

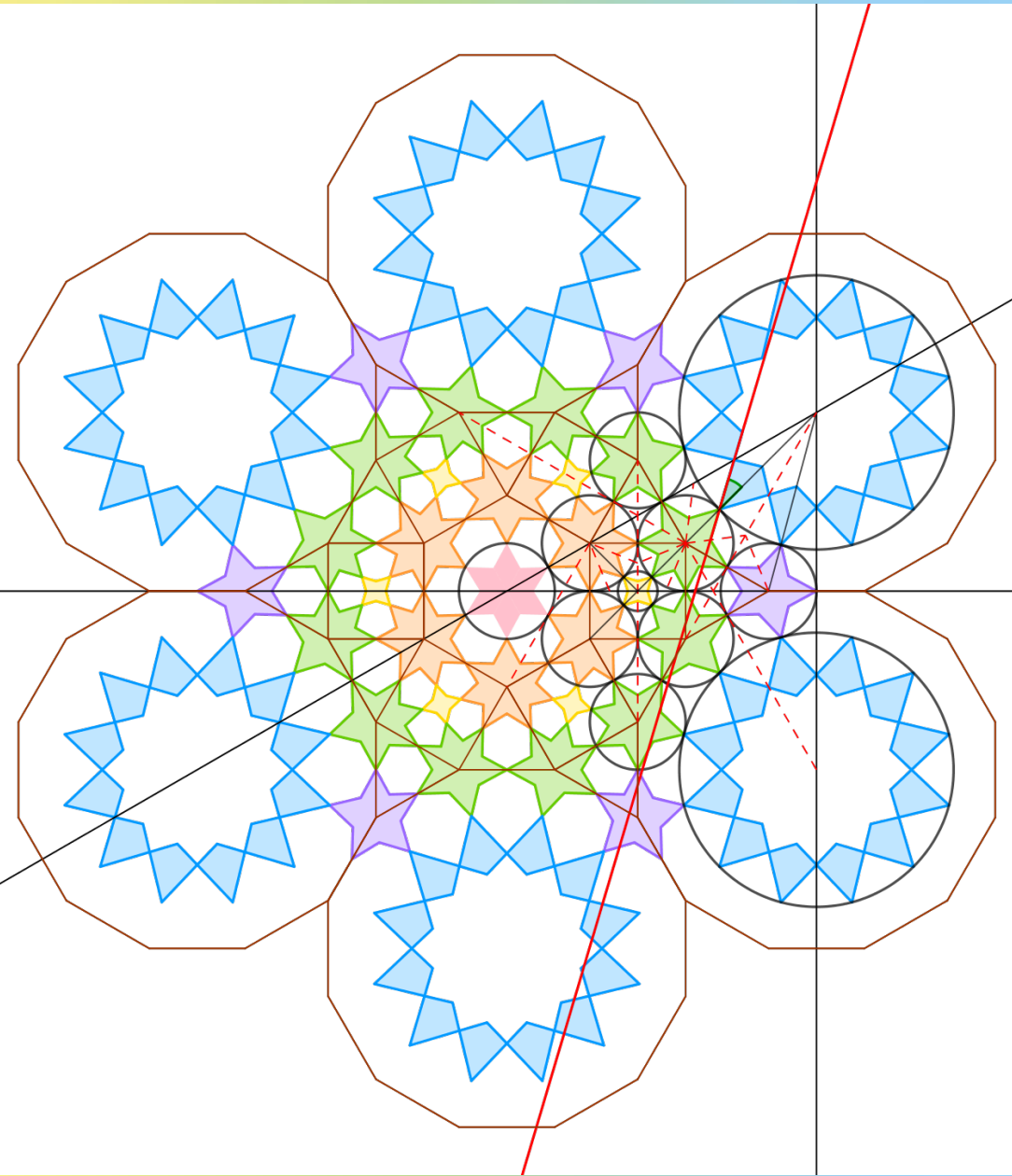
# Star rosettes in GeoGebra

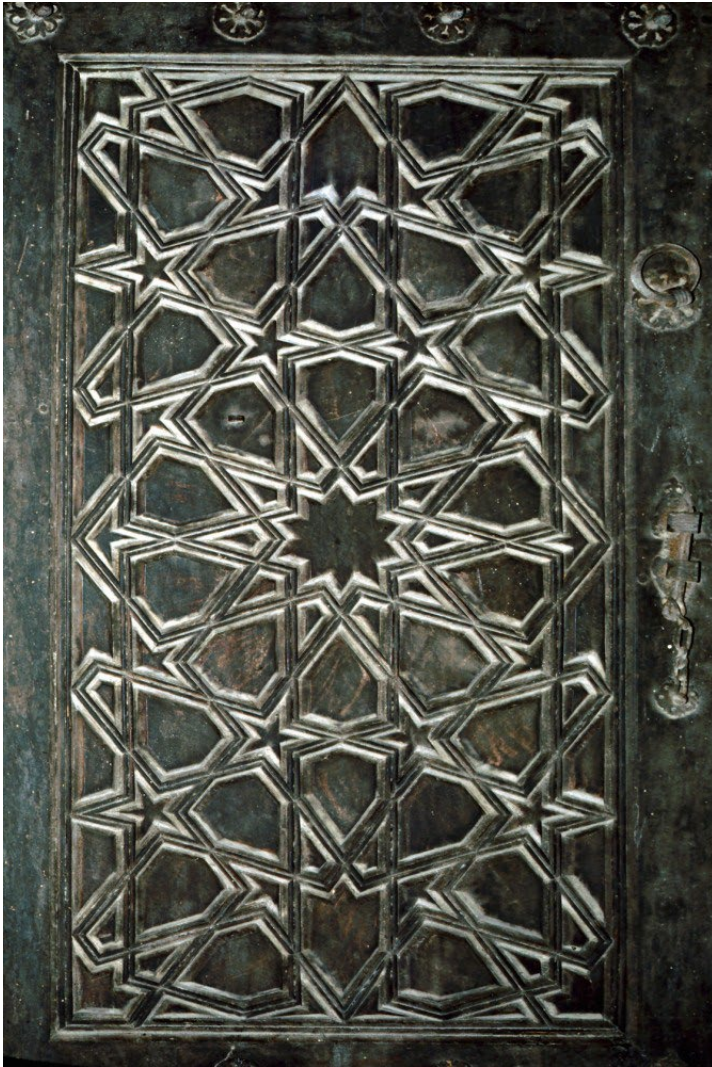
constructing traditional patterns  
with contemporary technologies

Sarah Brewer

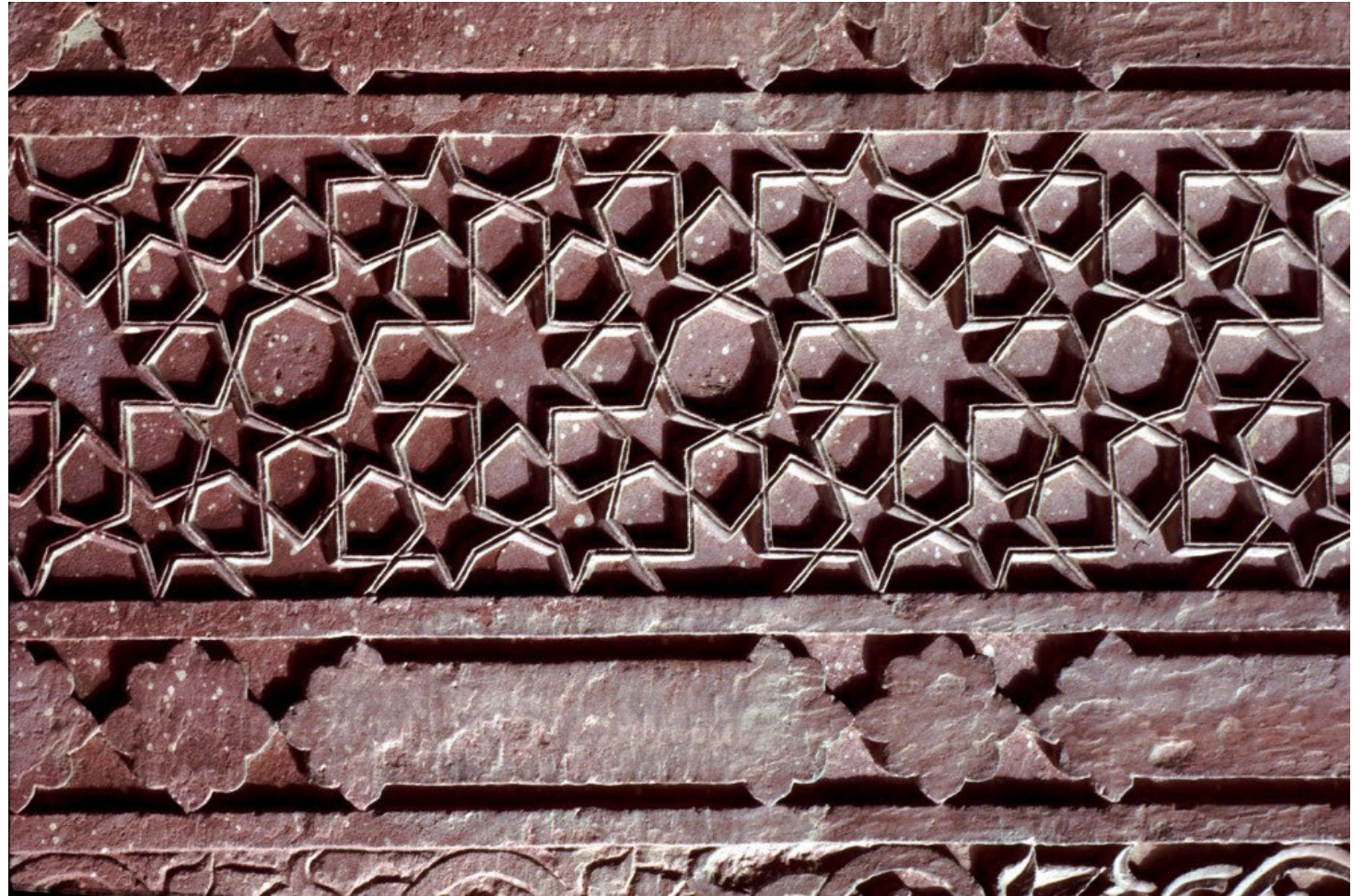
University of Toronto Undergraduate Seminar  
26 October 2022

reworked from a talk “Generating Families of  
Islamic Star Rosette Patterns Based on  $k$ -Uniform  
Tilings” given at Bridges Aalto, 01 August 2022

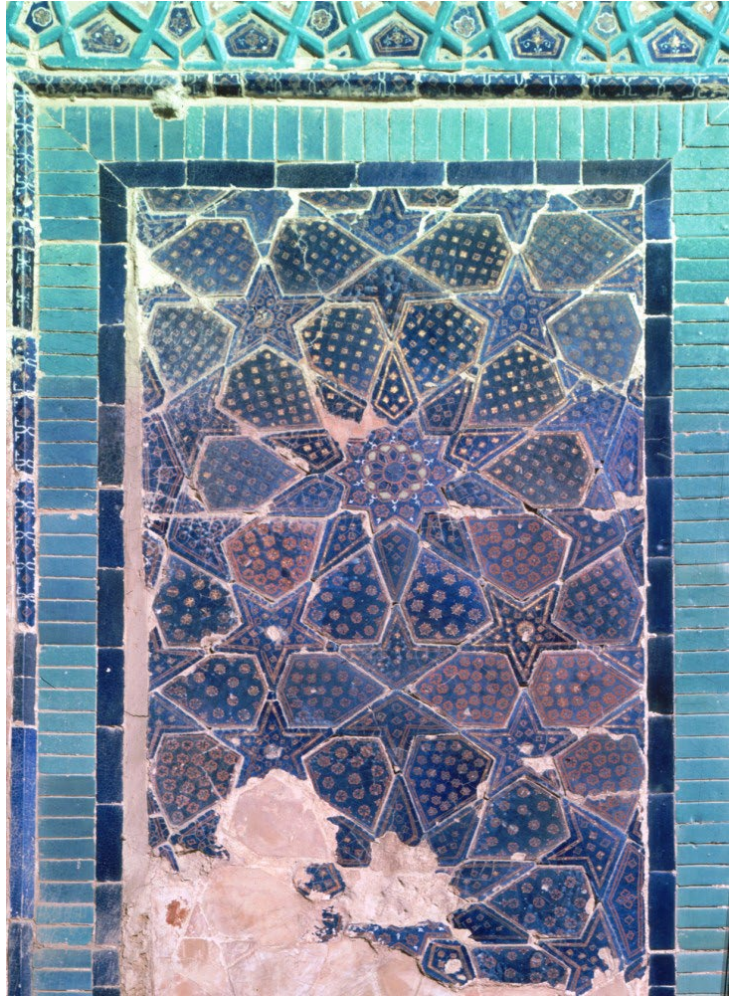




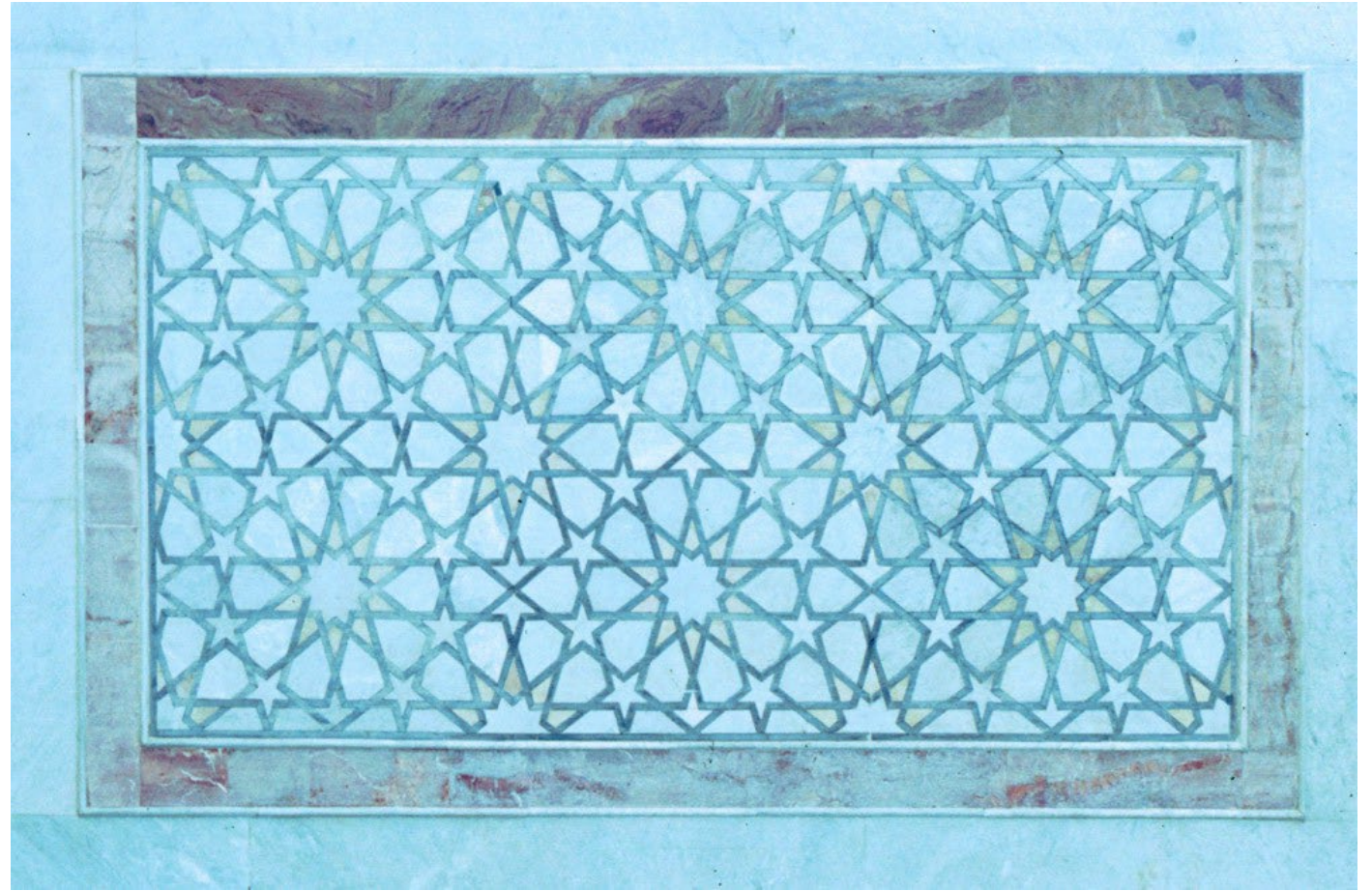
Wooden Door, Eski Mosque, Edirne, Turkey.  
Wade Photo Archive [TUR 0124](#).



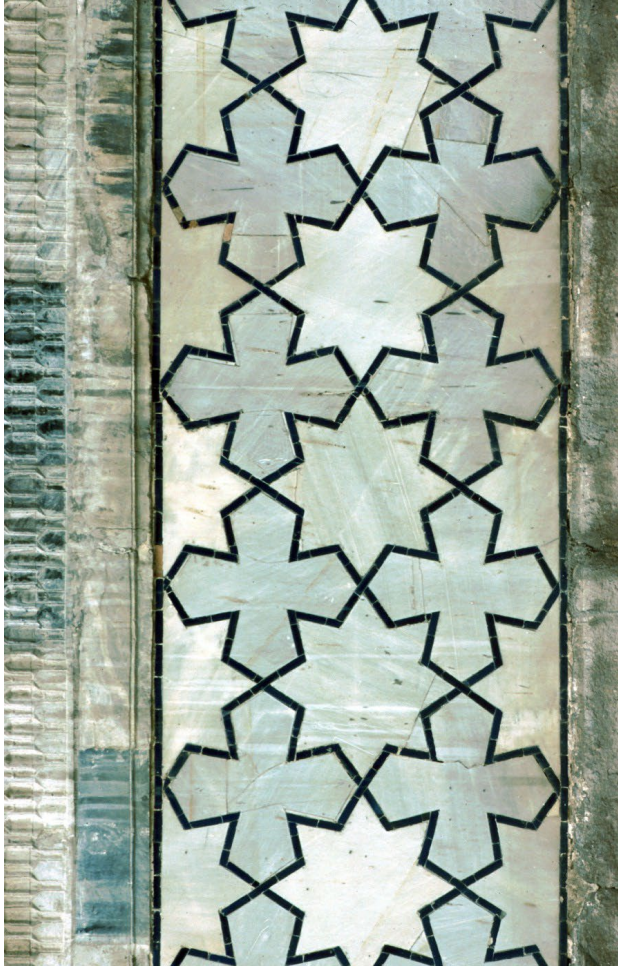
Stone Panel, Agra Fort, Agra, India.  
Wade Photo Archive [IND 0326](#).



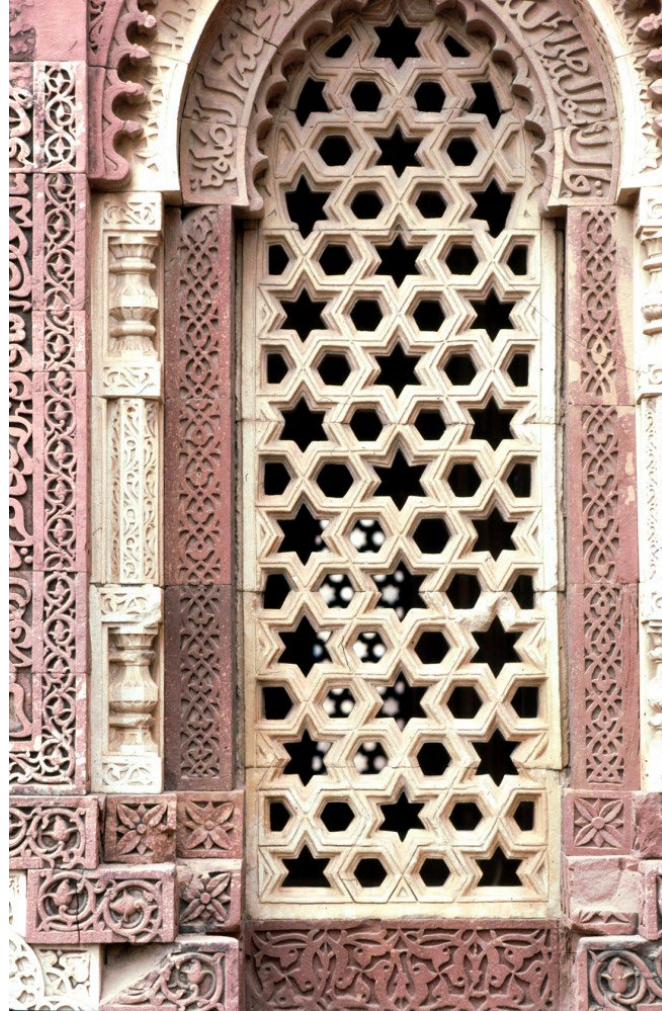
Ceramic Panel, Shakh-i-Zindeh complex,  
Samarkand, Uzbekistan.  
Wade Photo Archive [TRA 0205](#).



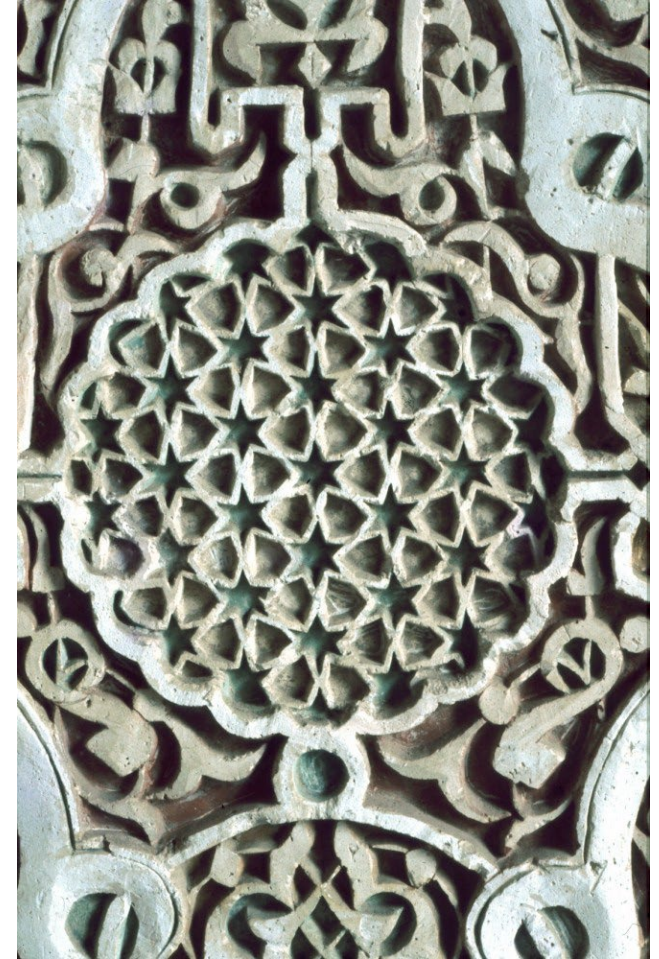
Stone Inlay Panel, Great Mosque, Damascus, Syria.  
Wade Photo Archive [SYR 0126](#).



Stone Inlay Panel, Bibi Khanum,  
Samarkand, Uzbekistan.  
Wade Photo Archive [TRA 0305](#).



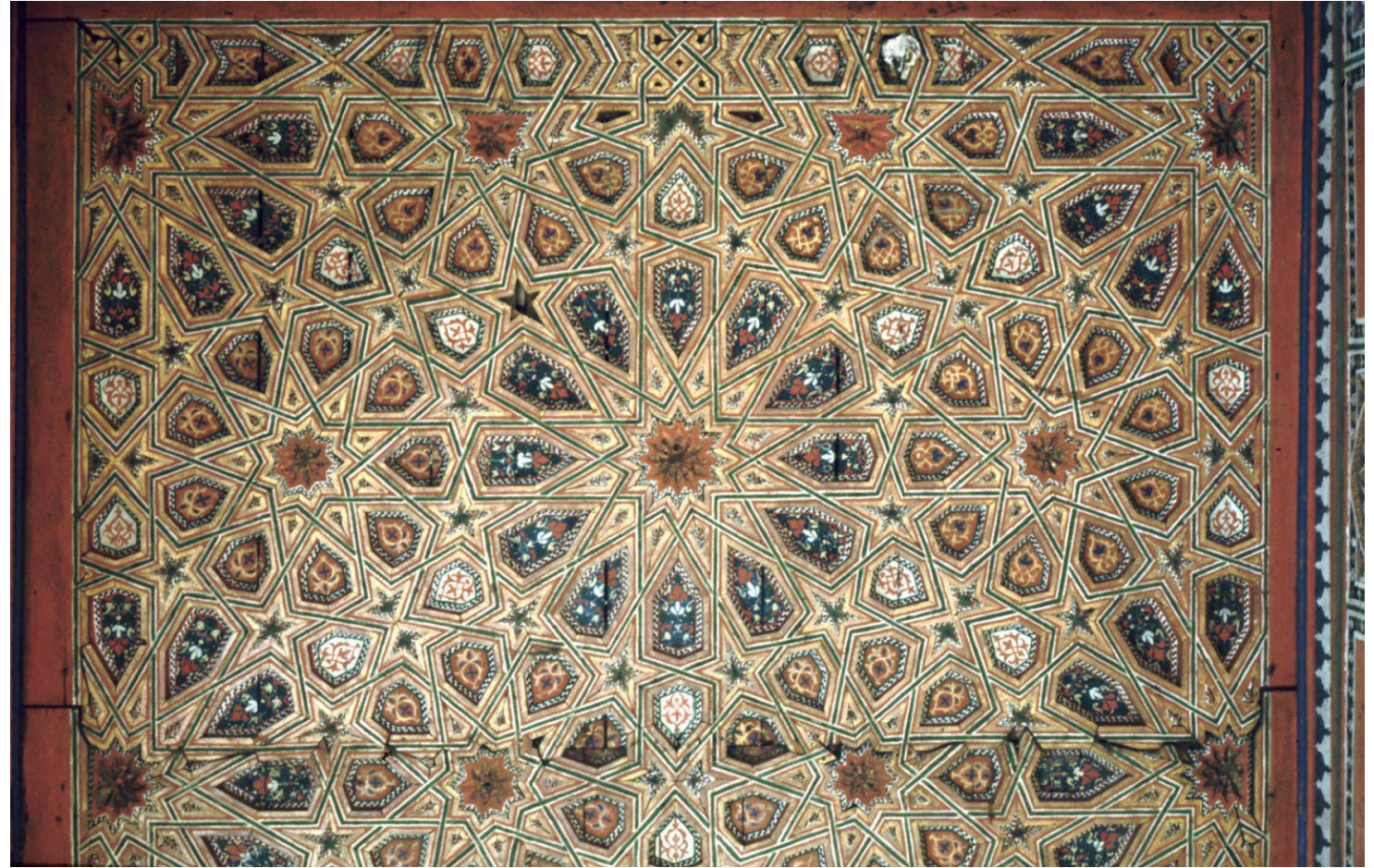
Stone Screen, Alai Darwaza, Delhi, India.  
Wade Photo Archive [IND 0106](#).



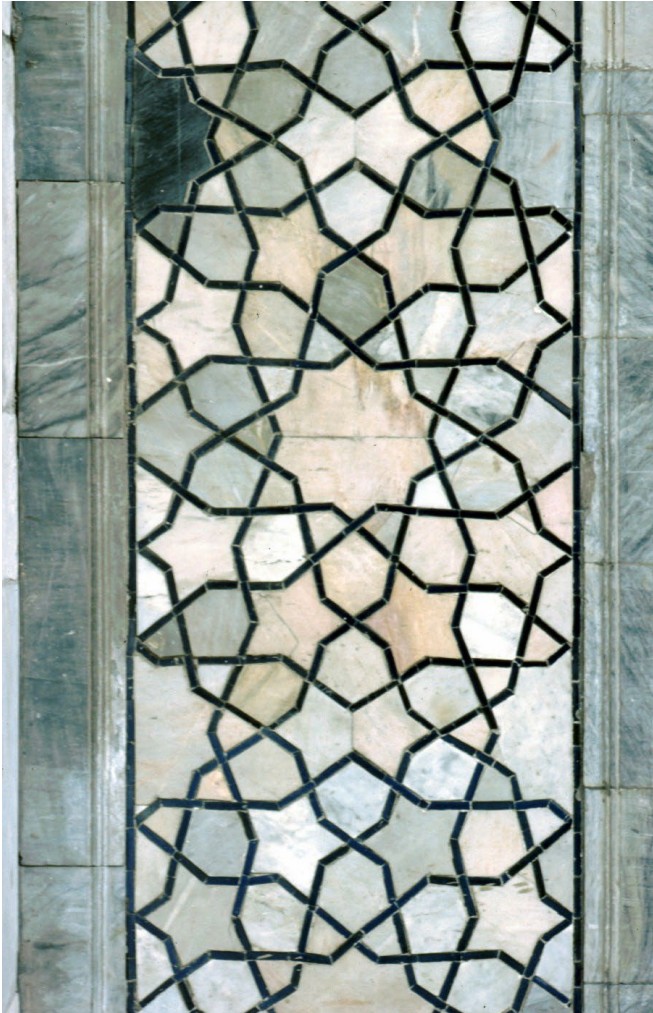
Stucco Fountain, Sultan's Palace,  
Tangier, Morocco.  
Wade Photo Archive [MOR 1508](#).



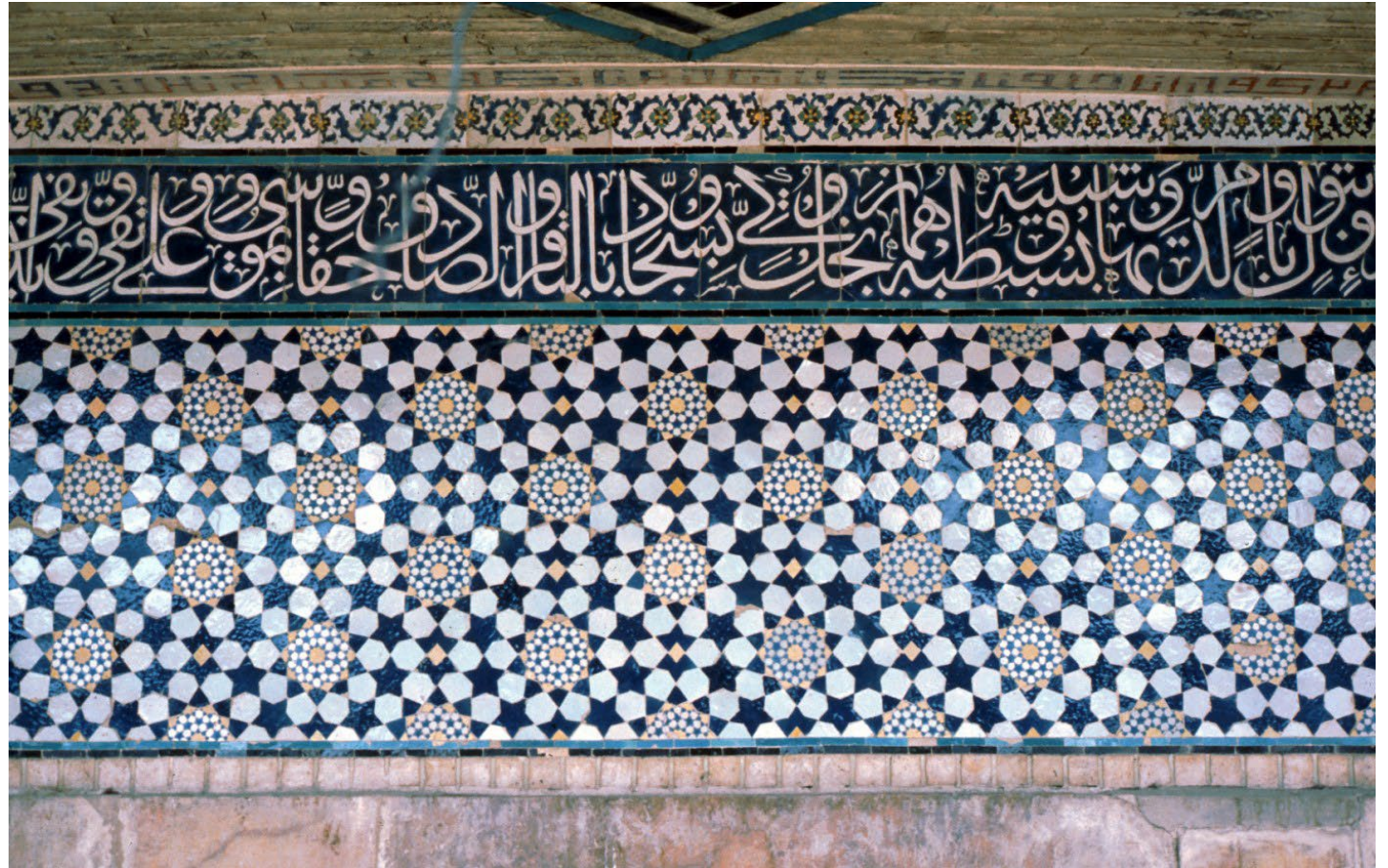
Carved Stone Panel, Altun Bogha Mosque,  
Aleppo, Syria.  
Wade Photo Archive [SYR 0426](#).



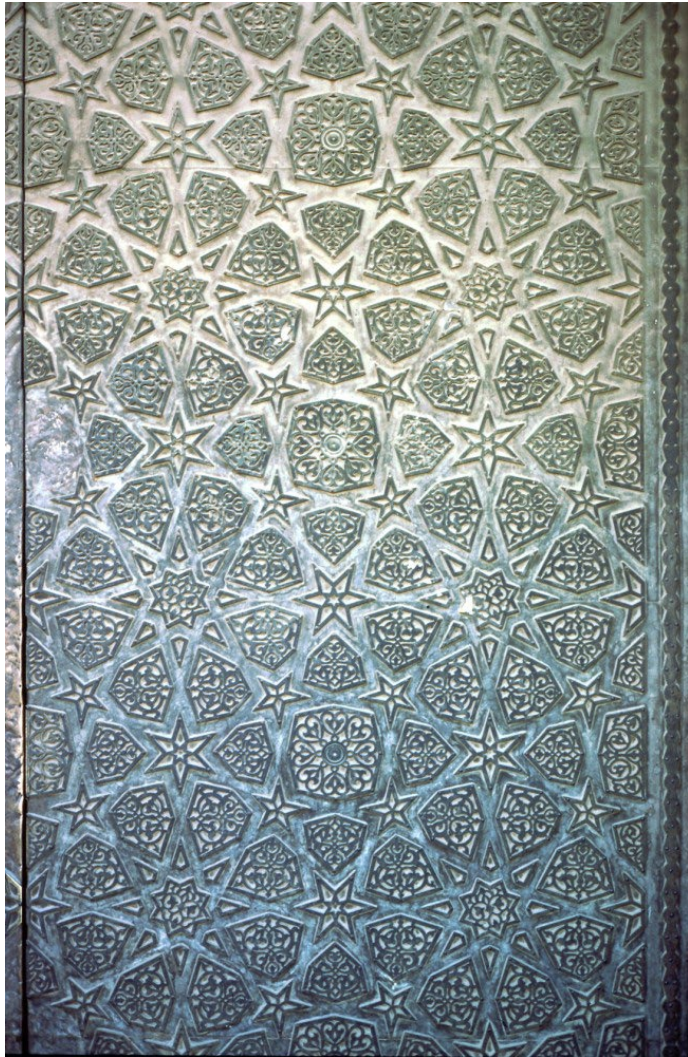
Wooden Ceiling, Tomb of Moulay Ishmael, Meknes, Morocco.  
Wade Photo Archive [MOR 0833](#).



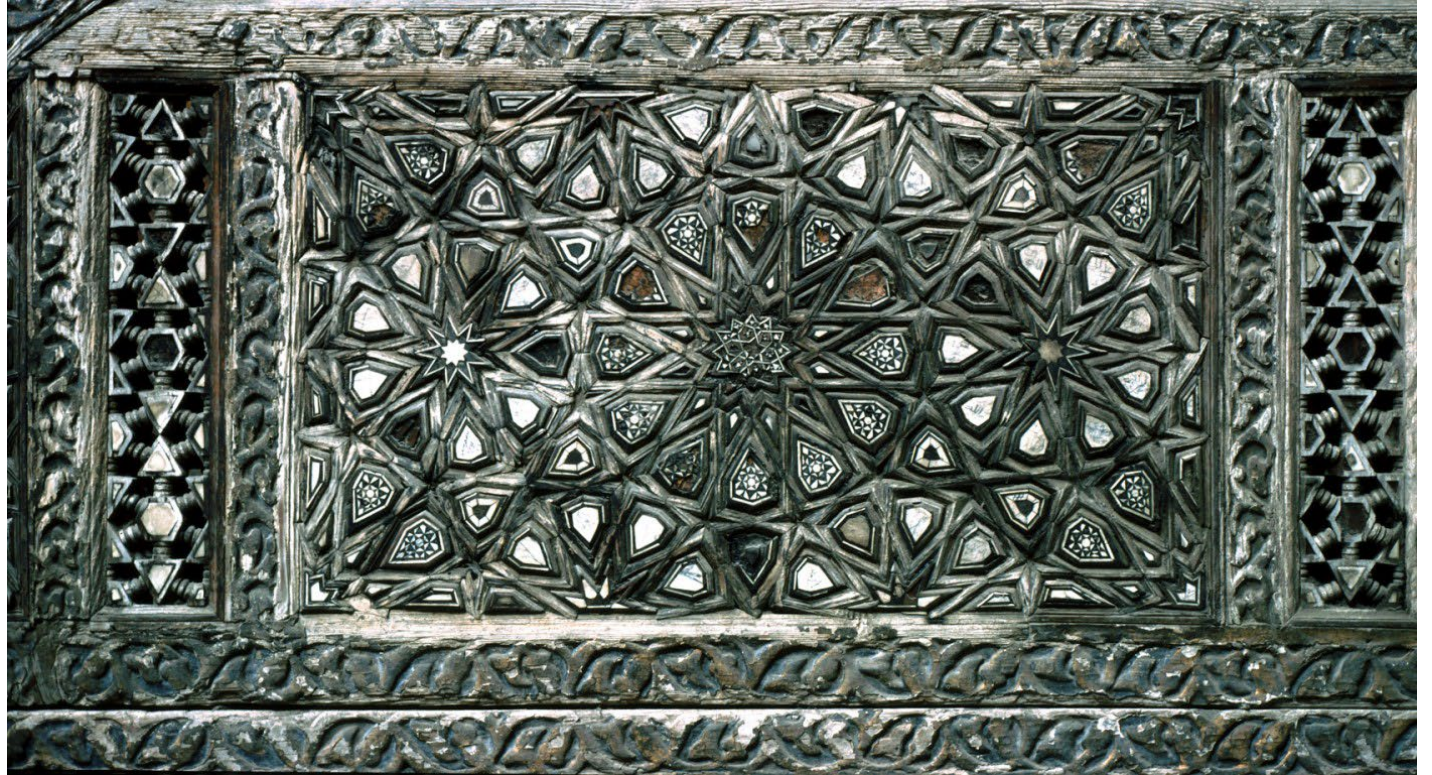
Stone Inlay Panel, Shirdar Madrassa,  
Samarkand, Uzbekistan.  
Wade Photo Archive [TRA 0335](#).



Ceramic Panel, Masjid-i-Jami, Isfahan, Iran.  
Wade Photo Archive [IRA 0517](#).

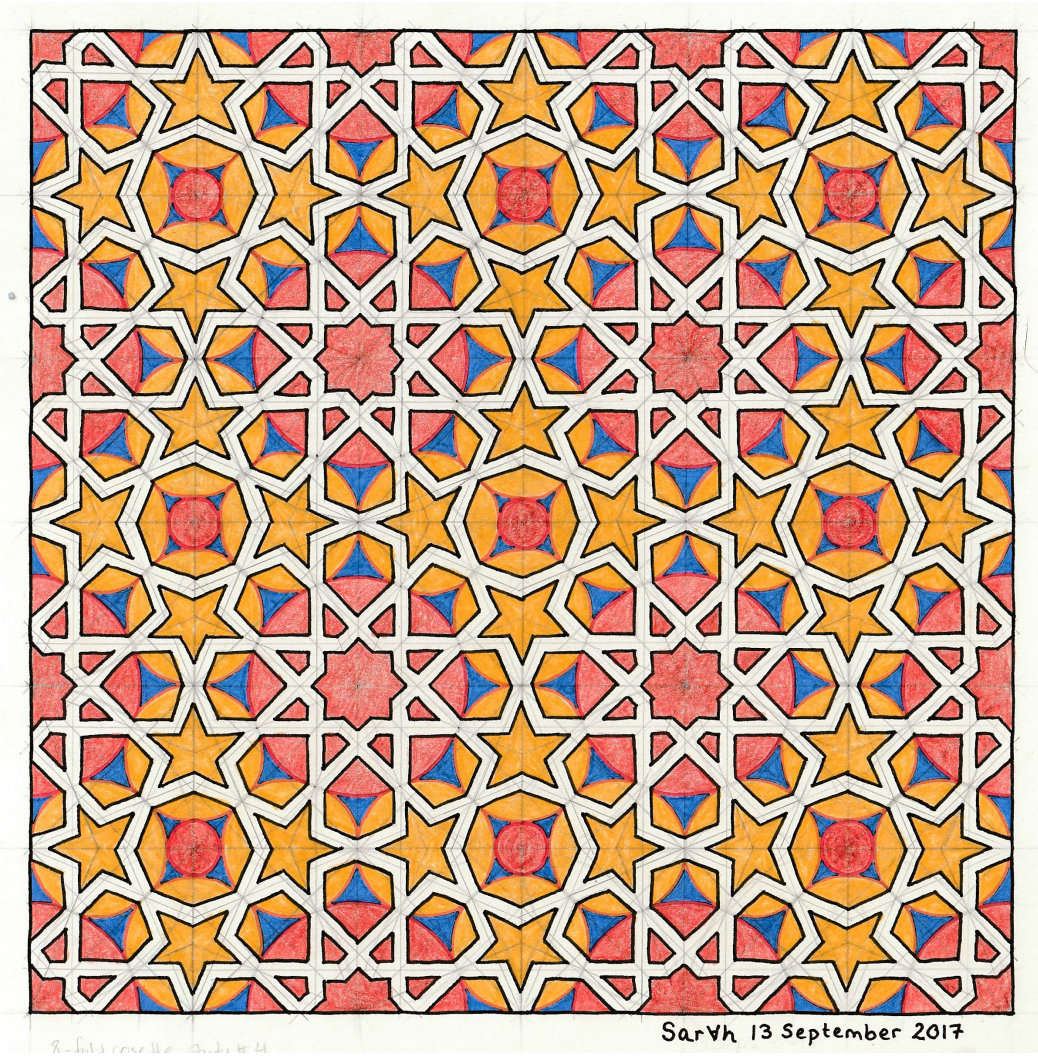


Metalwork Door, Mosque of al-Mu'ayyad, Cairo, Egypt.  
Wade Photo Archive [EGY 1514](#).

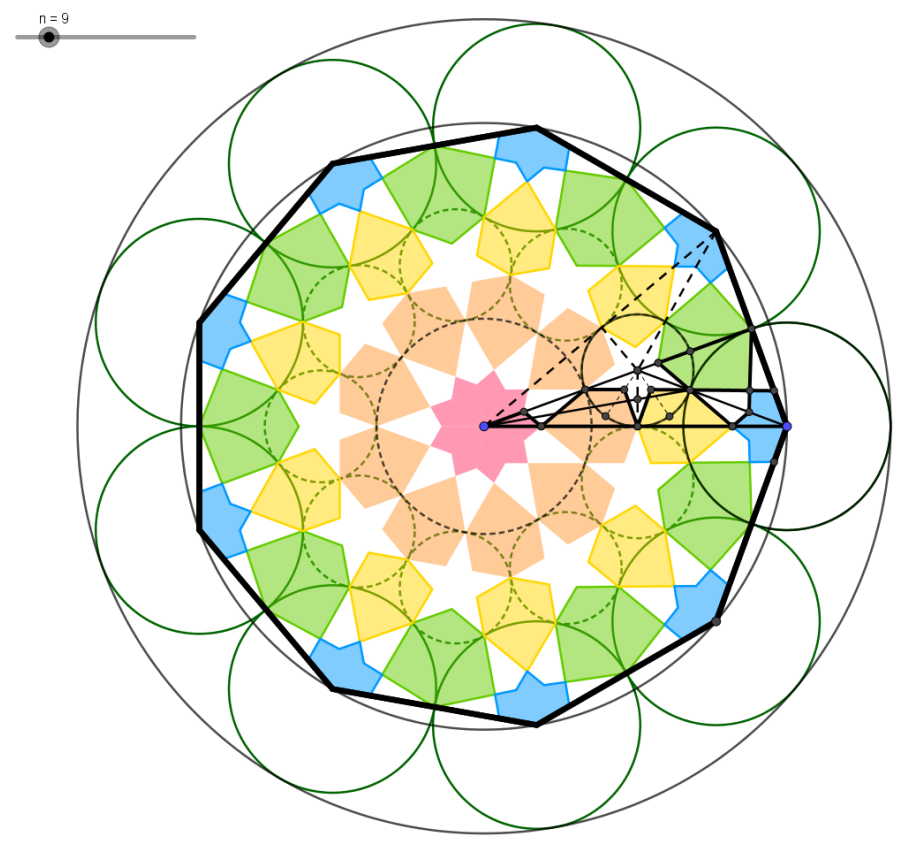


Wooden Minbar, Mosque of al-Mu'ayyad, Cairo, Egypt.  
Wade Photo Archive [EGY 1216](#).

# Anatomy of a Star Rosette



Ideal or Canonical 8-fold Rosette Pattern, drawn after Samira Mian.



9-fold Parallel Rosette pattern based on concentric circle packing



# Anatomy of a Star Rosette

*n.b. The labelling of points a, b, ..., e, f needs revision*

Monday, JANUARY 3, 1966 *sun 2 Nov 1975*

### Suggested Nomenclature for Type I Rosettes.

**Fig. 1** Usual method of location of points B.

**Fig. 2** Constituent cells of a Type I rosette.

1. Outer cell
2. Mid-cell
3. Inner cell or Central Star

$a$  = outer points, on circumcircle;  
 $b$  = outer midpoints, on outer mid-circle;  
 $c$  = inner midpoints, on inner mid-circle;  
 $d$  = inner points, on in-circle;  
 $e$  = centre of rosette.

The terminal cross or cap cross is drawn through each point  $a$ .

**Fig. 3** *Shoulder angle* ( $\theta$ ), *edge - secondary radius or intraradius*, *cap, or terminal segment*, *principal radius*, *lateral segment*, *lateral segments are here considered - radiating or parallel (same or divergent) (n.b.)*, *subterminal point*, *peripheral cell*.

**Fig. 4** The repetition  $n$  times round the centre  $e$  in an  $n$ -rayed rosette results in the formation of the outer, mid- and inner cells by multiple intersections of the lines of all  $n$  rays.

**Fig. 5** *Terminal segments collinear lateral segments parallel*

**Fig. 6** *Terminal segments collinear lateral segments parallel*

**Fig. 7** *Terminal segments acute lateral segments convergent*

**Fig. 8** Showing the manner of joining two geometrical rosettes in a "repeat path" i.e. "ae" is a straight line.

This shows a simple type I junction, at the shared terminal point  $a$ . If the rosettes are identical, obviously  $\theta' = \theta$ ; The latter relation usual holds when the rosettes are dissimilar, i.e. the four terminal segments form two lines crossing at point  $a$ . It is obvious that if the rosettes are unequal they cannot both have collinear terminal segments. If one of them is given collinear terminal segments it is usually the smaller of the two, in which case those of the larger rosettes will automatically become acute. The junction of terminal segments shown above may be termed "continuous". If  $\theta' \neq \theta$  the junction may be called "discontinuous" - this however produces a very ugly effect and is uncommon. (This becomes especially ugly when drawn as interlocking-bars).

**Stellated Rosette of Type I**

$f$  = stellate points or stellate circle.

**Simpler terminology:**  
 terminal segment = cap  
 lateral segment = side  
 (e.g. rays with collinear caps, sides parallel)

*n.b. The labelling of points a, b, ..., e, f needs revision*

Wednesday, JANUARY 5, 1966 *sun 3 Nov 1975*

*one really needs to refer to the shoulder, or point  $\theta$  here. See p.*

*There is uncertainty in that the two external segments are unequal.*

**Fig. 5** *Terminal segments collinear lateral segments parallel*

**Fig. 6** *Terminal segments collinear lateral segments parallel*

**Fig. 7** *Terminal segments acute lateral segments convergent*

**Fig. 8** Showing the manner of joining two geometrical rosettes in a "repeat path" i.e. "ae" is a straight line.

This shows a simple type I junction, at the shared terminal point  $a$ . If the rosettes are identical, obviously  $\theta' = \theta$ ; The latter relation usual holds when the rosettes are dissimilar, i.e. the four terminal segments form two lines crossing at point  $a$ . It is obvious that if the rosettes are unequal they cannot both have collinear terminal segments. If one of them is given collinear terminal segments it is usually the smaller of the two, in which case those of the larger rosettes will automatically become acute. The junction of terminal segments shown above may be termed "continuous". If  $\theta' \neq \theta$  the junction may be called "discontinuous" - this however produces a very ugly effect and is uncommon. (This becomes especially ugly when drawn as interlocking-bars).

29 *sun 2 June 1978* **(3x2) Type Ia general const.**

Saturday, FEBRUARY 5, 1966 *see p. 83 for revised type labels.*

**Fig. 9**  $m=12$ ,  $m=9$

**Fig. 10**  $m=10$ ,  $m=12$

**Fig. 11**  $m=10$ ,  $m=10$

*n-submedian point*, *m2.n1*, *m1.n1*, *median point*, *m2.n2*, *m1.n2*

**(3x2)12, 8/1a**

**(3x2)9, 12/1a**

**(3x2)10, 10/1a**

The 2nd cell triangle is indicated by shading.

point  $m1.n1$  is the incentre of the 2nd collateral triangle and the meeting point of the bisectors of its three angles.

In higher shoulders one might refer to the 1st, 2nd etc submedian points.

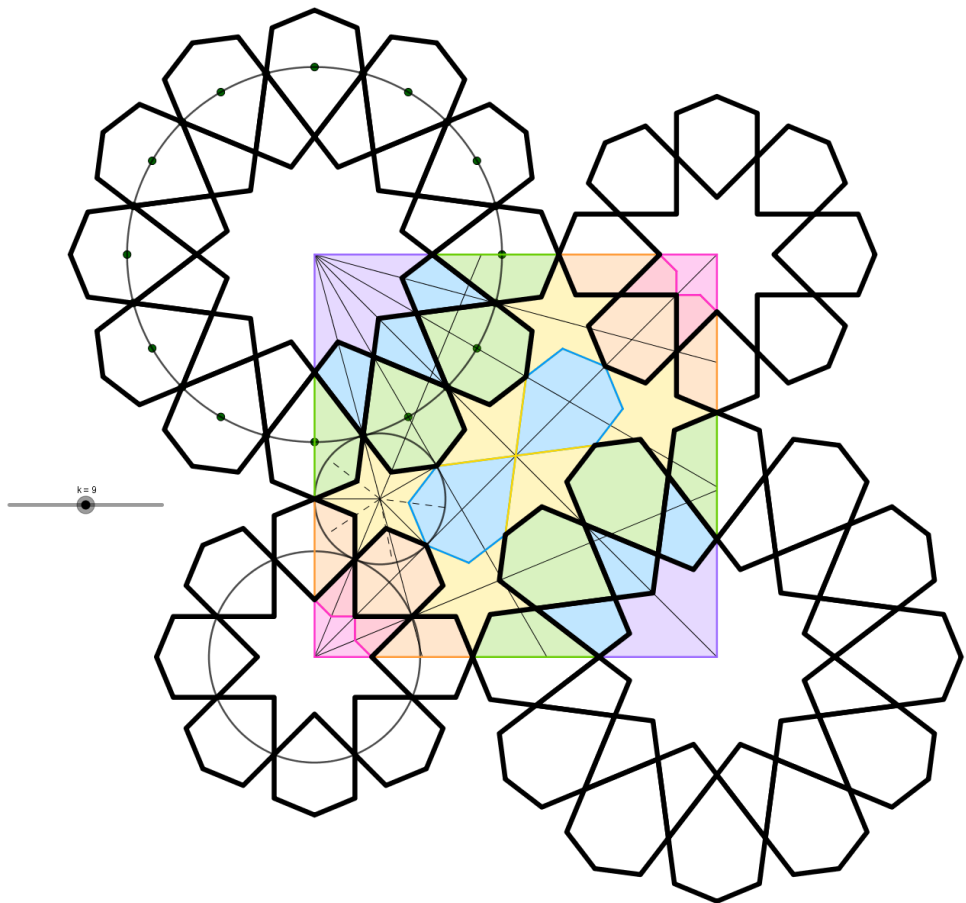
*Nov 5 June 1978*

Sunday, FEBRUARY 6, 1966

Note that the pattern within the 2nd collateral triangle is topologically equivalent to patterns of the same type in (2x2) thorns. The angle at  $m2.n2$  becomes a right angle in their deformation.

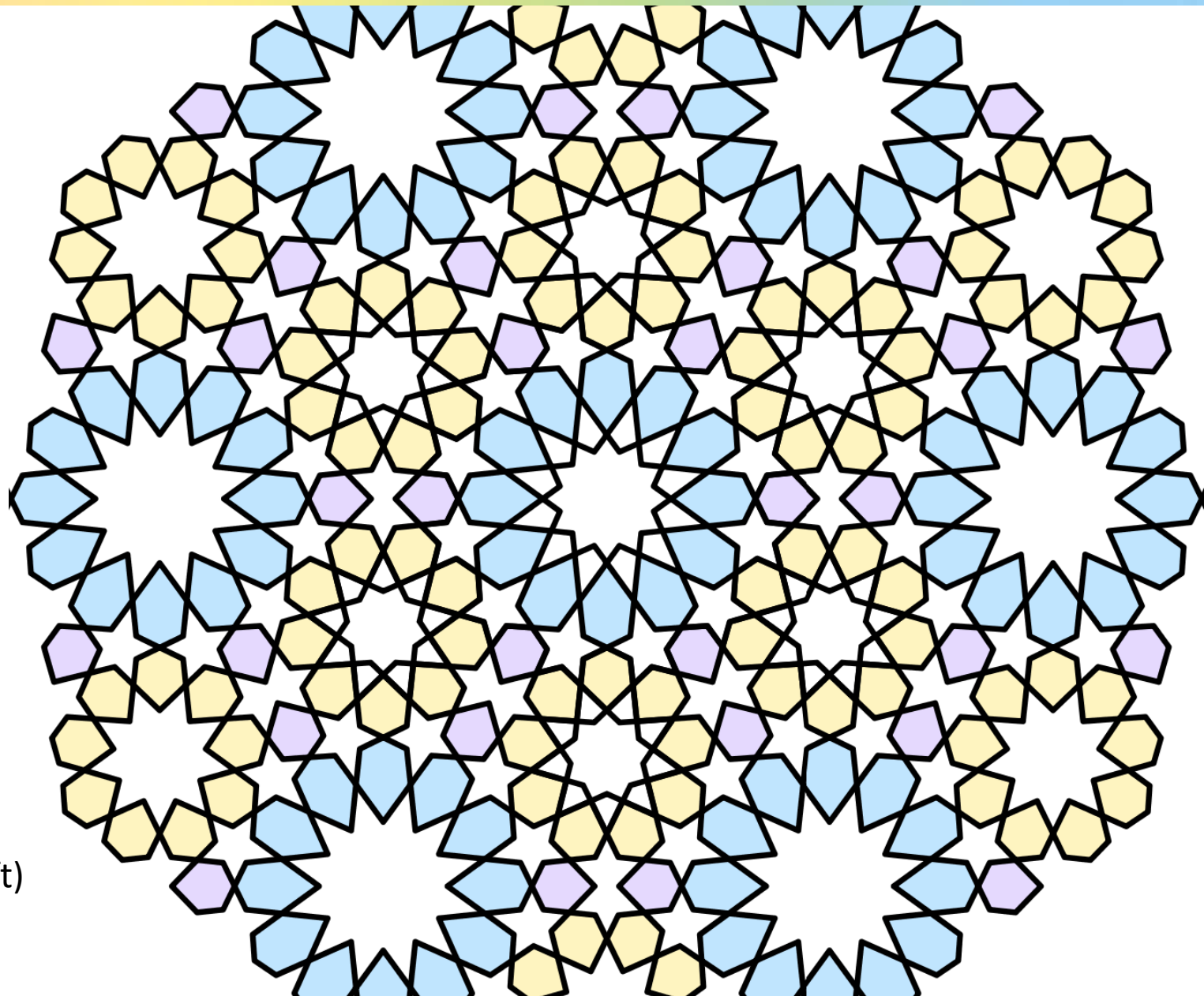
Pages 7, 9, and 33 from Part I of A.J. Lee's Notes on Islamic Star Patterns, <http://www.tilingsearch.org/tony/>

# Anatomy of a Star Rosette

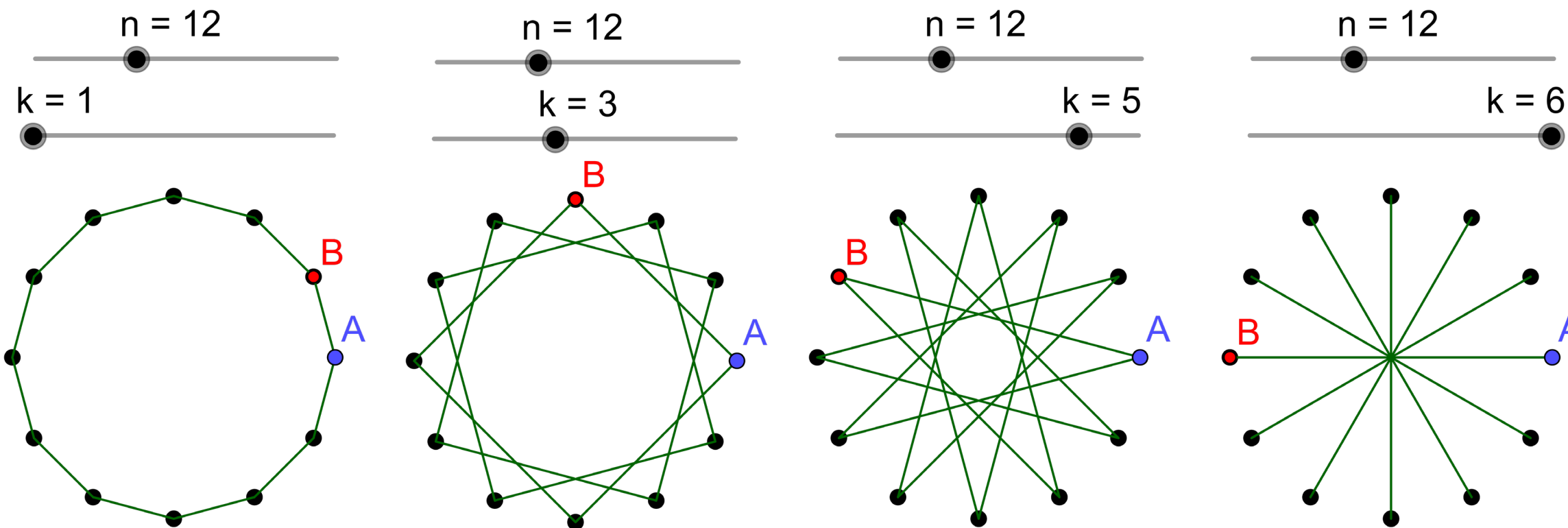


12 & 8 Star Rosettes via a  $\{24/9\}$  star polygon (left)

12&9 Star Rosettes at a  $26^\circ$  angle (right)

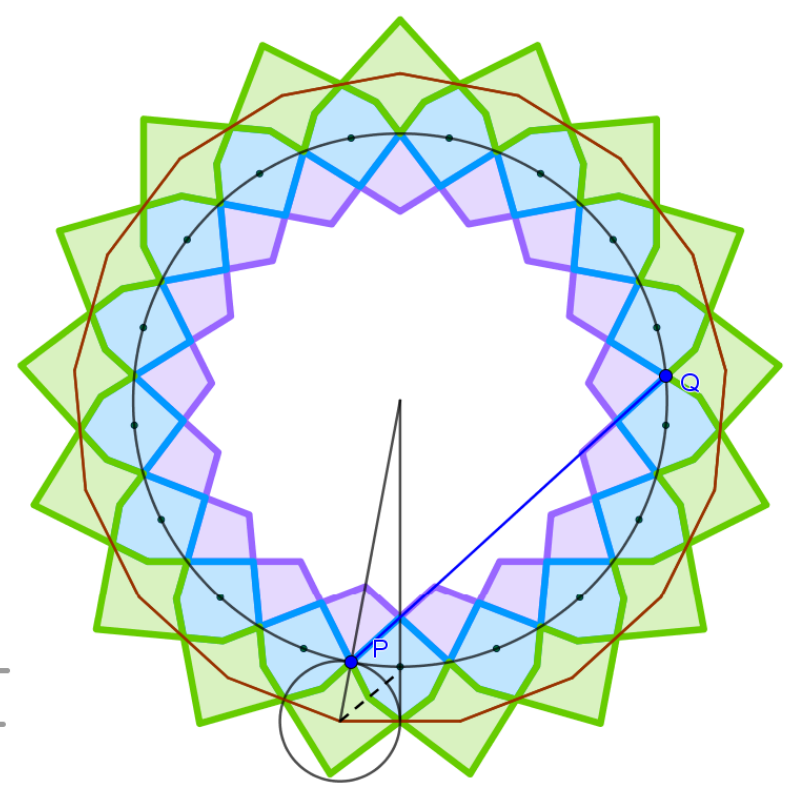


# Anatomy of a Star Rosette

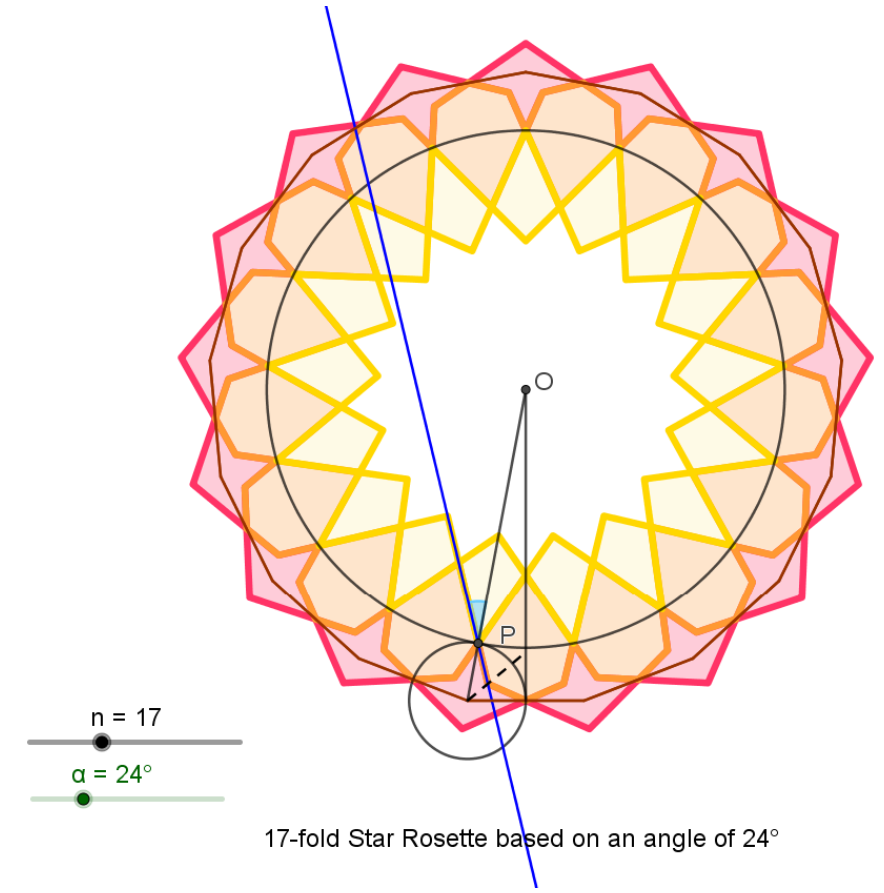


An  $\{n / 1\}$  star polygon is just a polygon; for  $n$  even, an  $\{n / \frac{n}{2}\}$  star polygon is an asterisk;  
 if  $k$  is a factor of  $n$ , then the  $\{n / k\}$  star polygon is a compound of smaller overlapping polygons;  
 if  $n$  and  $k$  are relatively prime, the  $\{n / k\}$  star polygon can be drawn without picking up your pencil.

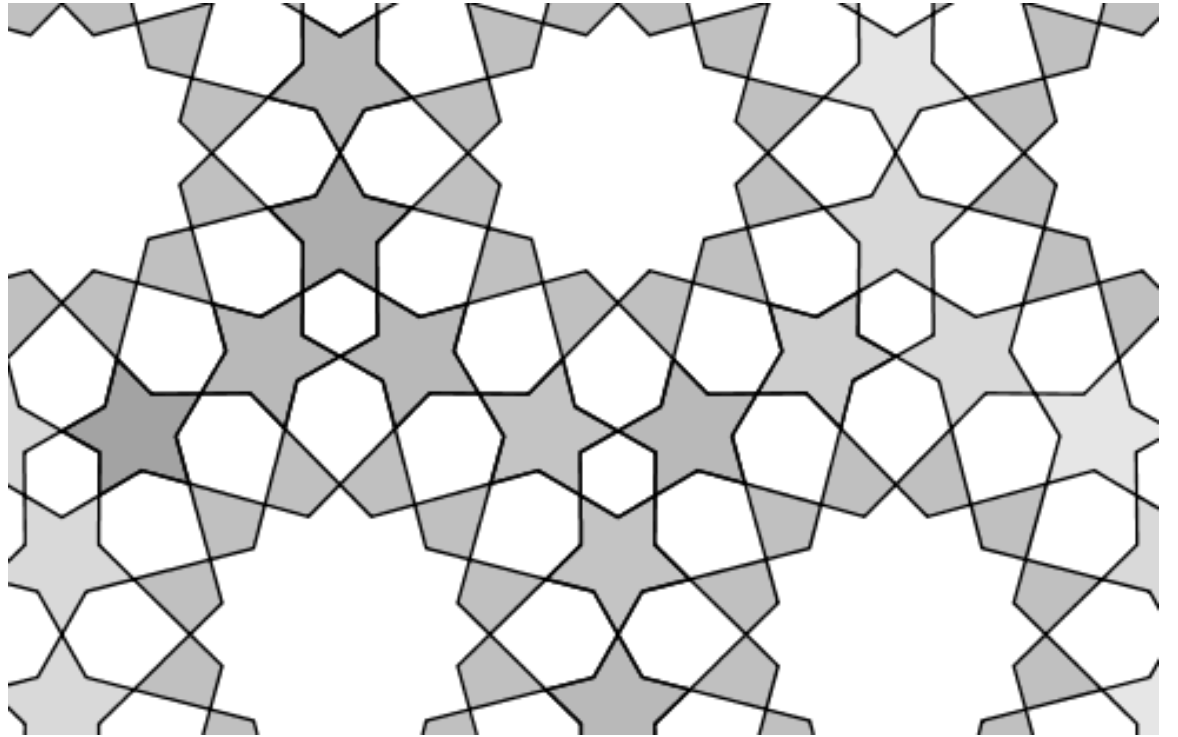
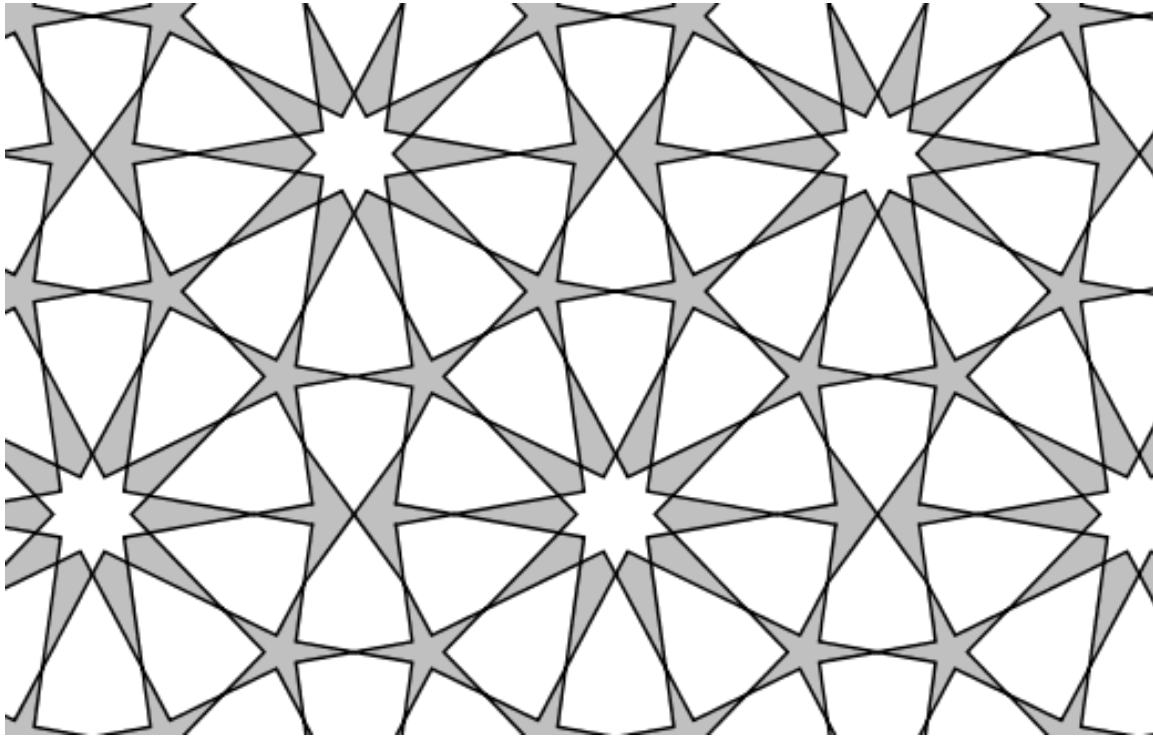
# Anatomy of a Star Rosette

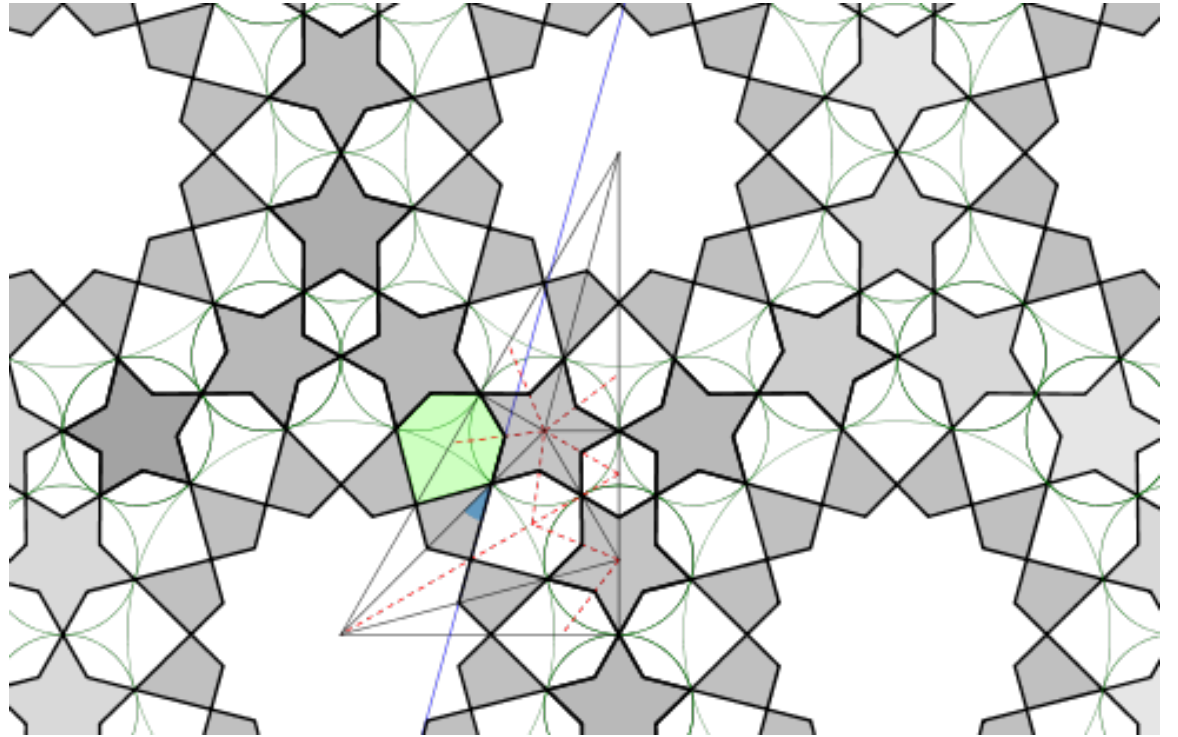
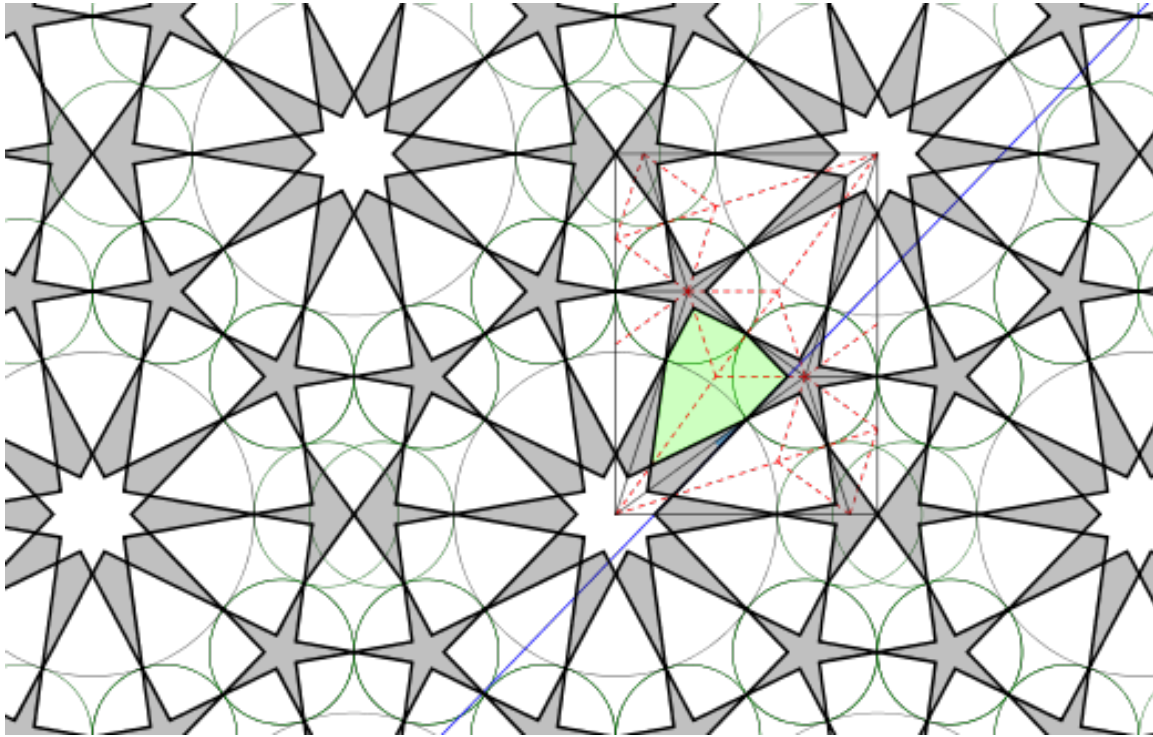


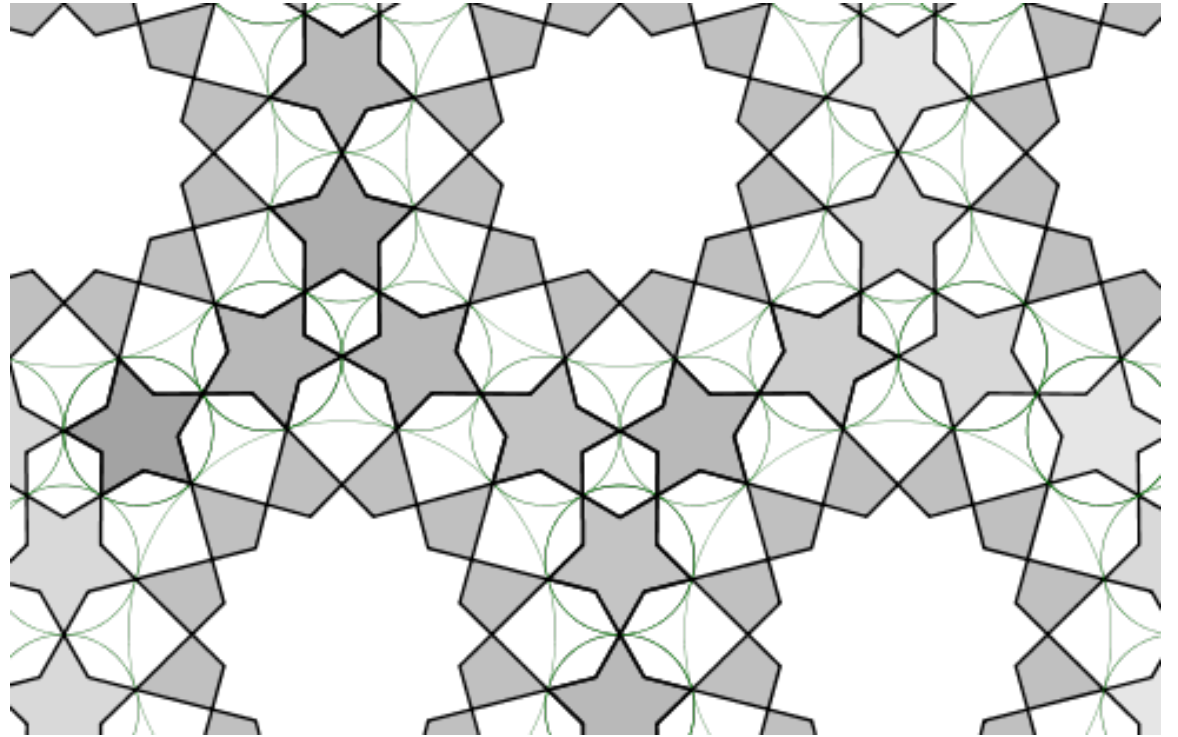
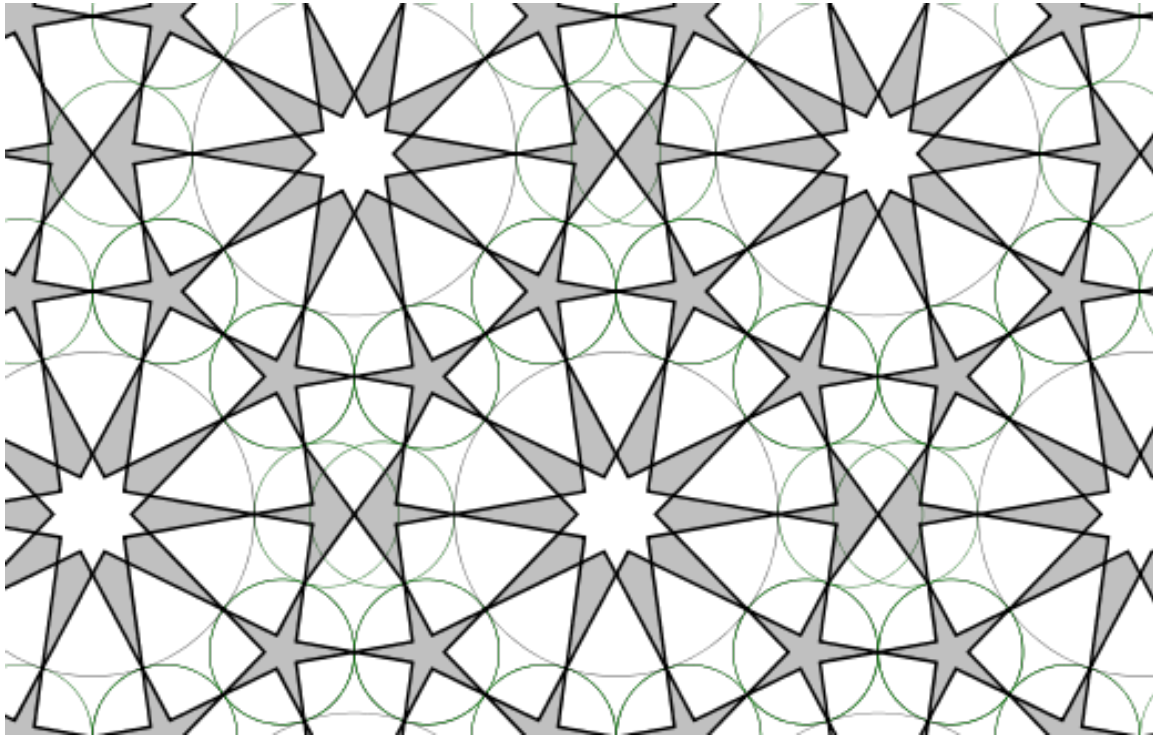
17-fold Star Rosette based on a  $\{34 / 10\}$  star polygon.

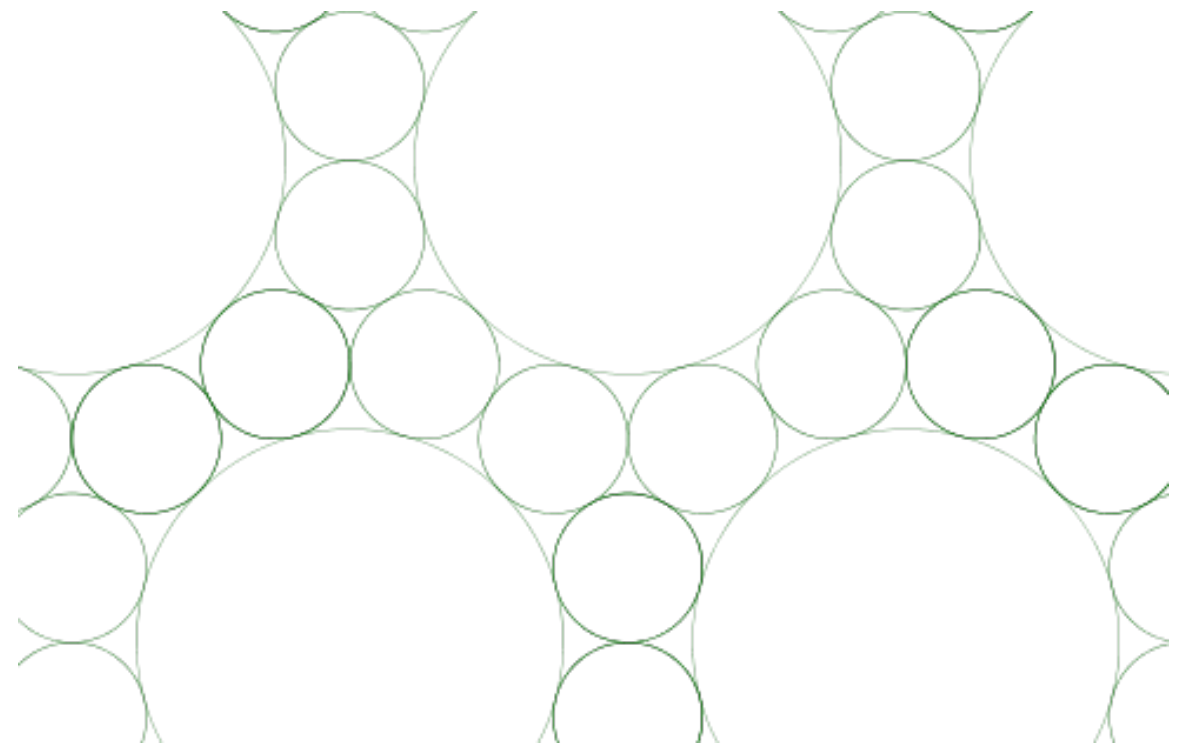
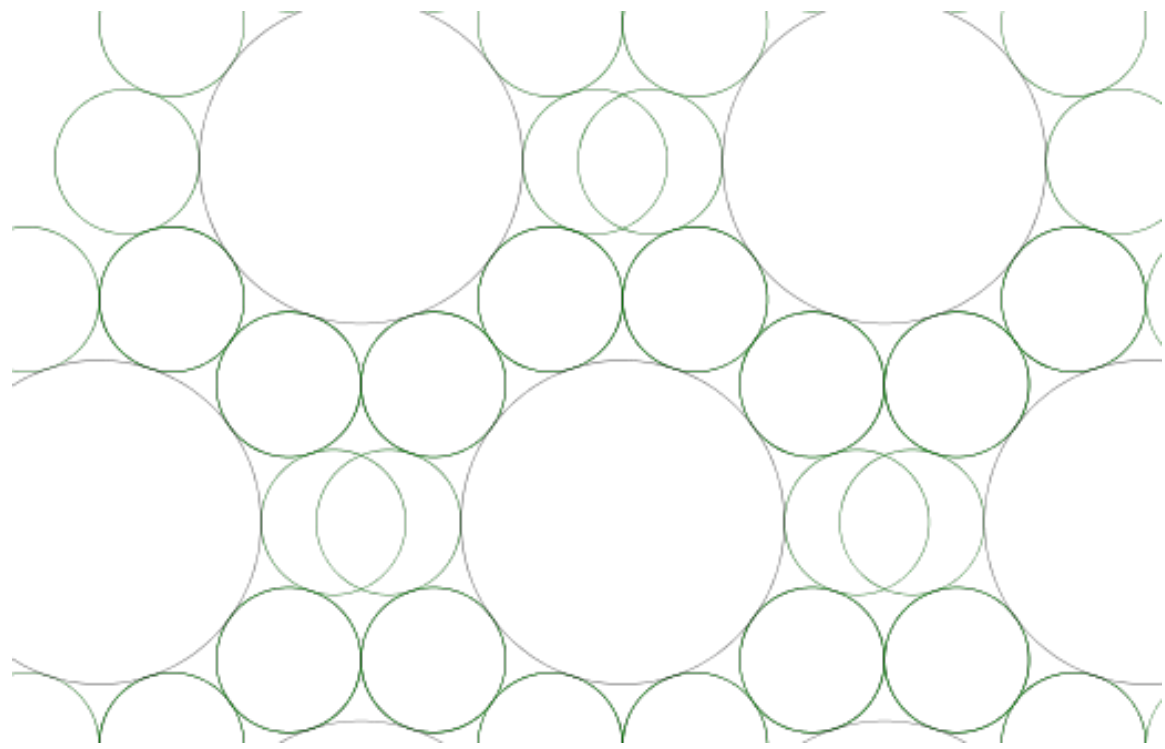


17-fold Star Rosette based on an angle of  $24^\circ$

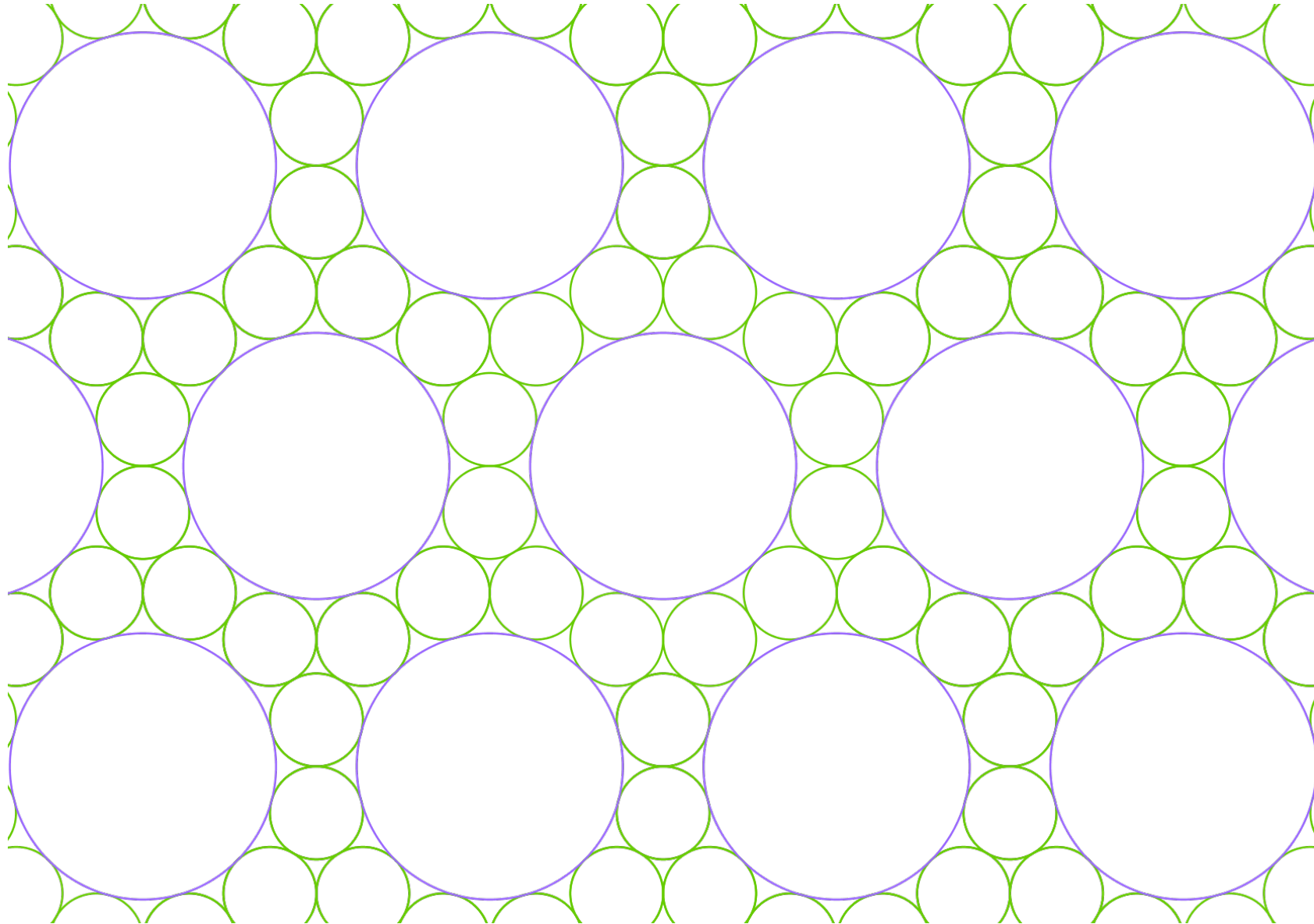


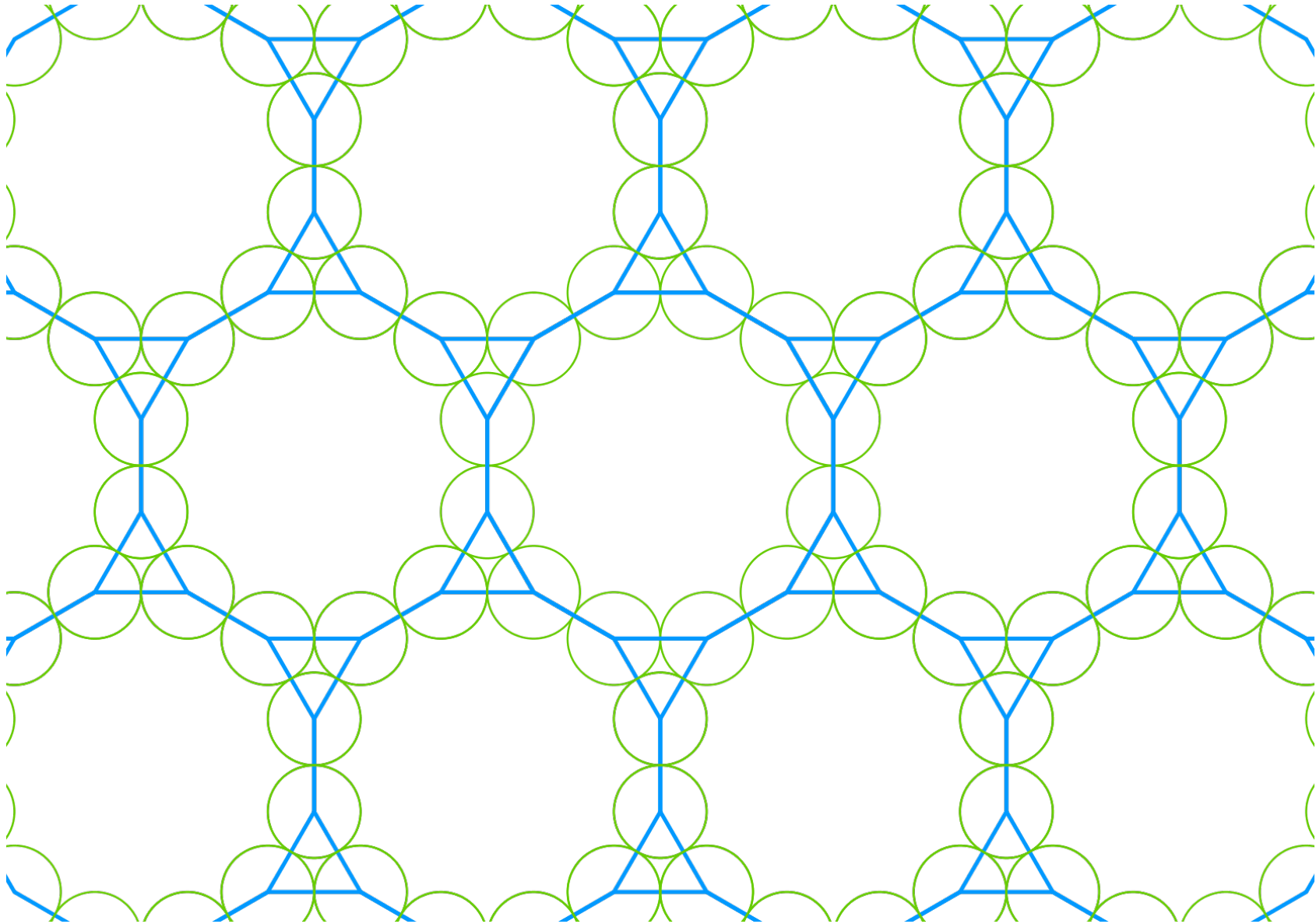


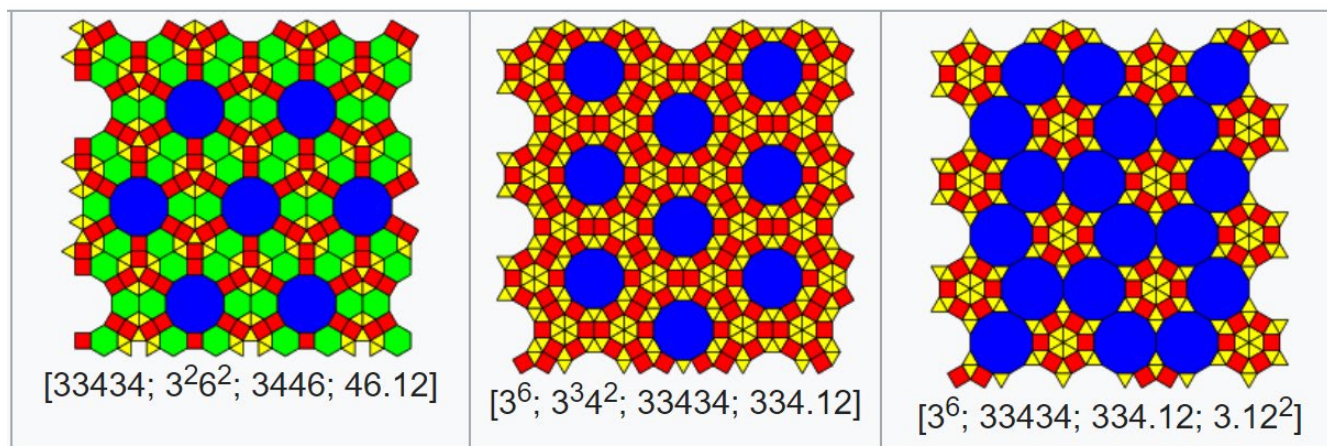
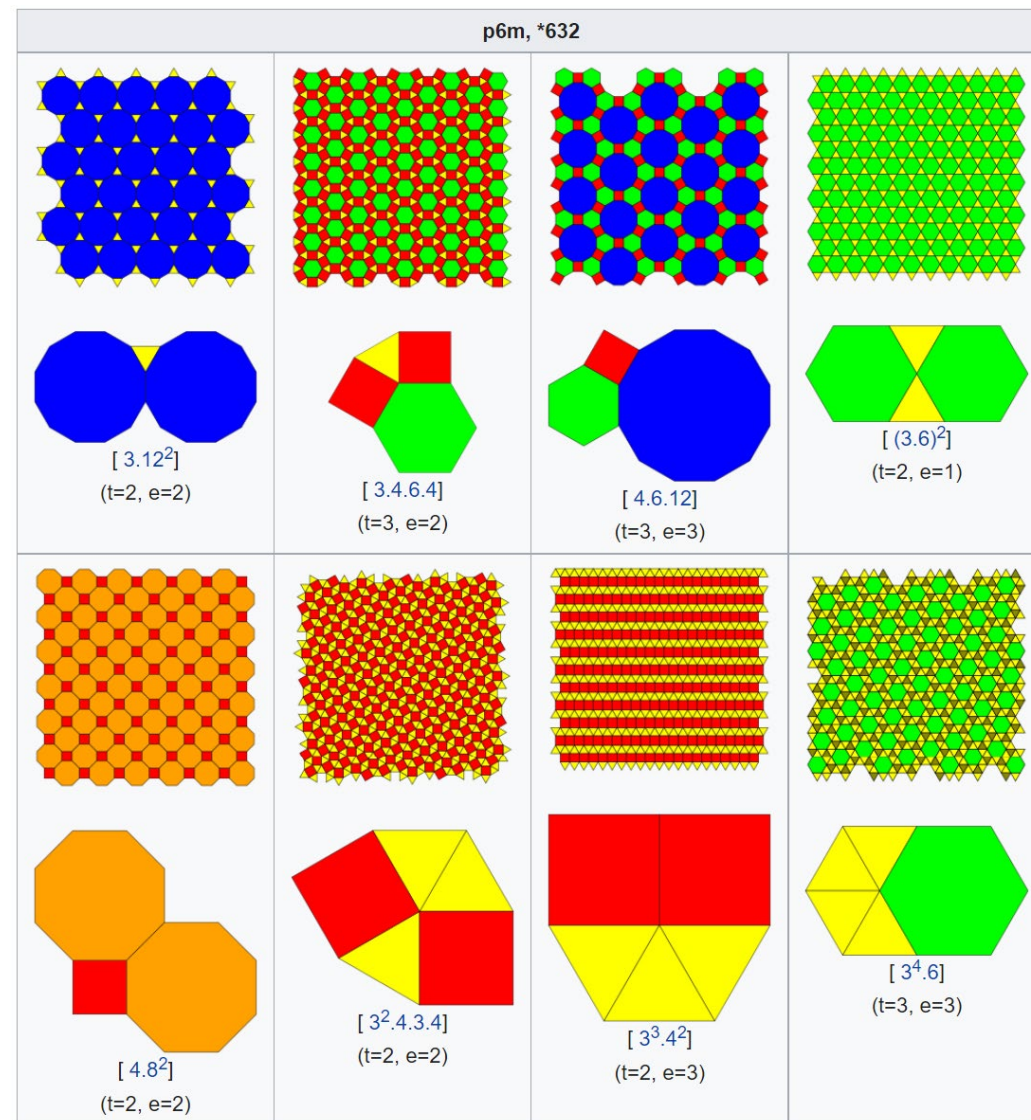
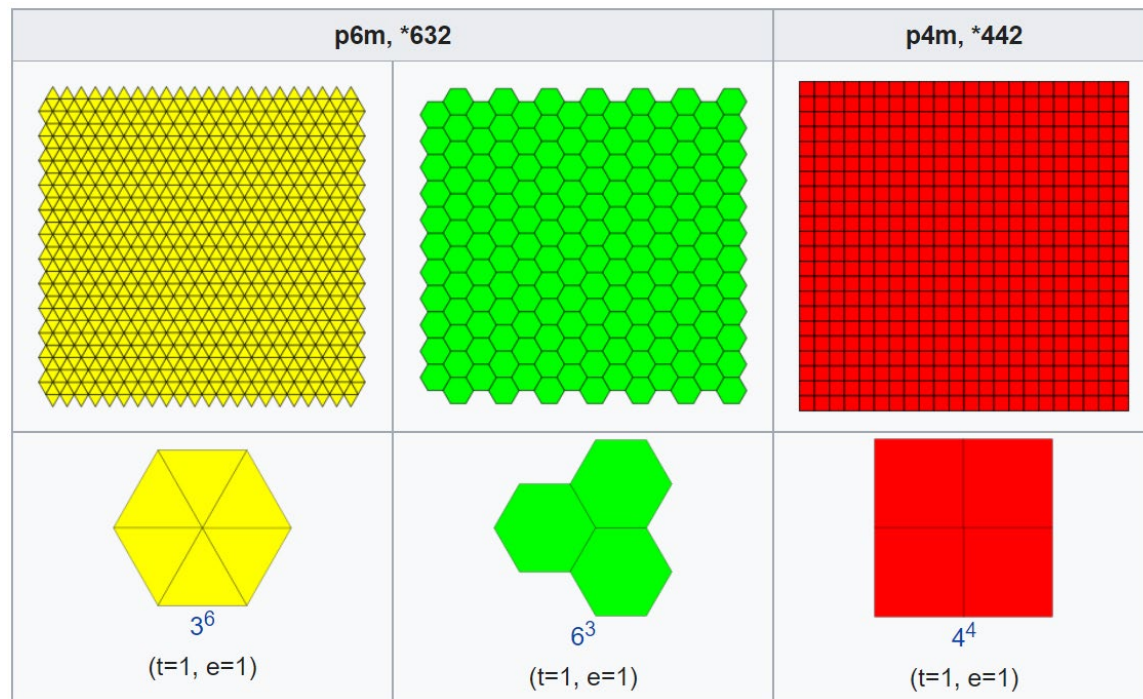


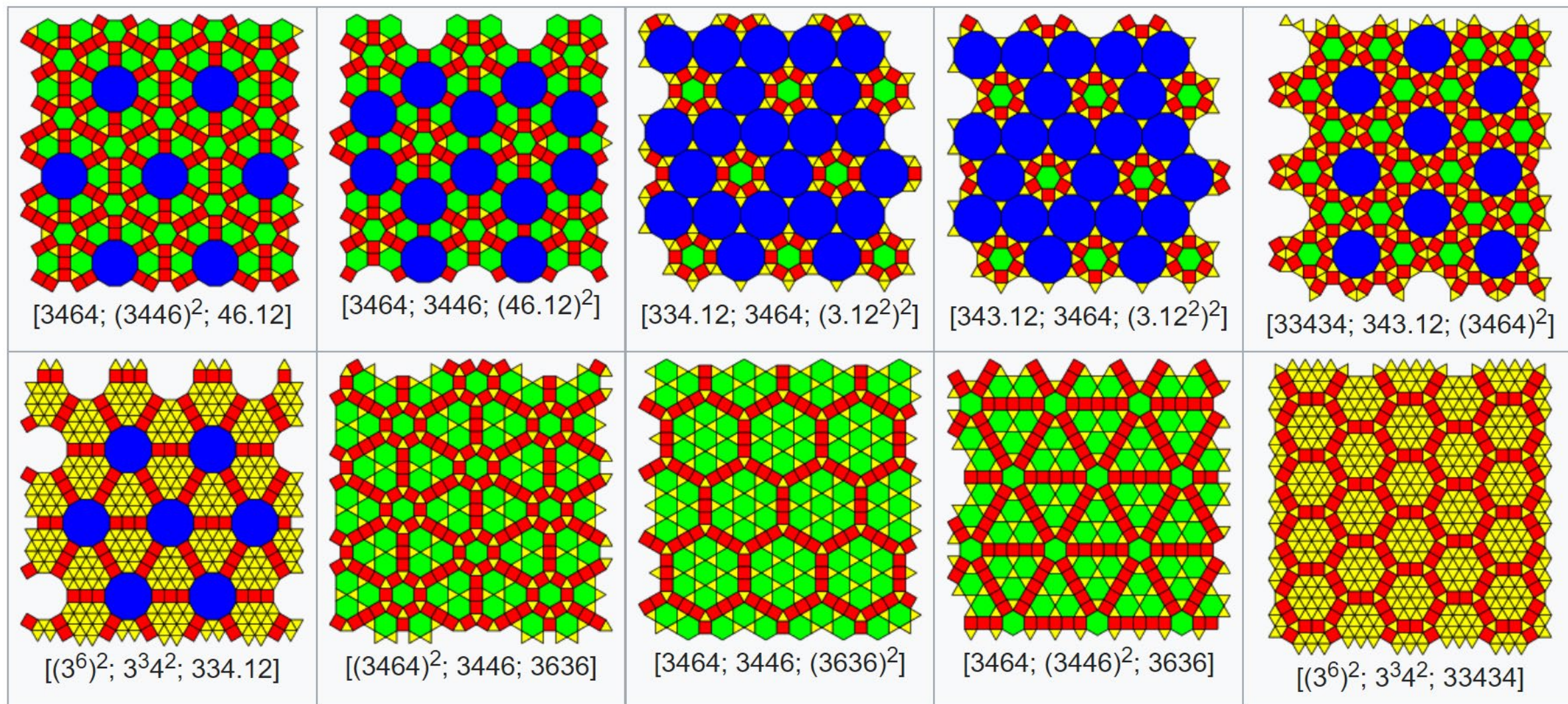




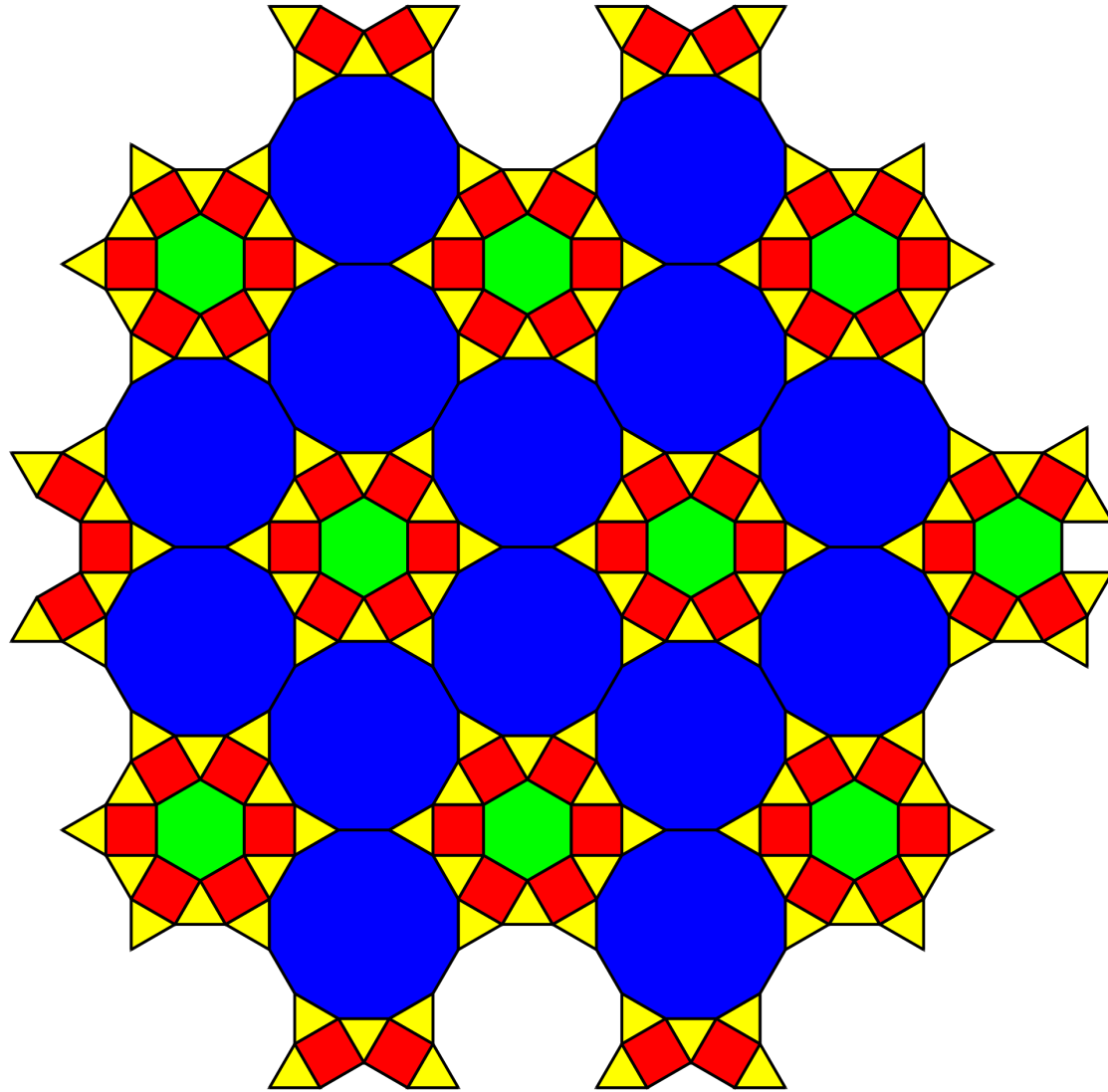








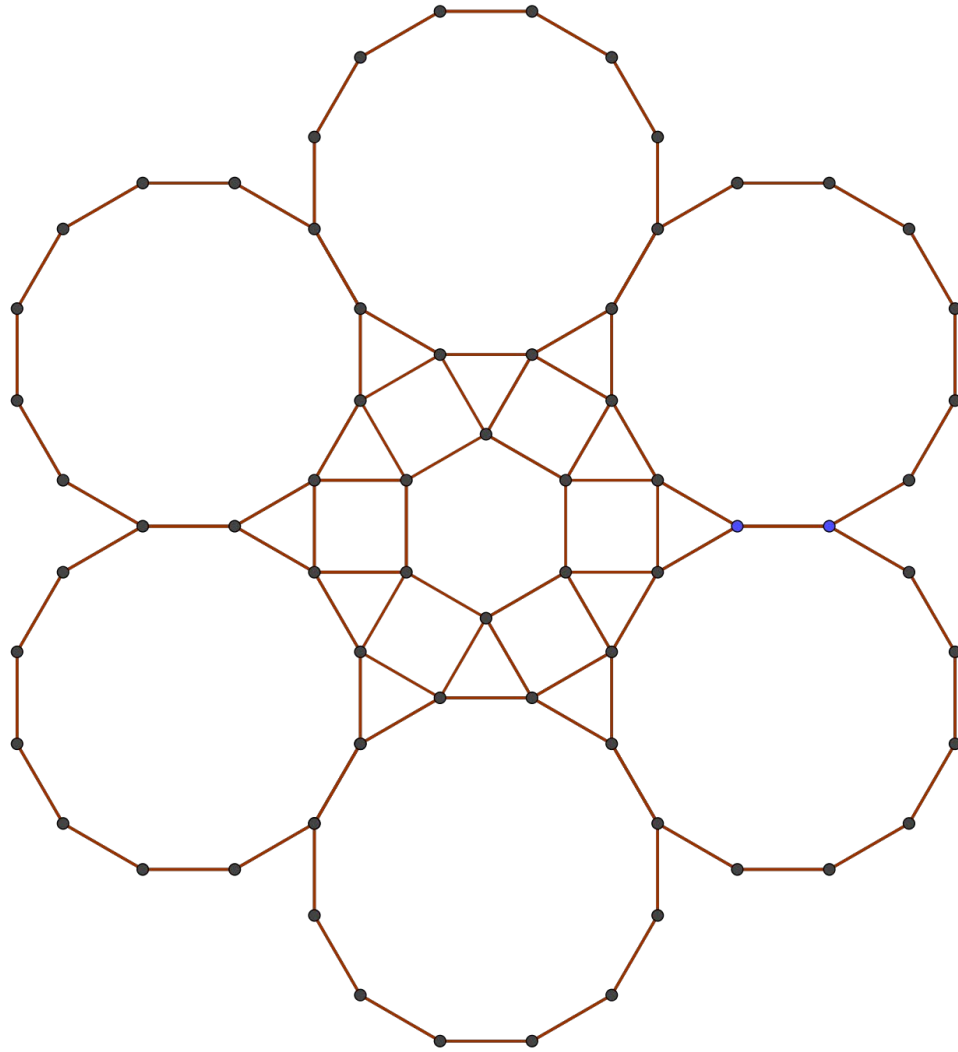
## From Polygons to Pattern



[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling of the plane.

[Image](#) by Tom Ruen

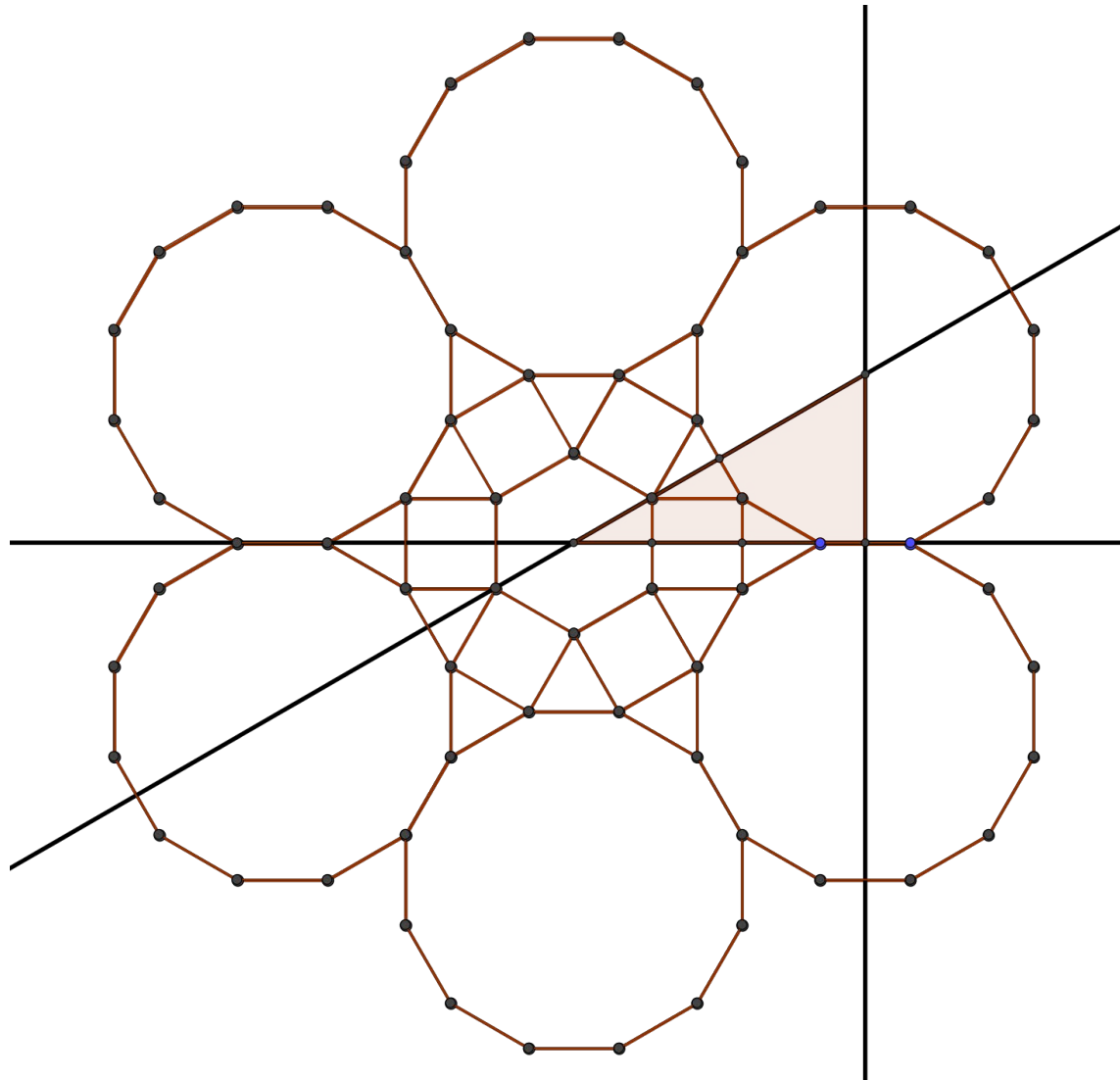
# From Polygons to Pattern



[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling  
base construction in GeoGebra

**Step 1:** Pick your favorite k-uniform tiling.

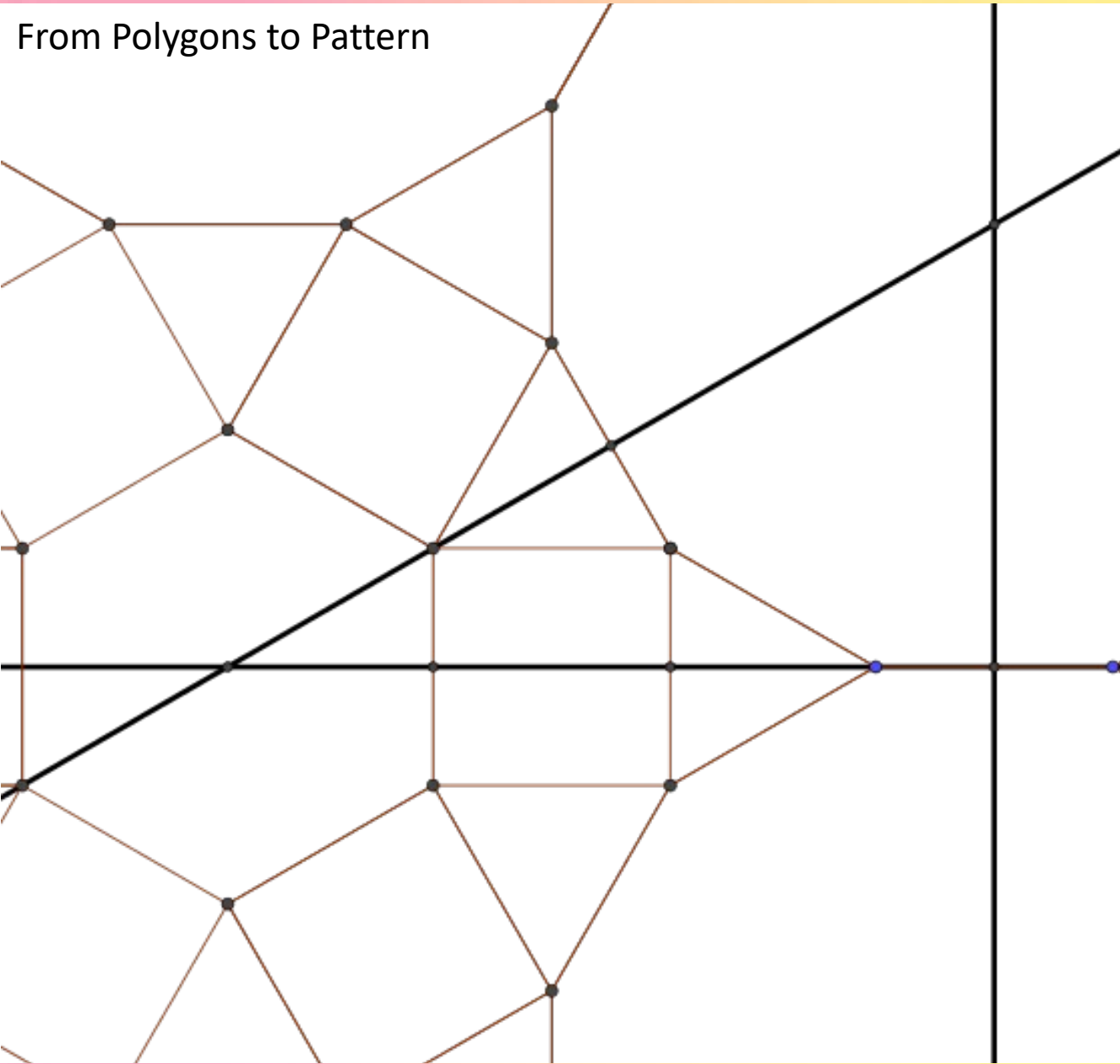
## From Polygons to Pattern



[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling  
base construction in GeoGebra

**Step 1:** Pick your favorite k-uniform tiling.

## From Polygons to Pattern

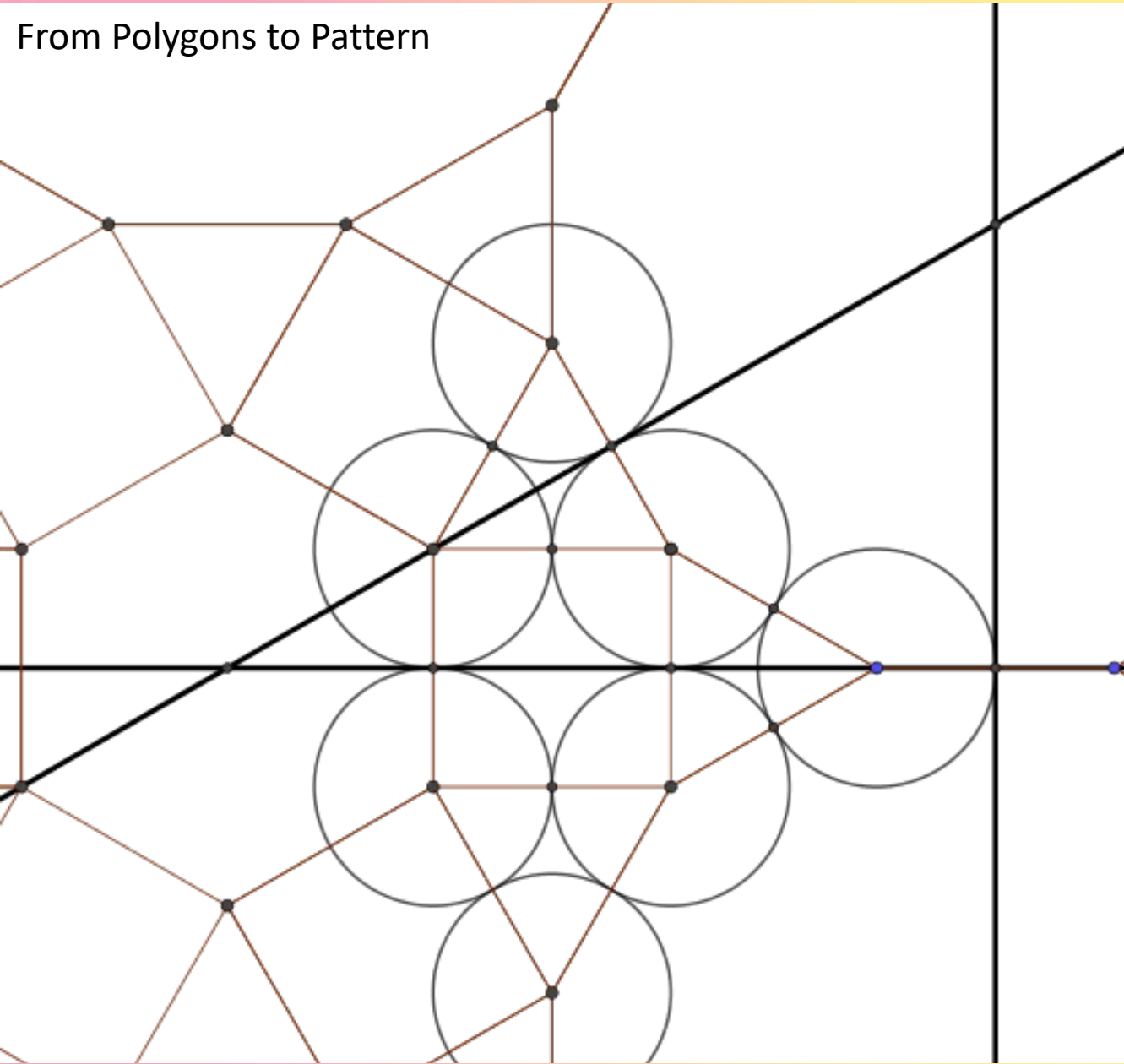


[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling  
minimal triangle

**Step 2:** locate the midpoints of the tiling edges,  
and construct circles with centers at each vertex  
whose radii are half the edge length



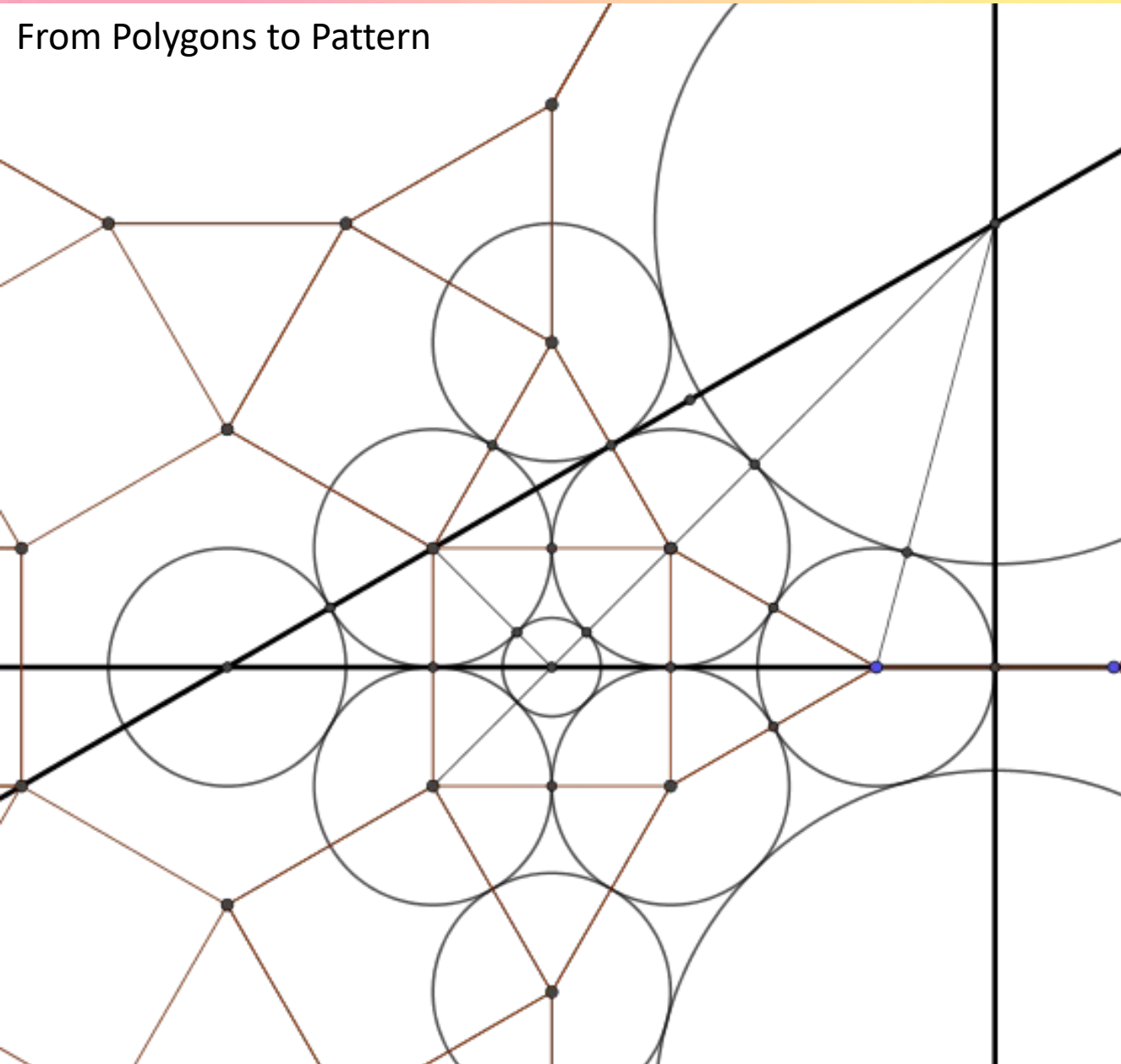
## From Polygons to Pattern



[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling  
minimal triangle with vertex circles

**Step 2:** locate the midpoints of the tiling edges,  
and construct circles with centers at each vertex  
whose radii are half the edge length

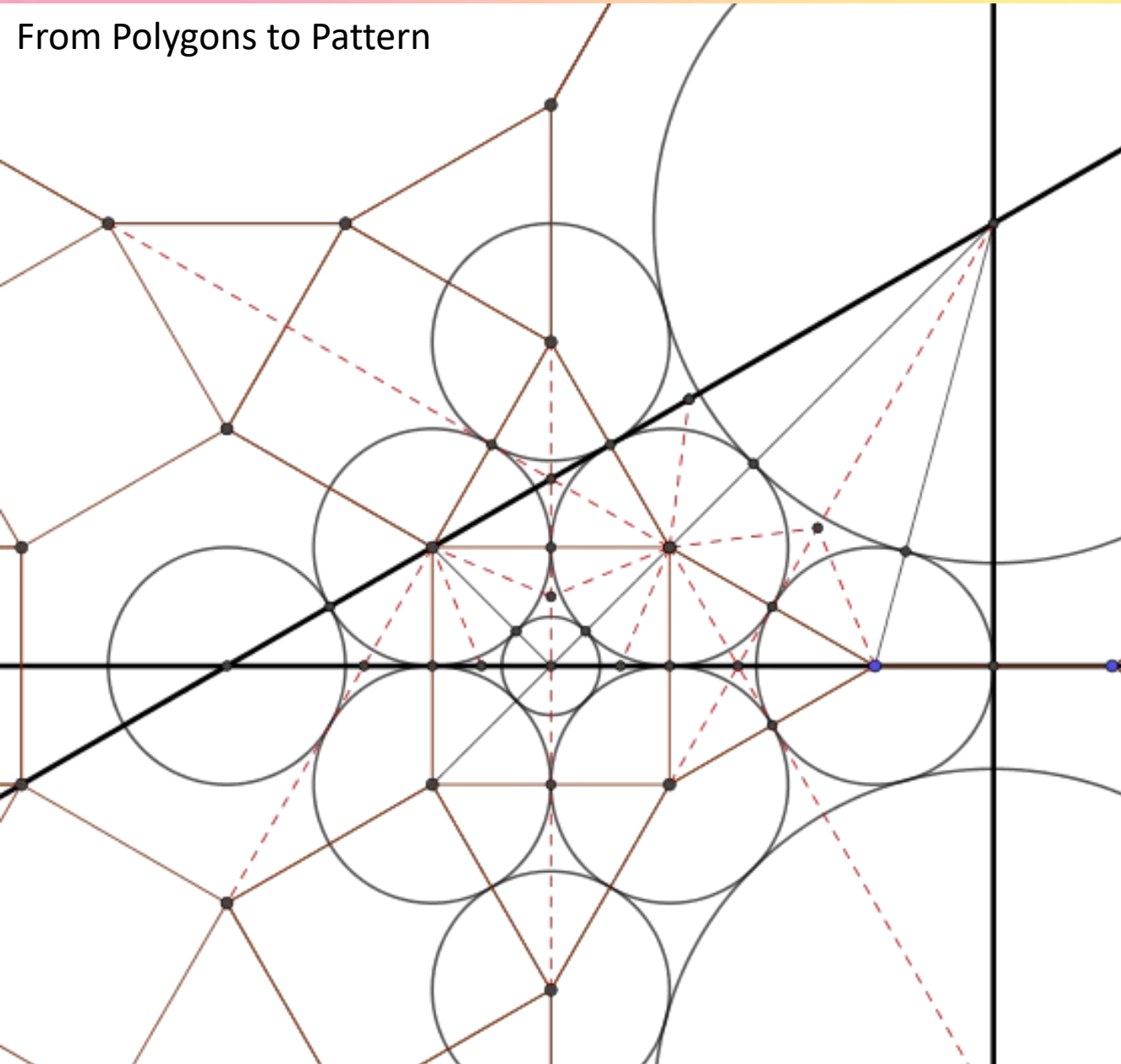
## From Polygons to Pattern



[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling  
minimal triangle with circle packing  
and radii of tangency

**Step 3:** complete the circle packing by filling in  
a circle in each non-triangular tile

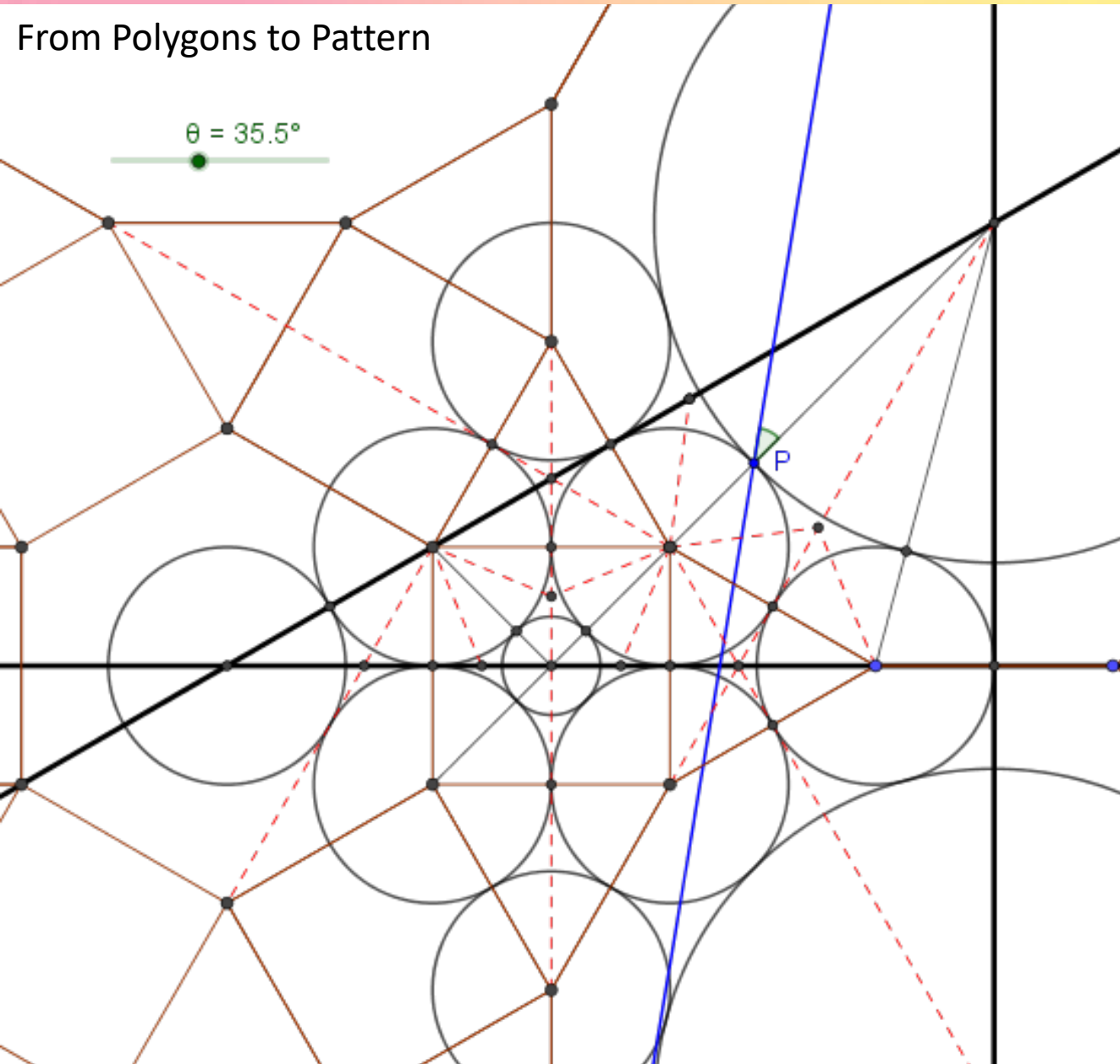
## From Polygons to Pattern



[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling  
minimal triangle with circle packing,  
radii of tangency, and angle bisectors

**Step 4:** construct the angle bisectors at each  
circle center between each adjacent pair of  
radii of tangency

## From Polygons to Pattern

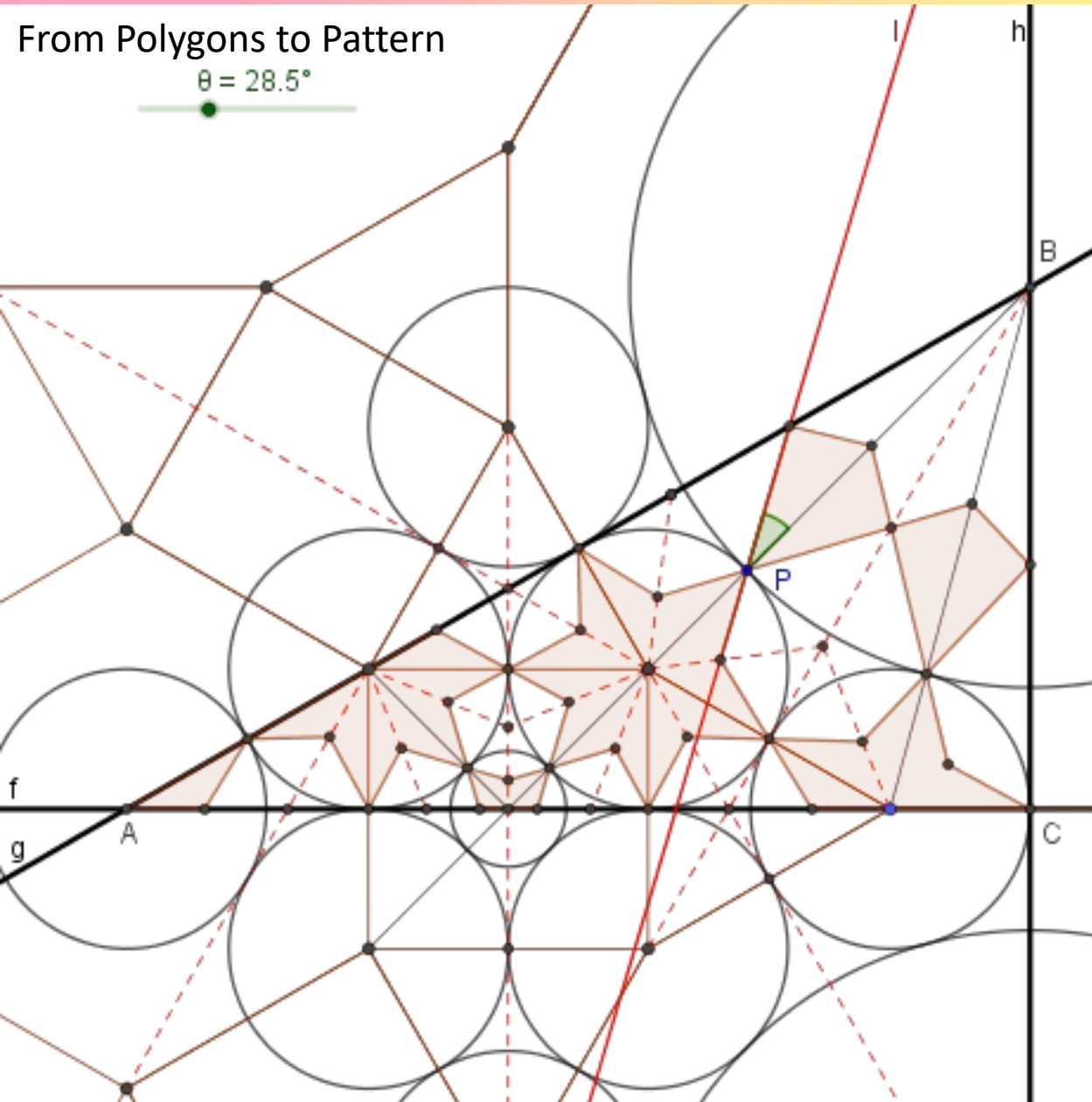


[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling  
minimal triangle with circle packing,  
radii of tangency, angle bisectors, and  
generating line

**Step 5:** Construct a line at one of the points of  
circle tangency at a variable angle to the radius.

# From Polygons to Pattern

$$\theta = 28.5^\circ$$

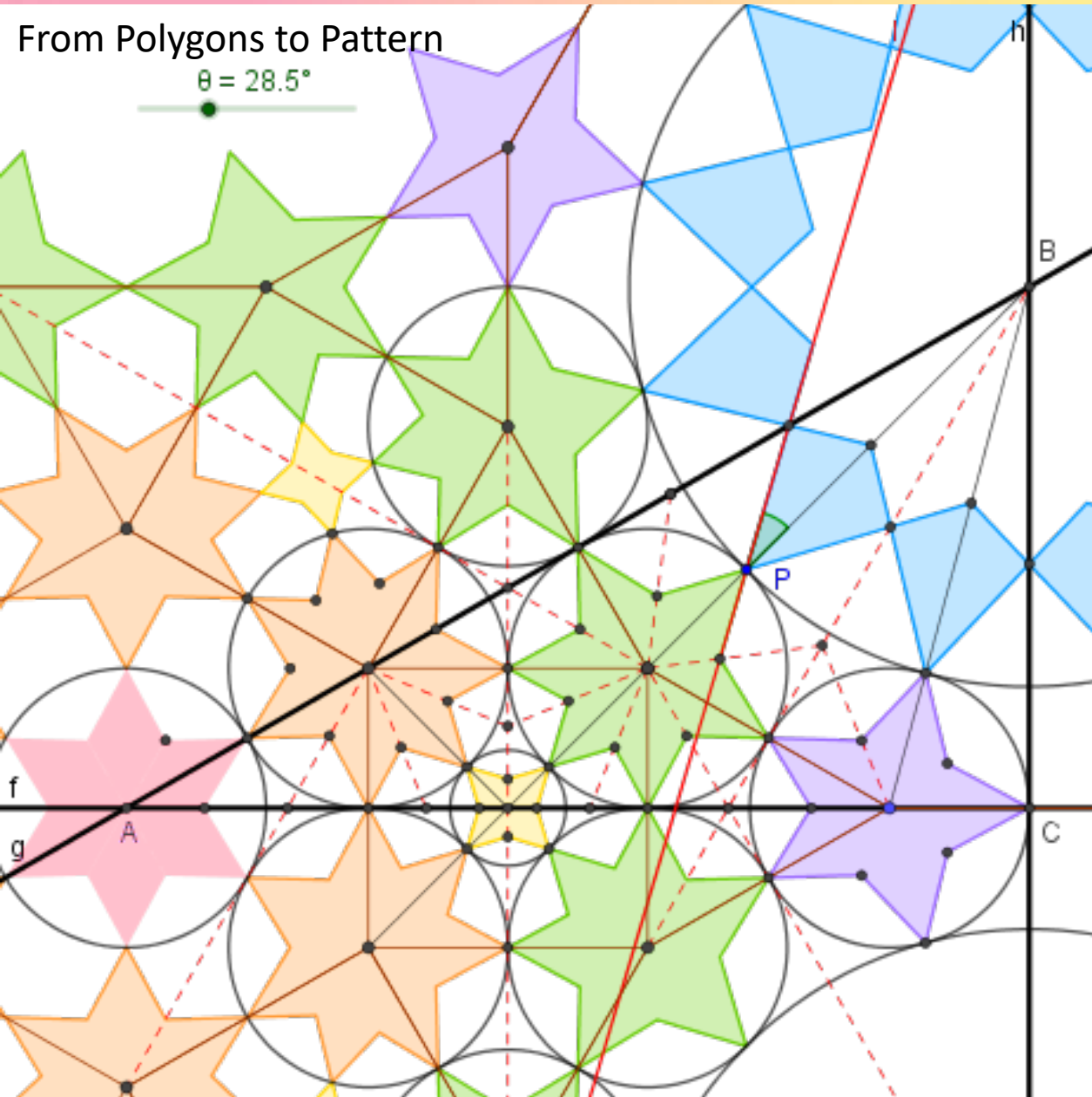


[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling  
minimal triangle with circle packing,  
radii of tangency, angle bisectors, and  
star polygons generated by reflections of  
the generating line over radii of tangency  
and angle bisectors.

**Step 6:** Generate the star rosette pattern by  
alternately reflecting this line over radii of  
tangency and angle bisectors.

## From Polygons to Pattern

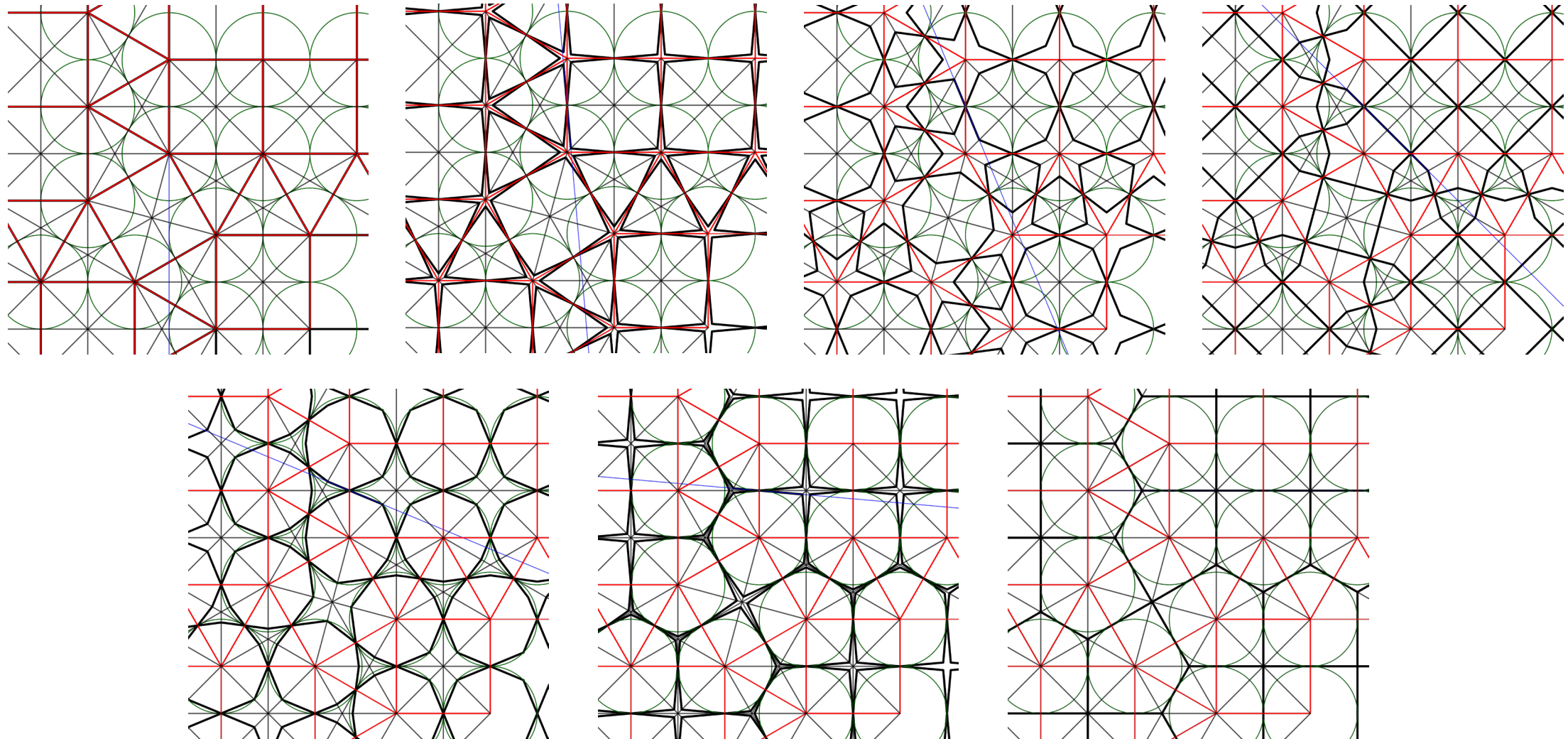
$$\theta = 28.5^\circ$$

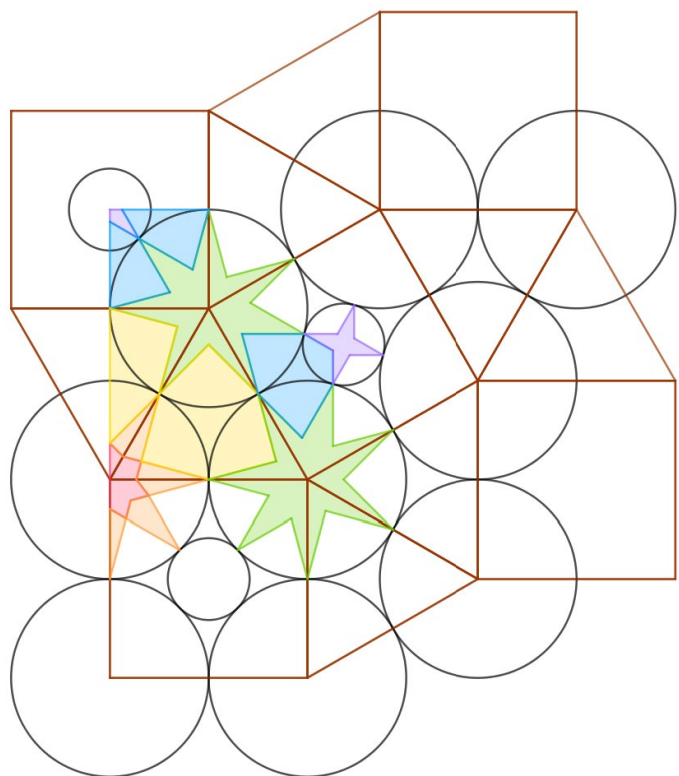


[3.4.3.12; 3.4.6.4; 3.12.12] 3-uniform tiling  
minimal triangle with circle packing,  
radii of tangency, angle bisectors, and  
star polygons generated by reflections of  
the generating line over radii of tangency  
and angle bisectors.

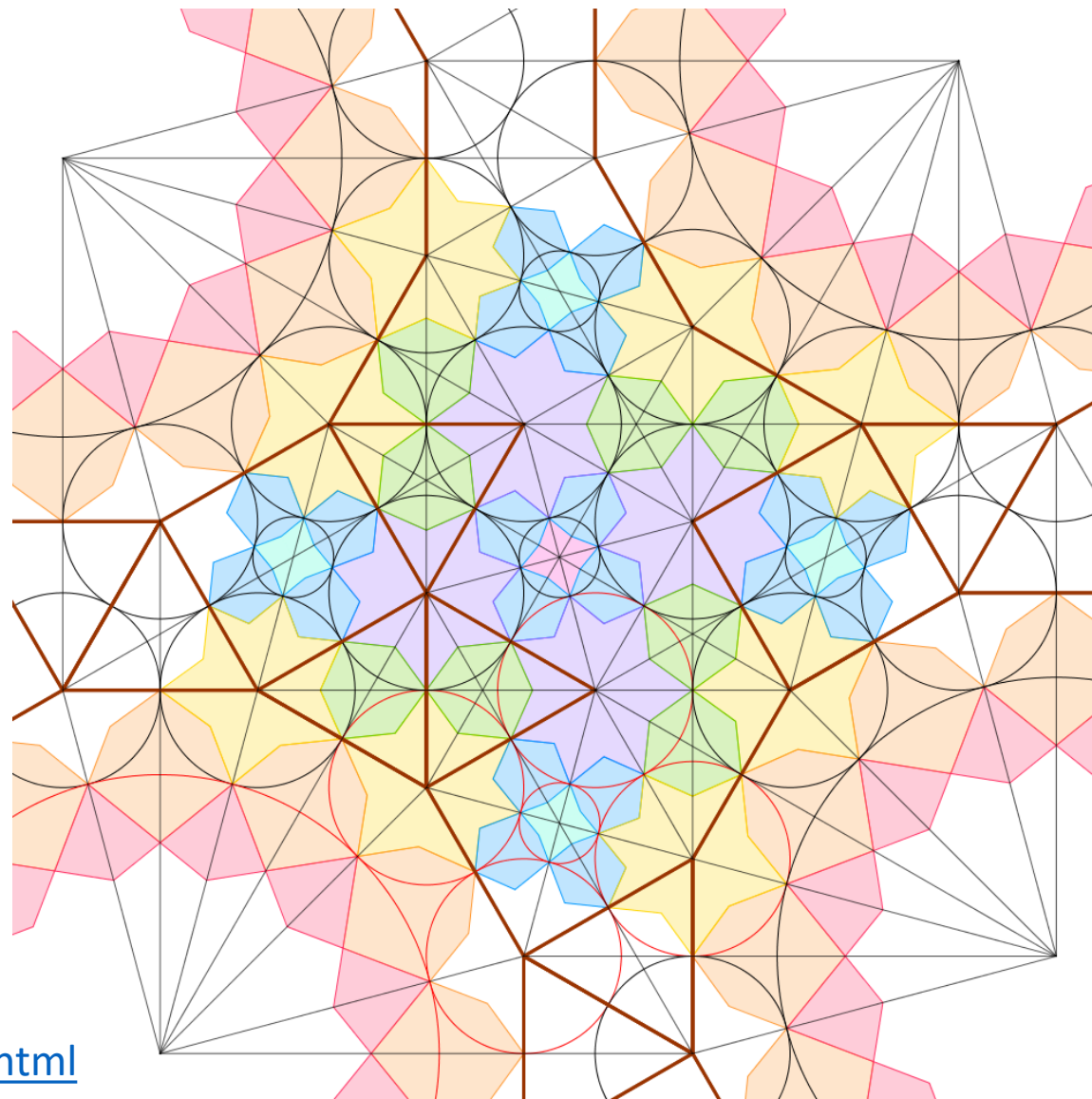
**Step 7:** Complete the pattern by reflecting and/or  
rotating the motifs in the minimal triangle to your  
desired pattern size.

# Variable-angled Star Rosette Pattern Family as transitions from a k-uniform tiling to its dual





- Animations on Instagram @[mathemartiste](https://www.instagram.com/mathemartiste)
- Interactive GeoGebra files:  
<https://www.geogebra.org/u/sarahgbrewer>
- Bridges paper:  
<https://archive.bridgesmathart.org/2022/bridges2022-391.html>





## References

- J. Bonner. *Islamic Geometric Patterns: Their Historical Development and Traditional Methods of Construction*. Springer, 2017.
- B. Grunbaum and G. Shephard. “Tilings by Regular Polygons.” *Mathematics Magazine*, vol. 50, no. 5, Mathematical Association of America, 1977, pp. 227-47. <https://www.jstor.org/stable/2689529>.
- A. J. Lee. “Generalities in a Scientific Study of Islamic Star Patterns” (A. Adams, transcriber). In *Islamic Star Patterns - Notes* [Unpublished manuscript], 1985. <http://www.tilingsearch.org/tony/>.
- G. Necipoğlu, ed. *The Arts of Ornamental Geometry*. Brill, 2017. Facsimile of original manuscript available at <https://gallica.bnf.fr/ark:/12148/btv1b52503120k/f393.item.zoom>.
- G. Necipoğlu, ed. *The Topkapi Scroll: Geometry and Ornament in Islamic Architecture*. Getty Center for the History of Art and the Humanities, 1995. <https://www.getty.edu/publications/virtuallibrary/9780892363353.html>.
- K. Stephenson. “Circle Packing: A Mathematical Tale.” *Notices of the AMS*, vol. 50, no. 11, December 2003, pp. 1376-1388. <http://www.ams.org/notices/200311/fea-stephenson.pdf>.
- D. Sutton. *Islamic Design: A Genius for Geometry*. Bloomsbury, 2007.