Width, Ricci Curvature, and Bisecting Surfaces

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June 15, 2016

Outline

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Acknowledgements

- Alex Nabutovsky, Regina Rotman, and Robert Young
- Yevgeney Liokumovich and Zhifei Zhu
- Alfonso Gracia-Saz and Raymond Grinnell
- Almut Burchard and Kasra Rafi
- Stefan Bilaniuk, Marcus Pivato, David Poole, and Reem Yassawi.
- The Entire Tenth Floor of Huron
- Megan Shaw
- KC and Lesley Wynne
- Kathleen Schmidt-Hertzberg, Norman Taylor, Raja Rajagopal, and John Karsemeyer
- Sam Chapin, Derek Krickhan, and Nick Saika

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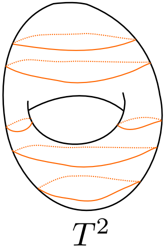
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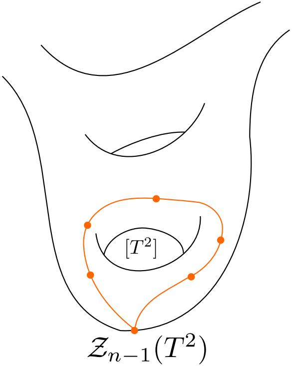
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(The Width-Volume Inequality doesn't hold for general manifolds.)

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- Desingularize the cycle to obtain a surface.

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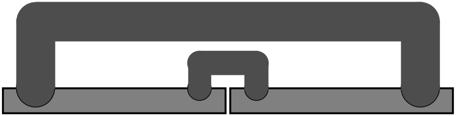

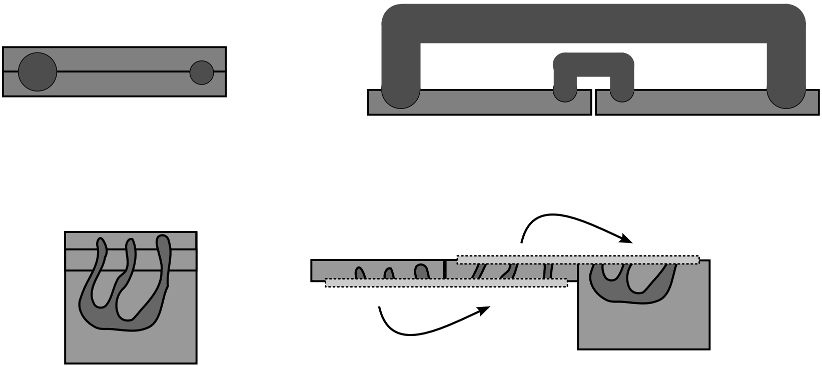

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