Euler's Identity Exercises

Name: ______________________ Student Number: ________________

This worksheet does not have enough space for the full solutions, so you will need to write your solutions on paper or your device. Good luck! Enjoy!

Q1. Plot the following complex numbers.

1. $1 + i$
2. $-2 - 3i$
3. $\sqrt{2} - \sqrt{2}i$

Plot the series $z_n = (1 + i)^n$.

Q2. Calculate the following values.

1. $(1 + i)^4$
2. $\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)^2$

Q3. The **complex conjugate** of $a + bi$ is $\overline{a + bi} = a - bi$. We define the **magnitude** of z by $|z|^2 = z\overline{z}$. Alternatively, we call $|z|$ the **length** of z. Compute the length $|3 + 4i|$.

Q4. If z and w are complex numbers then $\overline{zw} = \overline{z}\,\overline{w}$. This implies $|zw| = |z||w|$.

Q5. Factor the polynomials $f(x) = x^2 + 1$ and $g(x) = x^3 - 1$ over the complex numbers.

Q6. Suppose that Euler's identity is true. Prove the following trigonometric identities.

1. $\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
2. $\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

Q7.

1. If $z \neq 1$ is a complex number and n is a whole number,
$$1 + z + z^2 + \cdots + z^n = \frac{1 - z^{n+1}}{1 - z}.$$
2. Suppose $|z| < 1$. As n tends to infinity, we get:
$$1 + z + z^2 + \cdots = \frac{1}{1 - z}.$$
3. Use this fact to tell a story about pouring two cups of tea.
$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = 2$$

Q8. If $f'(x) = 0$ then the function $f(x)$ is constant.

$$f(x) = e^{ix} \Longrightarrow f'(x) = ie^{ix} \qquad h'(x) = \frac{f(x)}{g(x)} \Longrightarrow h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Use these facts to prove Euler's identity.