Parker Adey November 2, 2025

The Mathematics of Braiding

Name: ______________________________

Q1. Take three pieces of string and braid the "standard 3-strand braid".
If you get stuck, please ask Parker or the volunteers for help!

Instructions: Let's call the three strands Left, Center, and Right.

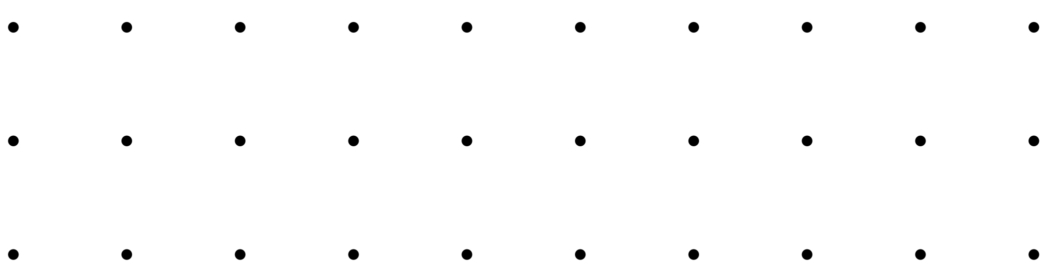
1. Pass the left strand over the center and leave it there.
2. Pass the right strand over the center and leave it there.
3. Repeat from Step 1.

Once you've completed your braid, complete the following:

I noticed...

I wondered...

Q2. Draw a picture of your 3-strand braid.

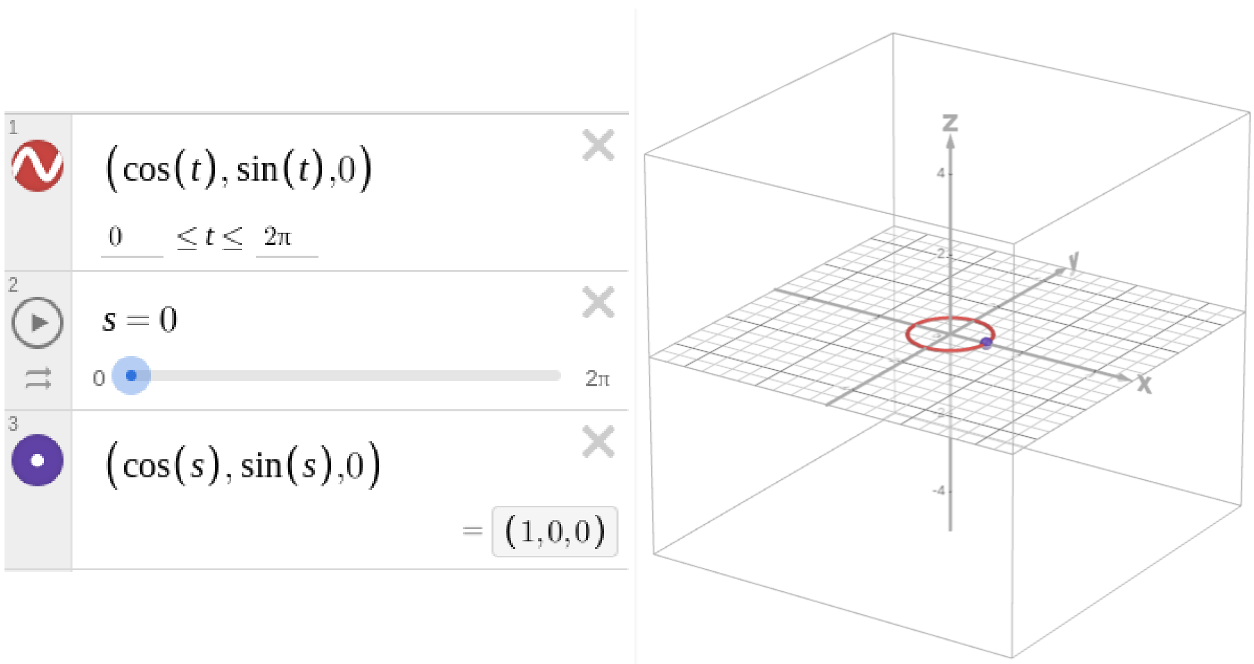


For the rest of the workshop, we'll use computers to simulate braids. To log in to your computer, enter the following username: and password:

Open a browser, navigate to `https://www.desmos.com/3d` and graph the unit circle in the xy-plane:

1. Create a new equation:
$$(\cos(t), \sin(t), 0)$$
Desmos will automatically create a parameter $0 \leq t \leq 1$.
2. Adjust the bounds to $0 \leq t \leq 2\,$`pi`.
3. Add a slider s by typing `ss` in a blank equation and clicking "Add slider".
4. Set the slider bounds to $0 \leq s \leq 2\,$`pi`.
5. Set the slider to "Repeat in One Direction".
6. In a new equation, create a sliding point $(\cos(s), \sin(s), 0)$.

If you click the play button, this will show a point gliding around the unit circle.

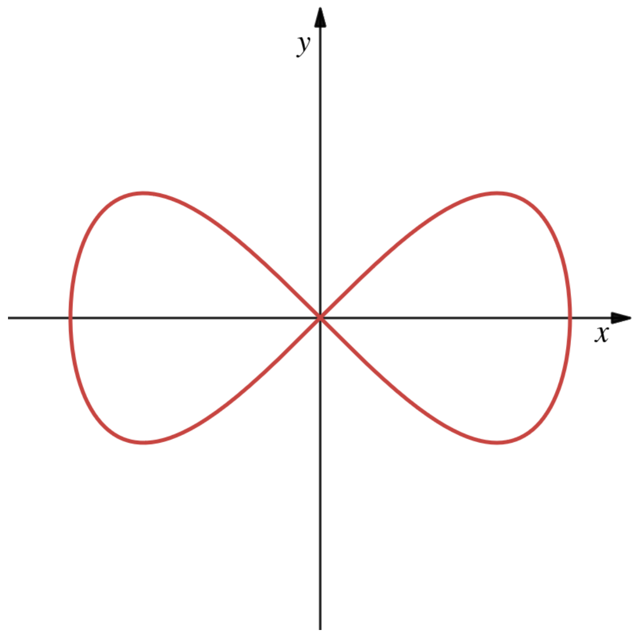


We add a bit more structure to create three equally spaced points spinning around the circle.

7. Create a list:
$$N = \left[0, \frac{2\ \texttt{pi}}{3}, \frac{4\ \texttt{pi}}{3}\right]$$
8. Modify your sliding point to $(\cos(s+N), \sin(s+N), 0)$.

At this point, you should have three points spinning around a circle.

The following approach to braiding is due to Noémi Speiser (1926-2025), a British-Swiss textile scholar working in Basel, Switzerland. She invented the theory of "track plans" which we now explore.



Q3. Our original equation $(\cos(t), \sin(t), 0)$ describes a unit circle.
Try to modify the y-coordinate so that you get the infinity shaped "track plan" shown above.
Your equation will be in the following format:

$$(\cos(t), \underline{\hspace{2cm}} \sin(t), 0).$$

Modify your Desmos equations $(\cos(t), \sin(t), 0)$ and $(\cos(s+N), \sin(s+N), 0)$ accordingly.

Q4. How can we modify our equations so that they *lift* off the xy-plane?

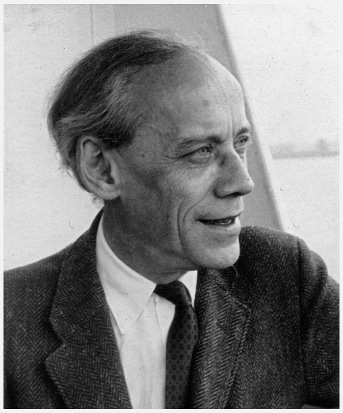
$$\text{The equation } (f(t), g(t), 0) \text{ becomes } (f(t), g(t), \underline{\hspace{3cm}}).$$

Modify your Desmos equations appropriately, and watch the braid spring to life.

Q5. How could we generalize this construction from three strands to five strands?

(a) The values of N divide up the circle in the 3 equal parts. How would you generalize this to 5 parts?
The equation `N = [0, 2pi/3, 4pi/3]` becomes `N = [ 0, _____, _____, _____, _____ ]`.

(b) *Spicy*: Can you get Desmos to do this with n parts?
Let n be a slider that goes from $n = 1$ to $n = 11$ with "Step: 2".



Mathematicians have a special language for describing braids. It was invented in the 1920s by Emil Artin (1898-1962), an Austrian-Armenian mathematician who worked at Princeton. In this language, a braid is written down crossing-by-crossing. The standard 3-strand braid becomes a "braid word" $(\sigma_1\sigma_2^{-1})^n$.

Q6. Use your computer model to sketch a five strand braid.

Once you've completed your braid, complete the following: (about braids, the workshop, anything)

I noticed...

I wondered...

For more information about braiding, see:

1. Speiser, N. (1983). *The manual of braiding.*
2. Artin, E. (1959). The theory of braids. *Mathematics Teacher*, 52(5), 328-333.

Feel free to ask me about mathematics / braiding / university: `parker.glynn.adey@utoronto.ca`
A quick e-mail: "Hey Parker! I was at your braiding workshop. I wanted to ask: ...." goes a long way!