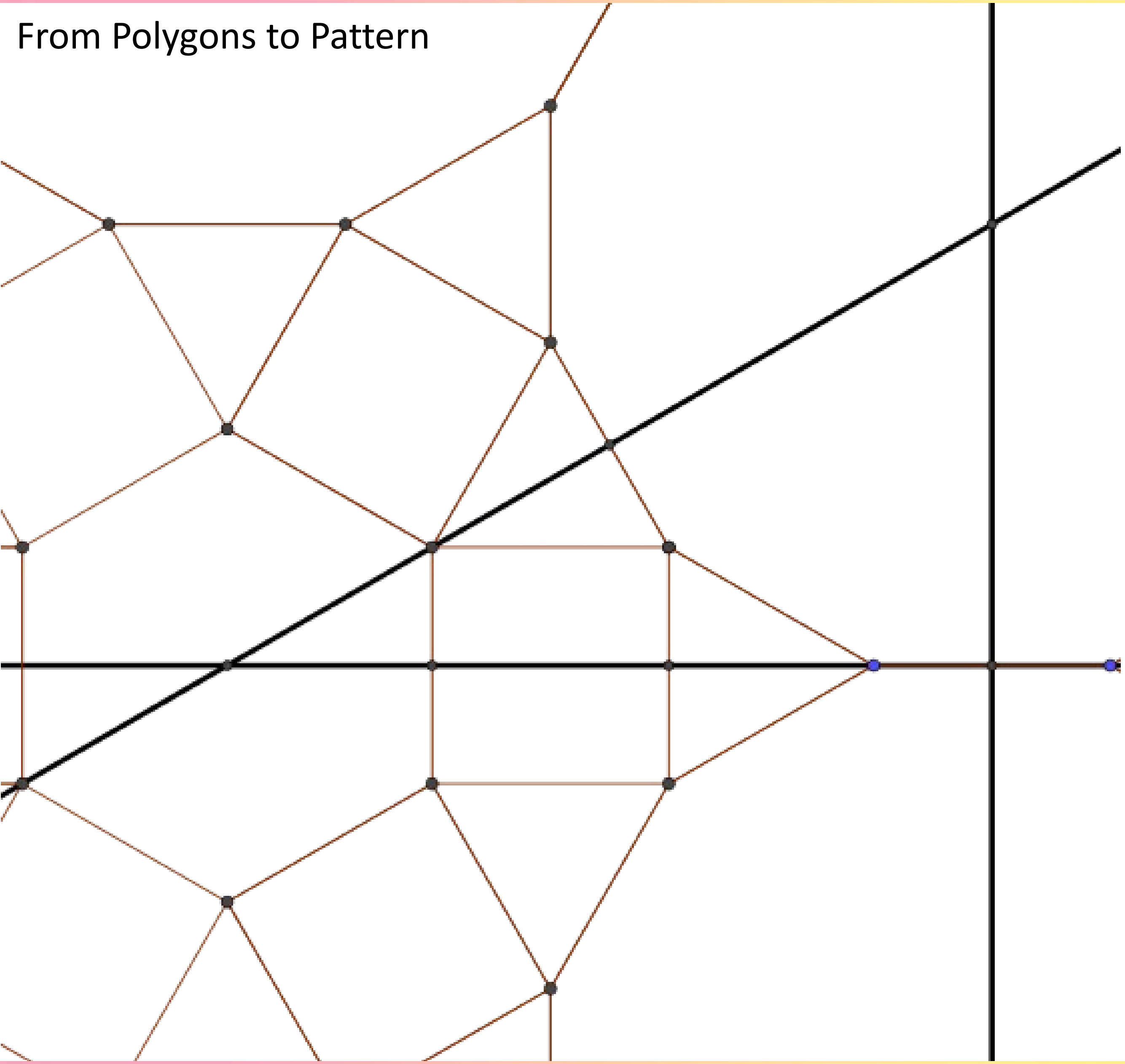
[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling
base construction in GeoGebra

**Step 1:** Pick your favorite k-uniform tiling.



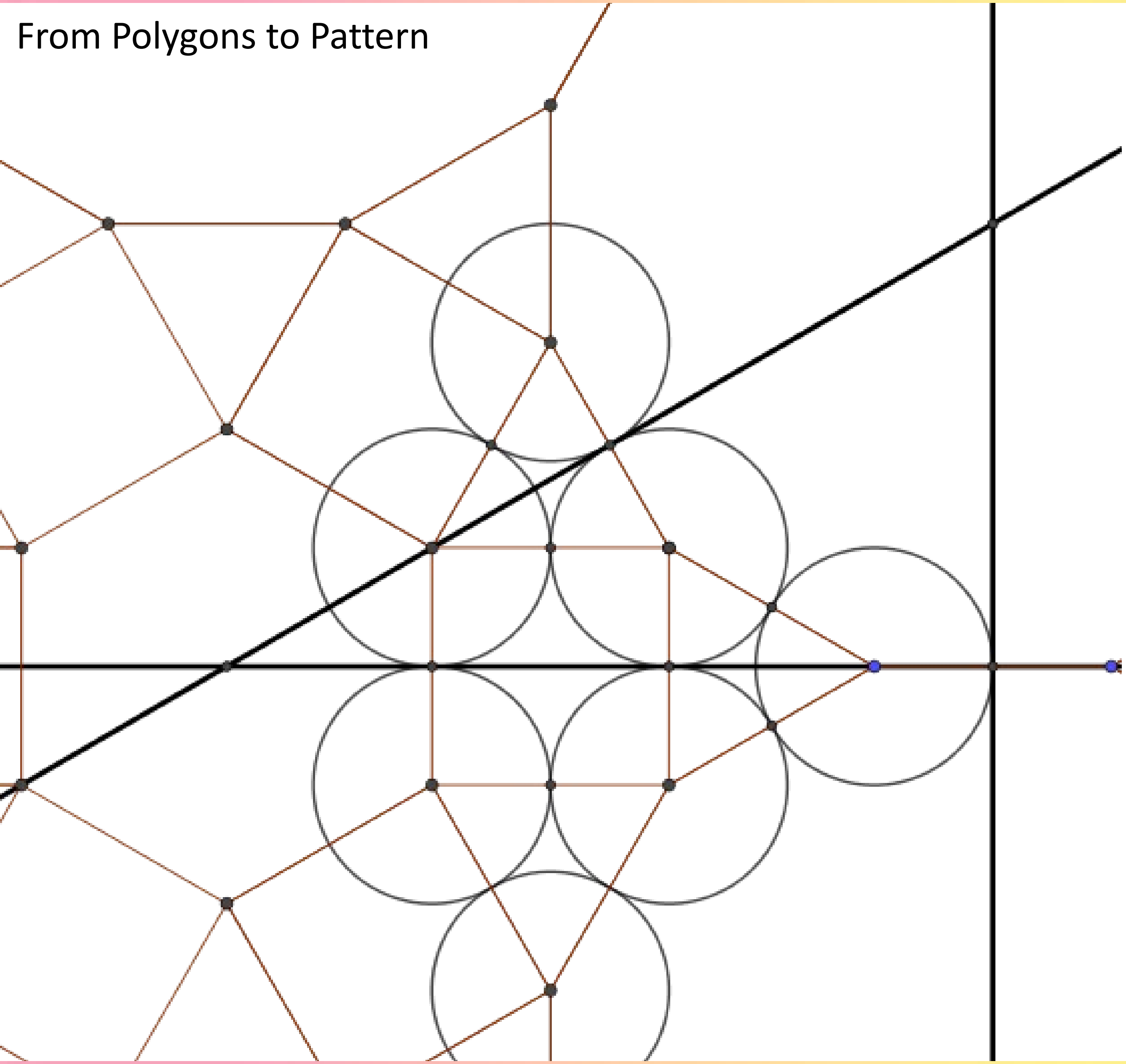
[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling base construction in GeoGebra

**Step 1:** Pick your favorite k-uniform tiling.



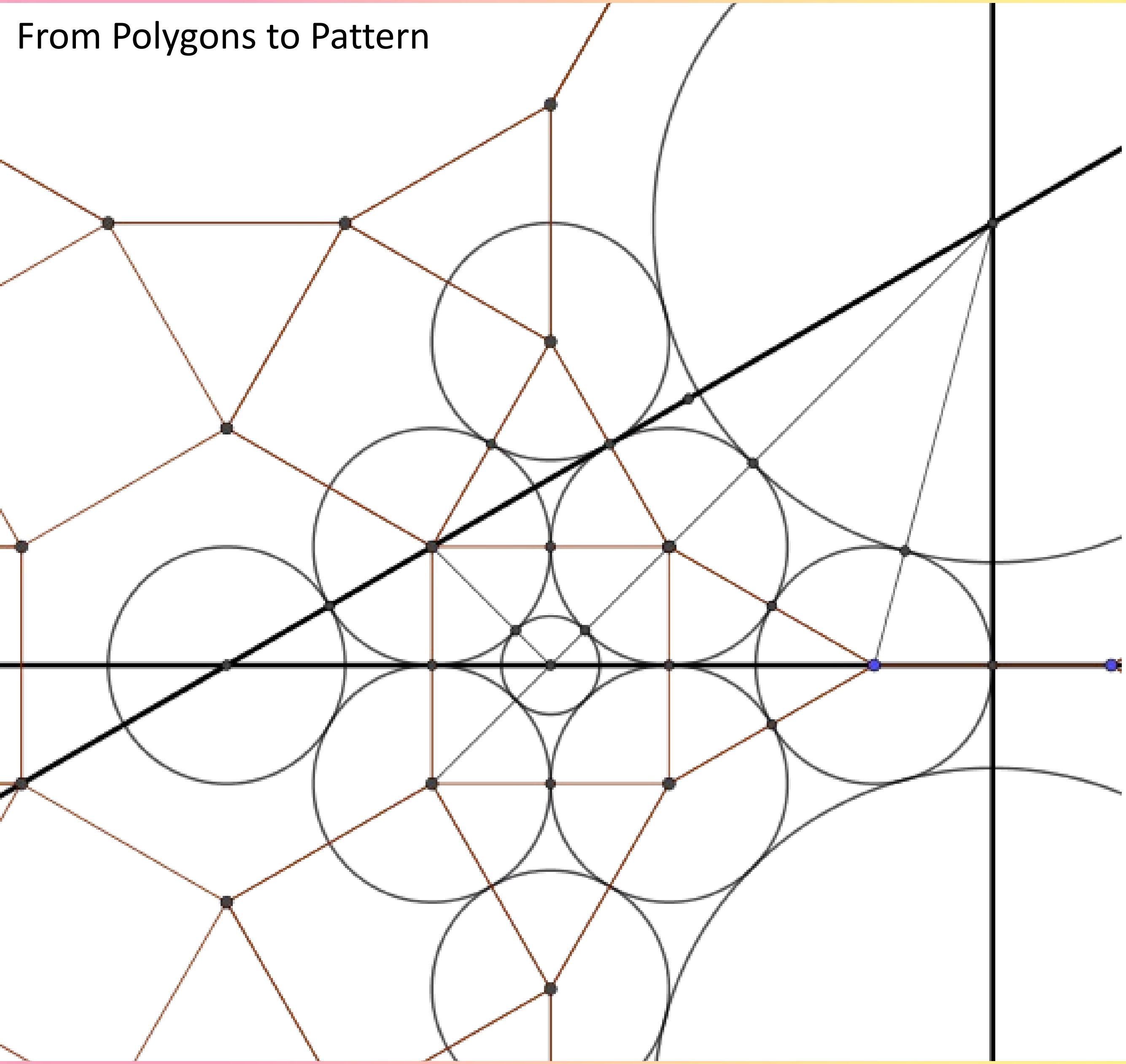
[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling
minimal triangle

**Step 2:** locate the midpoints of the tiling edges, and construct circles with centers at each vertex whose radii are half the edge length



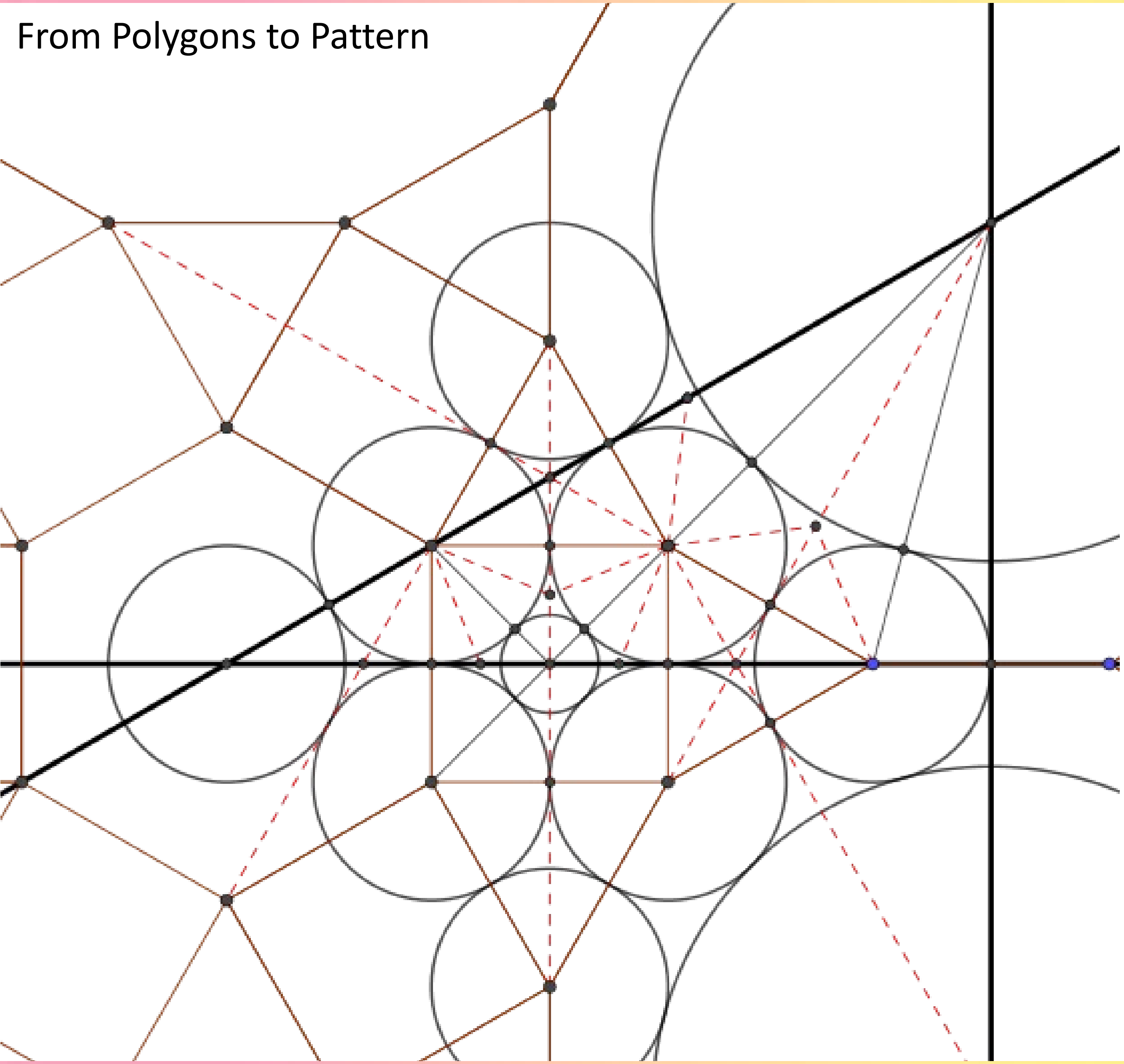
[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling
minimal triangle with vertex circles

**Step 2:** locate the midpoints of the tiling edges, and construct circles with centers at each vertex whose radii are half the edge length



[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling
minimal triangle with circle packing
and radii of tangency

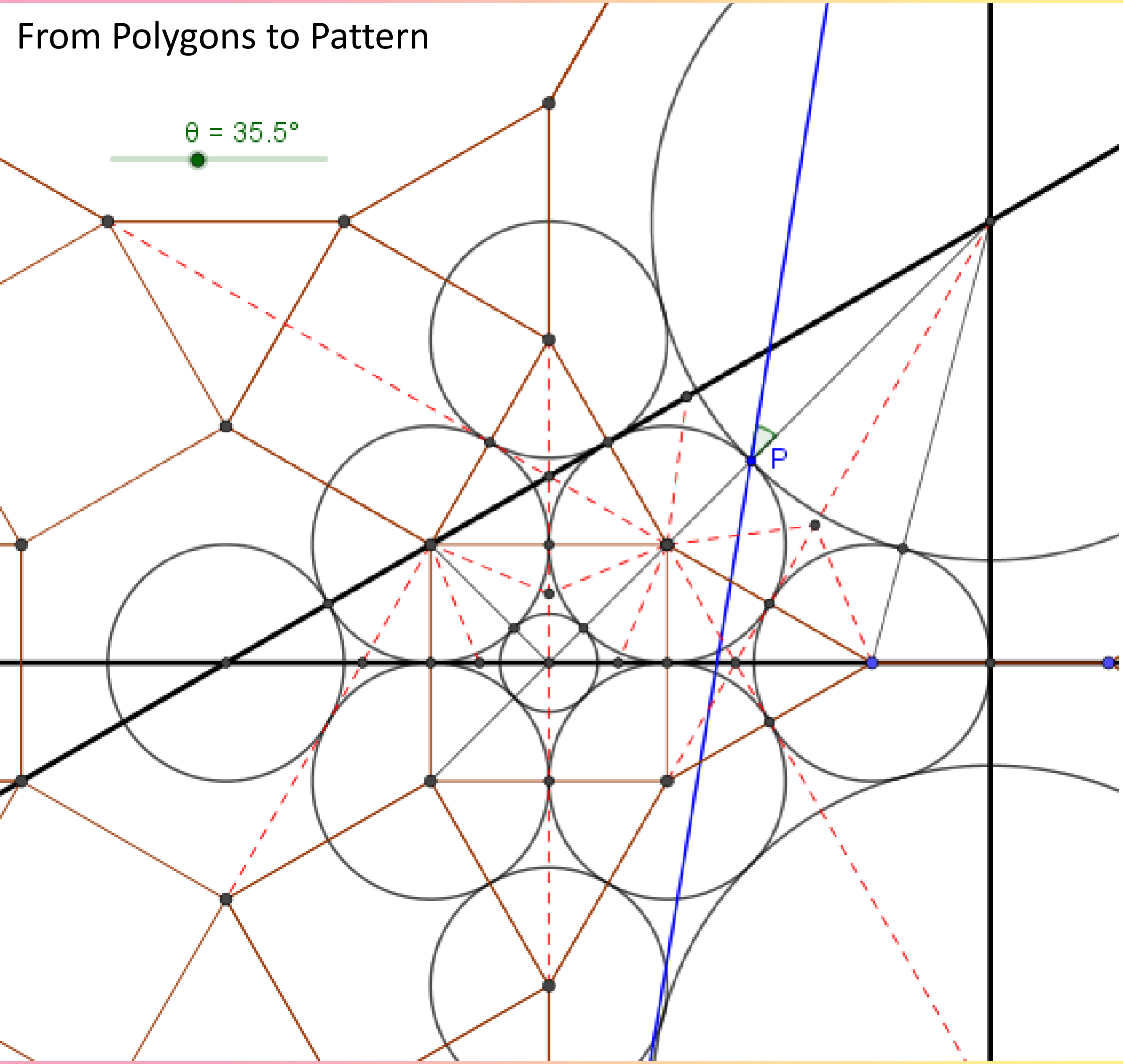
**Step 3:** complete the circle packing by filling in a circle in each non-triangular tile



[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling
minimal triangle with circle packing,
radii of tangency, and angle bisectors

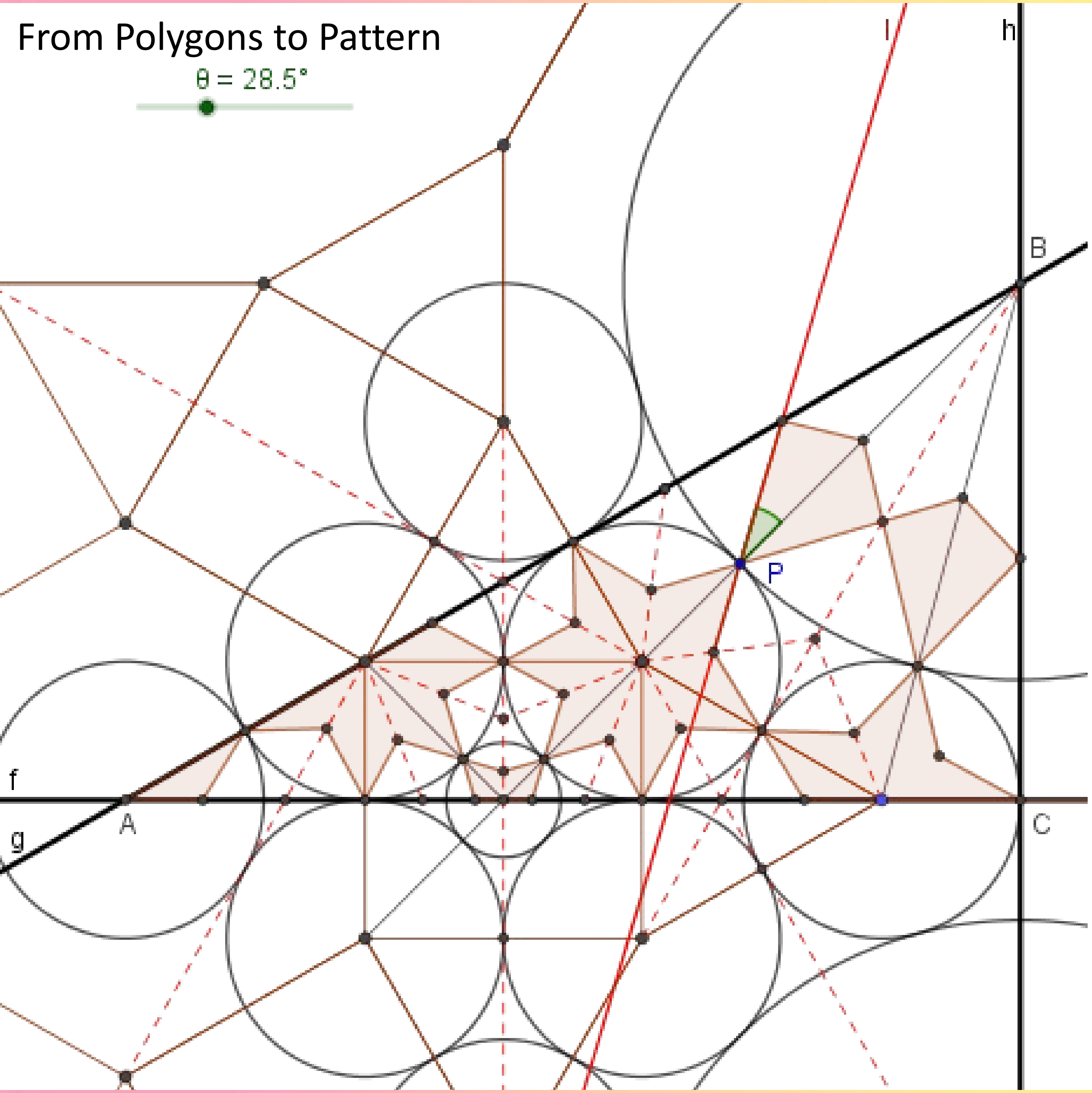
**Step 4:** construct the angle bisectors at each circle center between each adjacent pair of radii of tangency

From Polygons to Pattern




[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling minimal triangle with circle packing, radii of tangency, angle bisectors, and generating line

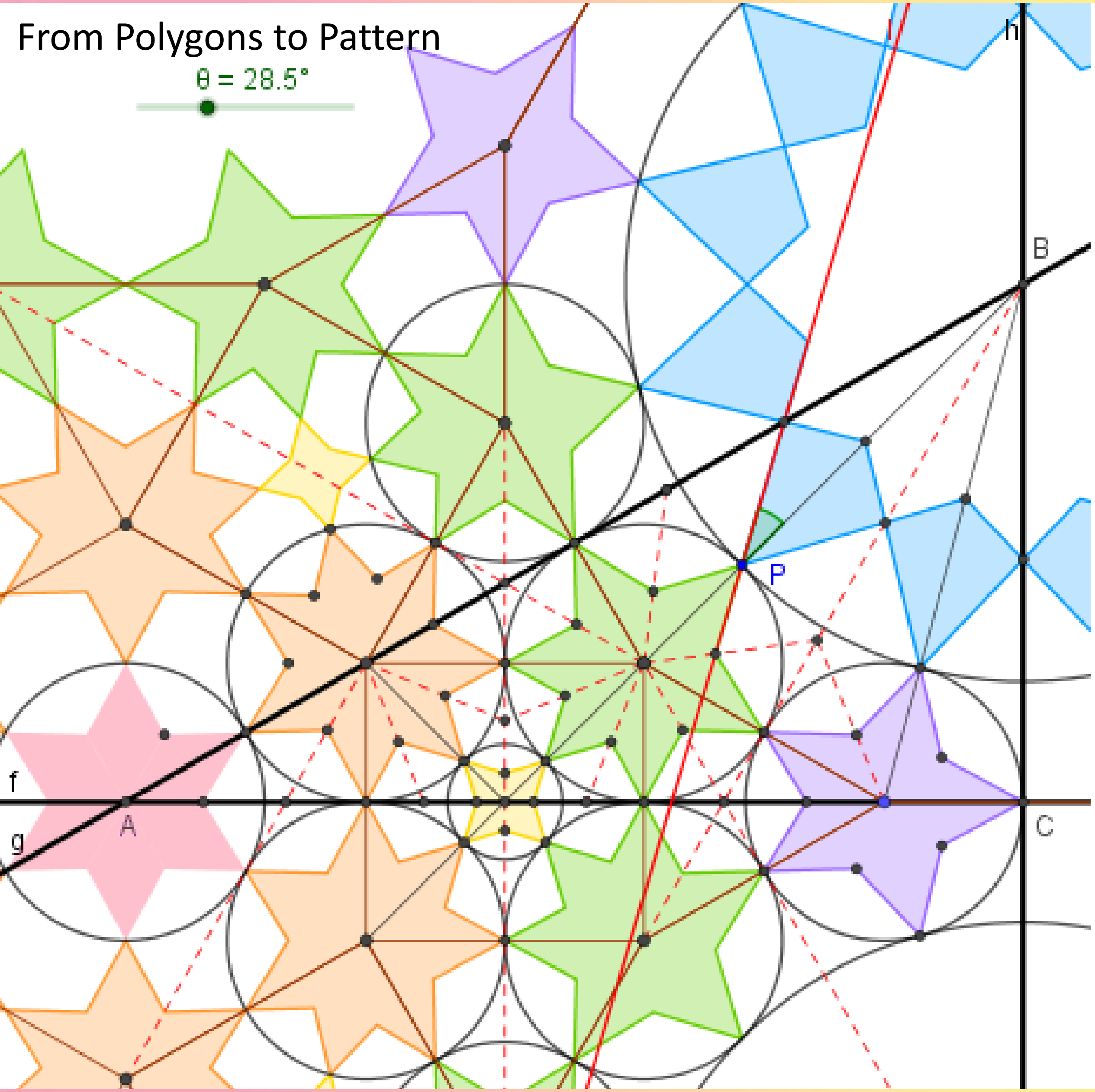
**Step 5:** Construct a line at one of the points of circle tangency at a variable angle to the radius.




[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling minimal triangle with circle packing, radii of tangency, angle bisectors, and star polygons generated by reflections of the generating line over radii of tangency and angle bisectors.

**Step 6:** Generate the star rosette pattern by alternately reflecting this line over radii of tangency and angle bisectors.

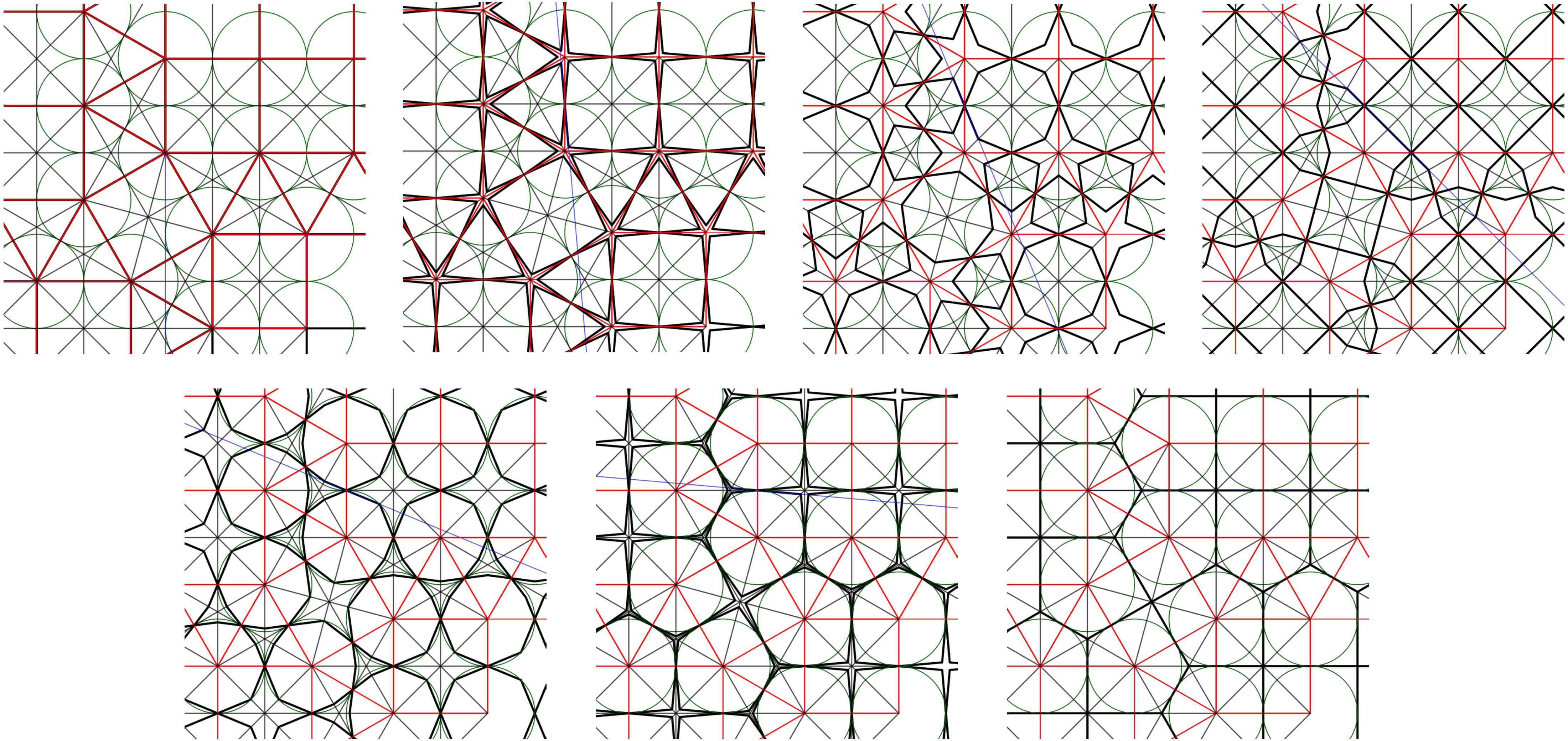
From Polygons to Pattern

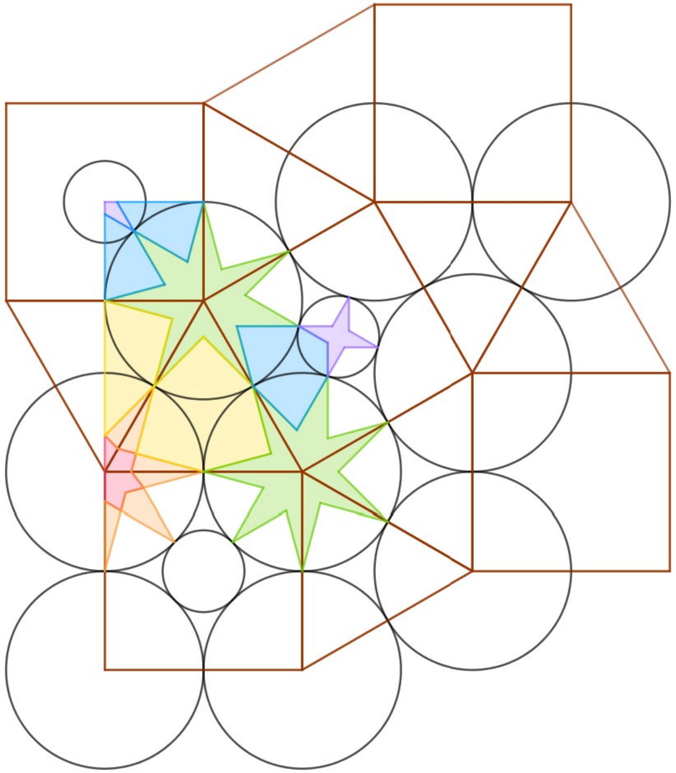


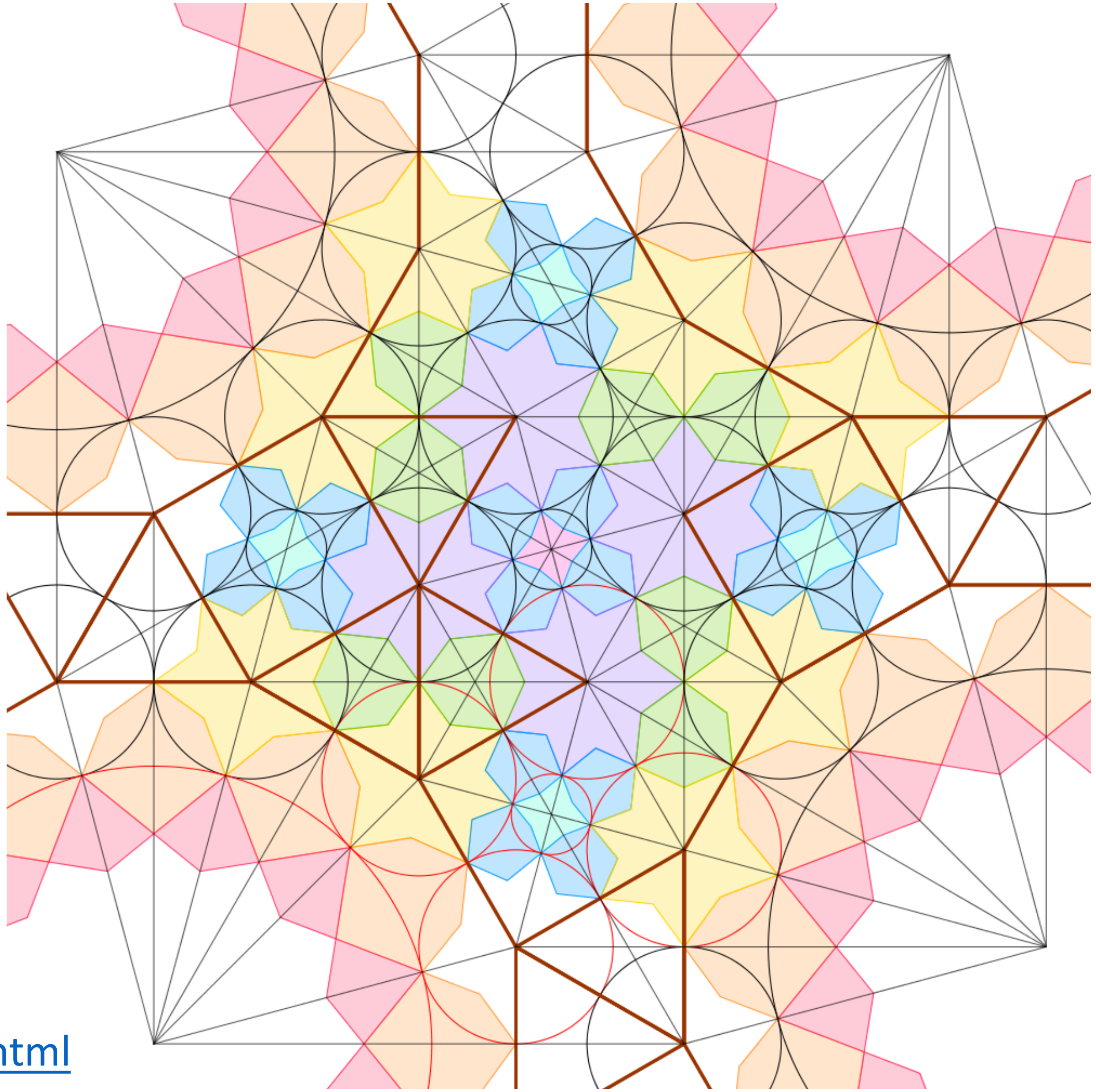

[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling minimal triangle with circle packing, radii of tangency, angle bisectors, and star polygons generated by reflections of the generating line over radii of tangency and angle bisectors.

**Step 7:** Complete the pattern by reflecting and/or rotating the motifs in the minimal triangle to your desired pattern size.

## Variable-angled Star Rosette Pattern Family as transitions from a k-uniform tiling to its dual







- Animations on Instagram @mathemartiste
- Interactive GeoGebra files: https://www.geogebra.org/u/sarahgbrewer
- Bridges paper: https://archive.bridgesmathart.org/2022/bridges2022-391.html

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